2026 EXAMINATION

CBSE QUESTION & CONCEPT BANK

Chapter-wise & Topic-wise

CLASS 12

Chapter-wise
CONCEPT MAPS

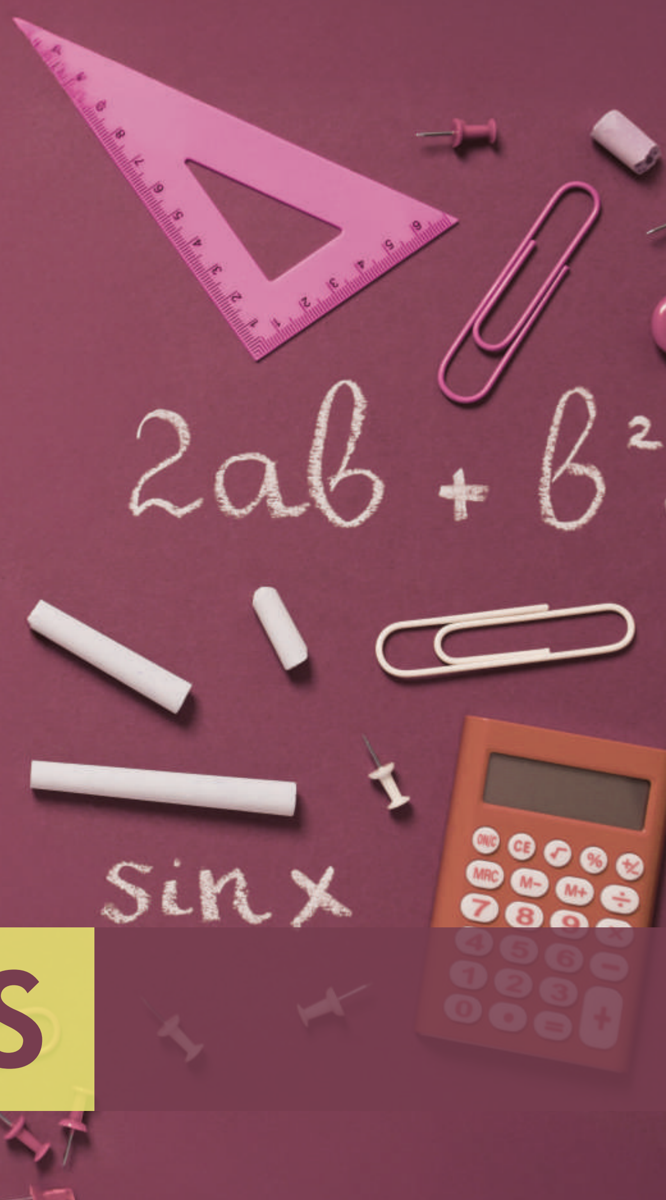
Important Questions with Detailed Explanations
NCERT & EXEMPLAR

Handpicked & High yield from Past 10 Years
PYQs

Revision Blue Print & Solved Questions
COMPETENCY FOCUSED

CBSE 2025 Past Year & SQP Solved Papers
LATEST CBSE PAPERS

As per Latest Pattern
MOCK TESTS



MATHEMATICS

Chapter-wise Weightage and Trend Analysis of
CBSE Past 6 Years' Papers

CHAPTERS	2020		2022		2023		2024		2025	
	DL	ODL	DL	ODL	DL	ODL	DL	ODL	DL	ODL
Relations and Functions	3	7	-	-	5	5	5	6	5	7
Inverse Trigonometric Functions	5	1	-	-	3	3	3	2	3	1
Matrices	3	3	-	-	9	7	9	9	2	5
Determinants	7	7	-	-	1	3	1	1	8	5
Continuity and Differentiability	8	7	-	-	8	4	10	6	9	6
Application of Derivatives	8	9	-	4	6	10	11	10	15	8
Integrals	8	8	8	9	11	11	4	9	5	11
Application of Integrals	6	6	4	-	5	5	6	5	4	6
Differential Equations	5	5	6	5	5	5	4	5	2	4
Vector Algebra	6	4	5	5	7	5	5	7	6	6
Three Dimensional Geometry	8	10	9	9	7	9	9	7	8	8
Linear Programming	5	5	-	-	5	5	5	5	5	5
Probability	8	8	8	8	8	8	8	8	8	8
Total Marks	**80**	**80**	**40**	**40**	**80**	**80**	**80**	**80**	**80**	**80**

**The marks allotment mentioned above is chapter-wise and includes internal choice questions as well. Therefore, the total might not match the Maximum Marks of the respective Previous Year Paper.*

***For the year 2021, the exam was not conducted*

Question Typology

YEAR	Objective Questions		Subjective Questions				
	MCQs	A/R	Fill in the Blanks	VSA	SA	LA	Case-Based type
2025	18	2	-	5	6	4	3
2024	18	2	-	5	6	4	3
2023	18	2	-	5	6	4	3
2022 (Term-I)	45	-	-	-	-	-	1
2022 (Term-II)	-	-	-	6	4	3	1
2020	10	-	5	5	6	6	-

Evolving Trends in CBSE Exam Patterns

2020

- ❖ No overall choice in the question paper. However, an internal choice has been provided in Section A,B,C and D.
- ❖ More competency-based questions were added compared to 2019.
- ❖ MCQ (1 Mark), Fill in the Blanks (1 Mark), VSA (1 Mark), SA-I (2 Marks), SA-II (4 Marks), LA (6 Marks) 2020 onwards,

2022

Term I

- ❖ All sections contained only objective questions, and each question carried equal marks (1 M).
- ❖ 40 questions needed to attempt out of 50 questions.
- ❖ An internal choice has been provided in Section A,B and C.
- ❖ Introduction of case based MCQ questions.

Term II

- ❖ Rationalised Syllabus Onwards.
- ❖ Question paper contains 14 questions.
- ❖ Term II is descriptive paper which contains VSA (2 marks), SA (3 marks), LA (4 marks), and 1 Case based questions with 2 subparts of 2 Marks each and Internal choice has given in section A, B and C.

2023

- ❖ Rationalised Syllabus Onwards.
- ❖ Assertion/Reason questions are introduced from this year onwards
- ❖ 3 case based questions with internal choices are added.
- ❖ The number of sections in the paper increased to 5 sections (A, B, C, D and E) compared to 2022.
- ❖ Competency based questions are increased in number.
- ❖ Internal choice has given in section B,C,D and E.
- ❖ MCQ (1 Mark), VSA (2 Marks), SA (3 Marks), LA (5.Marks), Case Based (4 Marks) 2023 onwards.

2024

- ❖ No changes in exam pattern.

2025

- ❖ No changes in exam pattern

HOW TO USE THIS BOOK

This book is structured to support your learning journey of preparing for your board exams through a variety of engaging and informative elements. Here's how to make the most of it:

CBSE Solved Paper of 2025 with detailed solutions: *Get yourself updated with the latest Board Question Papers. With provided explanations, learn the effective answering techniques to achieve higher scores.*

CBSE Solved Paper

CBSE SOLVED PAPER 2025

SECTION – A

(This section comprise of 20 multiple choice questions (MCQs) of 1 mark each) **(20×1=20)**

1. The projection vector of vector $\vec{a}$ on vector $\vec{b}$ is *(1 M)*

   (a) $\left(\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$ (b) $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$ (c) $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$ (d) $\left(\frac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right)\vec{b}$

Ans. (a) The projection of vector $\vec{a}$ onto vector $\vec{b}$ is calculated using the formula, $\text{Proj}_{\vec{b}}\vec{a} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$

The dot product $\vec{a}\cdot\vec{b}$ is defined as $|\vec{a}||\vec{b}|\cos\theta$, where θ is the angle between the vectors. The correct representation of the projection vector is $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$

Other options are incorrect because they either represent scalar projection or involve incorrect magnitudes. The correct result is, $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$

2. The function $f(x) = x^2 - 4x + 6$ is increasing in the interval *(1 M)*

   (a) (0, 2) (b) $(-\infty, 2]$ (c) [1, 2] (d) $[2, \infty)$

At the start of every chapter, you'll find a thoughtfully chosen image and a quote that captures the main idea and motivation of the topic. This approach aims to get your interest and give you a glimpse of the theme ahead.

Before diving into the details, we outline the syllabus and analyze the weightage given to each topic over the past five years. This helps you prioritize your study focus based on the significance of each section.

Preview

"Military and ballistics use inverse trigonometry to calculate launch angles for projectiles. In battles and shooting, like when cannons or missiles are fired, getting the right launch angle greatly matters. Figuring out this angle is where inverse trigonometry jumps in. It helps experts decide how to aim these projectiles accurately."

SYLLABUS & WEIGHTAGE

List of Concept Names	Years				
	2020	2022	2023	2024	2025
Domain, Range ,Principal Value and Properties of Inverse Trigonometric Functions (Definition, range, domain, principal, value branch, Graphs of inverse trigonometric functions)	1 Q (1 M) 1 Q (2 M)	–	1 Q (1 M) 1 Q (2 M)	1 Q (1 M) 1 Q (2 M)	1 Q (1 M)

The concept map appears to be a comprehensive study aid that outlines key concepts in a structured format. Use it to understand the chapter's structure and as a quick reference to recall important highlights.

A QR Code to access One Shot Revision Video of the chapter.

Concept Map

CONCEPT MAP

To Access Detailed Revision Notes Scan This QR Code

- **Null Vector or Zero Vector:** If the initial and terminal points of a vector coincide, then it is called a zero vector. It is denoted by $\vec{0}$ or O. Its magnitude is zero and direction indeterminate.
- **Unit Vector:** A vector whose magnitude is of unit length along any vector $\vec{a}$ is called a unit vector in the direction of $\vec{a}$ and is denoted by $\hat{a} \Rightarrow \hat{a} = \frac{\vec{a}}{|\vec{a}|}$
- **Equal Vector:** Two non-zero vectors are said to be equal vectors if their magnitude is equal and directions are the same.
- **Collinear Vector:** Two or more non-zero vectors are said to be collinear vectors if these are parallel to the same line.
- **Like and Unlike Vector:** Collinear vectors having the same direction are known as like vectors, while those having opposite directions are known as, unlike vectors.
- **Coplanar Vector:** Two or more non-zero vectors are said to be coplanar vectors if these are parallel to the same plane.
- **Localised Vector and Free Vector:** A vector drawn parallel to a given vector through a specified point as the initial point, is known as a localised vector. If the initial point of a vector is not specified, it is said to be a free vector.
- **Negative of a Vector:** Let $\overrightarrow{AB}$ be a vector directed from A to B. then $-\overrightarrow{AB}$ is a vector which would be directed from B to A.

VECTOR ALGEBRA

Types of Vector

Position Vector

Let O be the origin and A be a point such that $\overrightarrow{OA} = \vec{a}$, then we say that the position vector of A is $\vec{a}$.

Product of two Vectors

Vector or cross product:
If $\vec{a}$ and $\vec{b}$ are two vectors and θ is the angle between them, then $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$, where $\hat{n}$ is the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ such that $\vec{a}, \vec{b}$ and $\hat{n}$ forms a right handed screw system.

Scalar or Dot product:
If $\vec{a}$ and $\vec{b}$ are two non-zero vectors and θ be the angle between them, then their scalar product (or dot product) is $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$, where $0 \le \theta \le \pi$.

Projection of a vector on a line

If $\vec{a}$ and $\vec{b}$ are two vectors then
∴ Projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

Section Formula

If the position vectors of the points A and B be $\vec{a}$ and $\vec{b}$ then, the position vectors of the points dividing the line AB in the ratio $m:n$ internally and externally are $\frac{m\vec{b}+n\vec{a}}{m+n}$ and $\frac{m\vec{b}-n\vec{a}}{m-n}$, respectively.

Vector joining two points

Let $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ be the two points. Joining the points A and B with respect to the origin O.
$\therefore |\overrightarrow{AB}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Components of a Vector

Let a vector $\overrightarrow{OP} = \vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$, Here x, y and z are called scalar components and $x\hat{i}$, $y\hat{j}$ and $z\hat{k}$ are called vector components of $\vec{a}$ along the axes.

Properties of Vector Addition

If $\vec{a}$ and $\vec{b}$ are two vectors
- $\vec{a}+\vec{b} = \vec{b}+\vec{a}$ (commutative)
- $(\vec{a}+\vec{b})+\vec{c} = \vec{a}+(\vec{b}+\vec{c})$ (associative)
- $\vec{a}+\vec{0} = \vec{a} = \vec{0}+\vec{a}$
- $\vec{a}+(-\vec{a}) = \vec{0} = (-\vec{a})+\vec{a}$
- $|\vec{a}+\vec{b}| \le |\vec{a}|+|\vec{b}|$
- $|\vec{a}-\vec{b}| \ge ||\vec{a}|-|\vec{b}||$

Direction ratios and direction cosines of a vector.

If $\vec{a} = x\hat{i}+y\hat{j}+z\hat{k}$ then x, y, z are called direction ratio and l, m and n are called direction cosines where,

$$l = \cos\alpha = \frac{x}{\sqrt{x^2+y^2+z^2}},\quad m = \cos\beta = \frac{y}{\sqrt{x^2+y^2+z^2}},\quad n = \cos\gamma = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

Multiplication of a Vector by Scalar

If $\vec{a}$ and $\vec{b}$ are vectors & m, n are scalars, then
- $m(\vec{a}) = (\vec{a})m = m\vec{a}$
- $m(n\vec{a}) = n(m\vec{a}) = (mn)\vec{a}$
- $(m+n)\vec{a} = m\vec{a}+n\vec{a}$
- $m(\vec{a}+\vec{b}) = m\vec{a}+m\vec{b}$

1 DOMAIN, RANGE ,PRINCIPAL VALUE AND PROPERTIES OF INVERSE TRIGONOMETRIC FUNCTIONS

Important Terms

- **Inverse of a Function:** If $f: X \rightarrow Y$ is one-one and onto (bijective) function, then there exists a unique function $f^{-1}: Y \rightarrow X$ which assigns each element $y \in Y$ to a unique element $x \in X$ such that $f(x) = y$ and is called inverse function of f.
  i.e., $f^{-1}(y) = x \Rightarrow f(x) = y, x \in X$ and $y \in Y$
- **Inverse Trigonometric Functions:** Inverse trigonometric functions are defined as the inverse of their respective trigonometric functions. For example, arcsin(x) gives the angle whose sine is x.
  Inverse trigonometric functions are often denoted as arcsin(x), arccos(x), and arctan(x), but sometimes you might also see them written as $\sin^{-1}(x)$, $\cos^{-1}(x)$, and $\tan^{-1}(x)$.

Important Concepts

- **Principal Value Branches:** Since trigonometric functions being periodic are in general not bijective (one-one onto) and thus for existence of inverse of trigonometric function we restrict their domain and co-domain to make it bijective. This restriction of domain and range gives principal value branch of inverse trigonometric function which are as follows:

Function	Domain	Range (Principal value branch)
$y = \sin^{-1} x$	$[-1, 1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \text{cosec}^{-1} x$	$R - (-1, 1)$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1} x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$y = \tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1} x$	R	$(0, \pi)$

- **Principal Values:** The value of an inverse trigonometric function which lies in its principal value branch is called principal value of the inverse trigonometric functions.
  (*a*) If $\sin^{-1} x = \theta$, then θ is its principal value when $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.
  (*b*) If $\cos^{-1} x = \theta$, then θ is its principal value when $0 \leq \theta \leq \pi$.
  (*c*) If $\tan^{-1} x = \theta$, then θ is its principal value when $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.

Note:
(*i*) If no branch of an inverse trigonometric function is mentioned, we mean the principal value branch of that function.
(*ii*) $\sin^{-1} x \neq \frac{1}{\sin x}$ or $(\sin x)^{-1}$ and same holds true for other trigonometric functions also.
(*iii*) If $\sin^{-1} x = y$ then x and y are the elements of domain and range of principal value branch of $\sin^{-1}$ respectively.
i.e., $x \in [-1,1]$ and $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
Similar fact is also applicable for other inverse trigonometric functions.

Important Terms:
Important terms often serve as foundational concepts upon which more complex ideas are built. Introducing them early ensures students have a solid understanding before delving into more advanced topics.

Important Concepts:
Familiarizing with key concepts in advance helps prepare cognitive framework for processing and integrating new information. By highlighting important concepts upfront, students are better equipped to identify connections and relationships between various ideas presented in the chapter.

Important Formulas

- **Property I:**
  (*i*) $\sin^{-1} \frac{1}{x} = \text{cosec}^{-1} x, x \geq 1$ or $x \leq -1$
  (*ii*) $\cos^{-1} \frac{1}{x} = \sec^{-1} x, x \geq 1$ or $x \leq -1$
  (*iii*) $\tan^{-1} \frac{1}{x} = \cot^{-1} x, x > 0$
- **Property II:**
  (*i*) $\sin^{-1} (\sin \theta) = \theta$, for all $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
  (*ii*) $\cos^{-1} (\cos \theta) = \theta$, for all $\theta \in [0, \pi]$
  (*iii*) $\tan^{-1} (\tan \theta) = \theta$, for all $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
  (*iv*) $\text{cosec}^{-1} (\text{coses } \theta) = \theta$, for all $\theta \in \left[\frac{\pi}{2}, \frac{\pi}{2}\right], \neq 0$
  (*v*) $\sec^{-1} (\sec \theta) = \theta$, for all $\theta \in [0, \pi], \theta \neq \frac{\pi}{2}$
  (*vi*) $\cot^{-1} (\cot \theta) = \theta$, for all $\theta \in (0, \pi)$
- **Property III:**
  (*i*) $\sin (\sin^{-1} x) = x$, for all $x \in [-1, 1]$
  (*ii*) $\cos (\cos^{-1} x) = x$, for all $x \in [-1, 1]$
  (*iii*) $\tan (\tan^{-1} x) = x$, for all $x \in R$
  (*iv*) $\text{cosec} (\text{cosec}^{-1} x) = x$, for all $x \in (-\infty, -1] \cup [1, \infty)$
  (*v*) $\sec (\sec^{-1} x) = x$, for all $x \in (-\infty, -1] \cup [1, \infty)$
  (*vi*) $\cot (\cot^{-1} x) = x$, for all $x \in R$
- **Property IV:**
  (*i*) $\sin^{-1} (-x) = -\sin^{-1} x, x \in [-1,1]$
  (*ii*) $\cos^{-1} (-x) = \pi - \cos^{-1} x, x \in [-1, 1]$
  (*iii*) $\tan^{-1} (-x) = -\tan^{-1} x, x \in \mathrm{R}$
  (*iv*) $\text{cosec}^{-1} (-x) = -\text{cosec}^{-1} x, |x| \geq 1$
  (*v*) $\sec^{-1} (-x) = \pi - \sec^{-1} x, |x| \geq 1$
  (*vi*) $\cot^{-1} (-x) = \pi - \cot^{-1} x, x \in \mathrm{R}$
- **Property V:**
  (*i*) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$, for all $x \in [-1,1]$
  (*ii*) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$, for all $x \in R$
  (*iii*) $\sec^{-1} x + \text{cosec}^{-1} x = \frac{\pi}{2}$, for all $x \in (-\infty, -1] \cup [1, \infty)$
- **Property VI:**
  (*i*) $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}, xy < 1$
  (*ii*) $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}\ xy > -1$
  (*iii*) $2\tan^{-1} x = \sin^{-1} \frac{2x}{1+x^2}, |x| \leq 1$
  (*iv*) $2\tan^{-1} x = \cos^{-1} \frac{1-x^2}{1+x^2}, x \geq 0$
  (*v*) $2\tan^{-1} = \tan^{-1} \frac{2x}{1-x^2}, -1 < x < 1$

Important Formulas:
Introducing important formulas upfront brings clarity to the chapter's objectives, guiding students' focus towards essential mathematical principles that will be explored further.

Real Life Applications

- **Optics And Camera Angles:** In photography and optics, understanding angles of view and lens positioning involves inverse trigonometry for capturing precise images and adjusting focus.

- **Determining Heights Using Shadows:** In architecture, inverse trigonometry aids in finding heights. Suppose an architect assesses the height of a building.
- **Astronomy and Stargazing:** In astronomy, angles to celestial objects are determined using inverse trigonometry. If an observer knows the distance to a star and the angle subtended by it, they can calculate the star's actual position.

Different Problem Types

Type I: Determining the Principal Value of the Given Inverse Trigonometric Function

Find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$.

Solution:

Step I: Let $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = y$ then, $\sin y = \frac{1}{\sqrt{2}}$

Step II: We know that the principal value branch of $\sin^{-1}$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ and $\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

Step III: Now, $\sin^{-1} \frac{1}{\sqrt{2}} = \frac{\pi}{4}$, since $\frac{\pi}{4} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

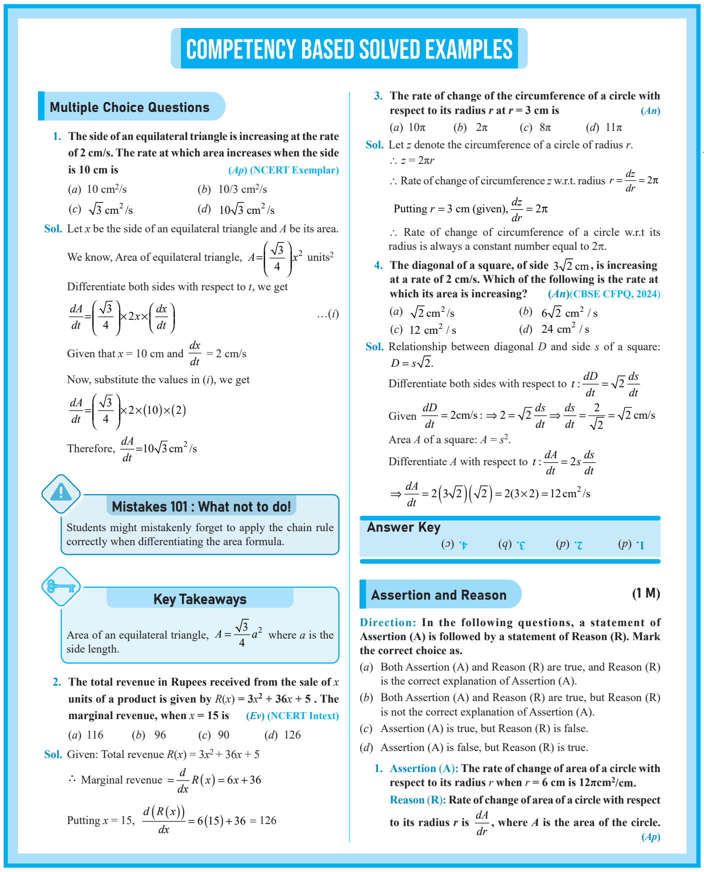
Therefore, Principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is $\frac{\pi}{4}$.

Real Life Applications:
Connecting abstract math to real scenarios deepens comprehension and aids in problem-solving.
Learning how math connects with other subjects shows that it's useful in many areas and helps us understand different topics better.

Different Problem Types:
Presenting different types of problems encourages critical thinking and creativity by challenging students to approach each problem uniquely, analyse it, develop strategies, and adapt their approaches to find solutions. Different problem types challenge students to analyse problems from diverse angles, fostering critical thinking skills essential for problem-solving.

Solved Examples

For each topic, solved examples are provided including tagging of Competencies, PYQs, CBSE SQPs, etc., that exemplify how to approach and solve questions. This section is designed to reinforce your learning and improve problem solving skills.


COMPETENCY BASED SOLVED EXAMPLES

Multiple Choice Questions

1. **The side of an equilateral triangle is increasing at the rate of 2 cm/s. The rate at which area increases when the side is 10 cm is** (Ap) (NCERT Exemplar)
(a) 10 cm^2/s (b) 10/3 cm^2/s
(c) $\sqrt{3}\ cm^2/s$ (d) $10\sqrt{3}\ cm^2/s$

Sol. Let x be the side of an equilateral triangle and A be its area.

We know, Area of equilateral triangle, $A=\left(\frac{\sqrt{3}}{4}\right)x^2$ units²

Differentiate both sides with respect to t, we get

$\frac{dA}{dt}=\left(\frac{\sqrt{3}}{4}\right)\times 2x\times\left(\frac{dx}{dt}\right)$ ...(i)

Given that x = 10 cm and $\frac{dx}{dt}$ = 2 cm/s

Now, substitute the values in (i), we get

$\frac{dA}{dt}=\left(\frac{\sqrt{3}}{4}\right)\times 2\times(10)\times(2)$

Therefore, $\frac{dA}{dt}=10\sqrt{3}\ cm^2/s$

Mistakes 101 : What not to do!
Students might mistakenly forget to apply the chain rule correctly when differentiating the area formula.

Key Takeaways
Area of an equilateral triangle, $A=\frac{\sqrt{3}}{4}a^2$ where a is the side length.

2. **The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue, when x = 15 is** (Ev) (NCERT Intext)
(a) 116 (b) 96 (c) 90 (d) 126

Sol. Given: Total revenue $R(x) = 3x^2 + 36x + 5$

$\therefore$ Marginal revenue $=\frac{d}{dx}R(x)=6x+36$

Putting x = 15, $\frac{d(R(x))}{dx}=6(15)+36=126$

3. **The rate of change of the circumference of a circle with respect to its radius r at r = 3 cm is** (An)
(a) 10π (b) 2π (c) 8π (d) 11π

Sol. Let z denote the circumference of a circle of radius r.
$\therefore z = 2\pi r$
$\therefore$ Rate of change of circumference z w.r.t. radius $r=\frac{dz}{dr}=2\pi$
Putting r = 3 cm (given), $\frac{dz}{dr}=2\pi$
$\therefore$ Rate of change of circumference of a circle w.r.t its radius is always a constant number equal to 2π.

4. **The diagonal of a square, of side $3\sqrt{2}$ cm, is increasing at a rate of 2 cm/s. Which of the following is the rate at which its area is increasing?** (An)(CBSE CFPQ, 2024)
(a) $\sqrt{2}\ cm^2/s$ (b) $6\sqrt{2}\ cm^2/s$
(c) 12 cm^2/s (d) 24 cm^2/s

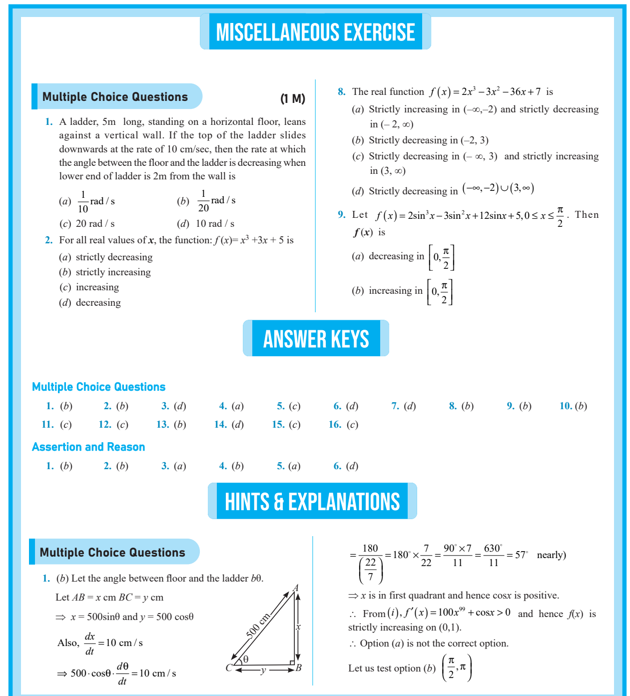
Sol. Relationship between diagonal D and side s of a square: $D=s\sqrt{2}$.
Differentiate both sides with respect to t: $\frac{dD}{dt}=\sqrt{2}\frac{ds}{dt}$
Given $\frac{dD}{dt}$ = 2cm/s : $\Rightarrow 2=\sqrt{2}\frac{ds}{dt}\Rightarrow\frac{ds}{dt}=\frac{2}{\sqrt{2}}=\sqrt{2}$ cm/s
Area A of a square: $A = s^2$.
Differentiate A with respect to t: $\frac{dA}{dt}=2s\frac{ds}{dt}$
$\Rightarrow\frac{dA}{dt}=2(3\sqrt{2})(\sqrt{2})=2(3\times 2)=12\ cm^2/s$

Answer Key
1. (d) 2. (d) 3. (b) 4. (c)

Assertion and Reason (1 M)

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.
(a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

1. **Assertion (A): The rate of change of area of a circle with respect to its radius r when r = 6 cm is $12\pi cm^2$/cm.**
Reason (R): Rate of change of area of a circle with respect to its radius r is $\frac{dA}{dr}$, where A is the area of the circle. (Ap)


MISCELLANEOUS EXERCISE

Multiple Choice Questions (1 M)

1. A ladder, 5m long, standing on a horizontal floor, leans against a vertical wall. If the top of the ladder slides downwards at the rate of 10 cm/sec, then the rate at which the angle between the floor and the ladder is decreasing when lower end of ladder is 2m from the wall is
(a) $\frac{1}{10}$ rad/s (b) $\frac{1}{20}$ rad/s
(c) 20 rad/s (d) 10 rad/s

2. For all real values of x, the function: $f(x) = x^3 + 3x + 5$ is
(a) strictly decreasing
(b) strictly increasing
(c) increasing
(d) decreasing

8. The real function $f(x)=2x^3-3x^2-36x+7$ is
(a) Strictly increasing in $(-\infty,-2)$ and strictly decreasing in $(-2,\infty)$
(b) Strictly decreasing in (–2, 3)
(c) Strictly decreasing in $(-\infty, 3)$ and strictly increasing in $(3,\infty)$
(d) Strictly decreasing in $(-\infty,-2)\cup(3,\infty)$

9. Let $f(x)=2\sin^3 x-3\sin^2 x+12\sin x+5, 0\le x\le\frac{\pi}{2}$. Then $f(x)$ is
(a) decreasing in $\left[0,\frac{\pi}{2}\right]$
(b) increasing in $\left[0,\frac{\pi}{2}\right]$

ANSWER KEYS

Multiple Choice Questions
1. (b) 2. (b) 3. (d) 4. (a) 5. (c) 6. (d) 7. (d) 8. (b) 9. (b) 10. (b)
11. (c) 12. (c) 13. (b) 14. (d) 15. (c) 16. (c)

Assertion and Reason
1. (b) 2. (b) 3. (a) 4. (b) 5. (a) 6. (d)

HINTS & EXPLANATIONS

Multiple Choice Questions

1. (b) Let the angle between floor and the ladder be θ.
Let $AB = x$ cm $BC = y$ cm
$\Rightarrow x = 500\sin\theta$ and y = 500 $\cos\theta$
Also, $\frac{dx}{dt}$ = 10 cm/s
$\Rightarrow 500\cdot\cos\theta\cdot\frac{d\theta}{dt}$ = 10 cm/s

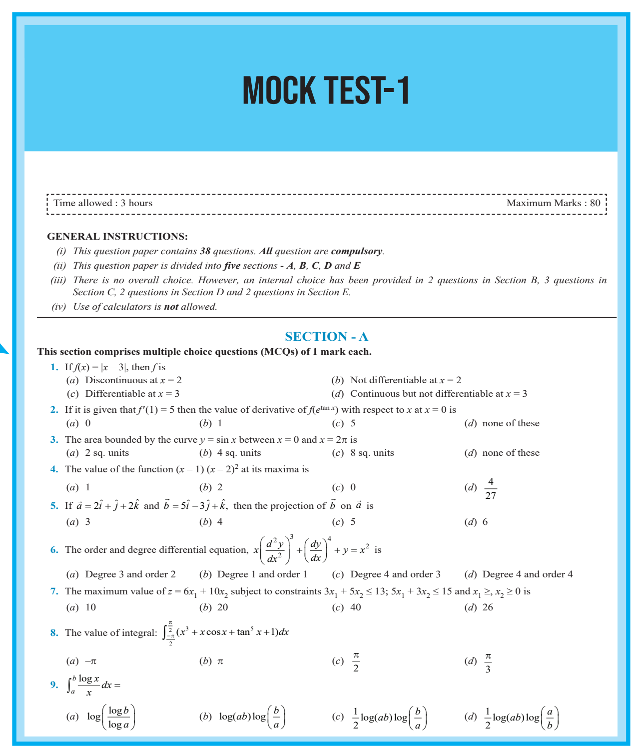
$=\frac{180}{\left(\frac{22}{7}\right)}=180^\circ\times\frac{7}{22}=\frac{90^\circ\times 7}{11}=\frac{630^\circ}{11}=57^\circ$ nearly)
$\Rightarrow x$ is in first quadrant and hence cos$x$ is positive.
$\therefore$ From (i), $f'(x)=100x^{99}+\cos x>0$ and hence $f(x)$ is strictly increasing on (0,1).
$\therefore$ Option (a) is not the correct option.
Let us test option (b) $\left(\frac{\pi}{2},\pi\right)$

At the end of each chapter, you'll find additional exercises intended to test your grasp of the material. These are great for revision and to prepare for exams.

Answer Key and Explanations including Topper's Explanations, Mistake 101, Nailing the right answer and Key takeaway to know how to write the ideal answer.

Answer Key

Mock Test


MOCK TEST-1

Time allowed : 3 hours Maximum Marks : 80

GENERAL INSTRUCTIONS:
(i) *This question paper contains **38** questions. **All** question are **compulsory**.*
(ii) *This question paper is divided into **five** sections - **A, B, C, D** and **E***
(iii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*
(iv) *Use of calculators is **not** allowed.*

SECTION - A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $f(x) = |x - 3|$, then f is
(a) Discontinuous at x = 2 (b) Not differentiable at x = 2
(c) Differentiable at x = 3 (d) Continuous but not differentiable at x = 3
2. If it is given that $f'(1) = 5$ then the value of derivative of $f(e^{\tan x})$ with respect to x at x = 0 is
(a) 0 (b) 1 (c) 5 (d) none of these
3. The area bounded by the curve $y = \sin x$ between x = 0 and $x = 2\pi$ is
(a) 2 sq. units (b) 4 sq. units (c) 8 sq. units (d) none of these
4. The value of the function $(x - 1)(x - 2)^2$ at its maxima is
(a) 1 (b) 2 (c) 0 (d) $\frac{4}{27}$
5. If $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=5\hat{i}-3\hat{j}+\hat{k}$, then the projection of $\vec{b}$ on $\vec{a}$ is
(a) 3 (b) 4 (c) 5 (d) 6
6. The order and degree differential equation, $x\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^4+y=x^2$ is
(a) Degree 3 and order 2 (b) Degree 1 and order 1 (c) Degree 4 and order 3 (d) Degree 4 and order 4
7. The maximum value of $z = 6x_1 + 10x_2$ subject to constraints $3x_1 + 5x_2 \le 13$; $5x_1 + 3x_2 \le 15$ and $x_1, x_2 \ge 0$ is
(a) 10 (b) 20 (c) 40 (d) 26
8. The value of integral: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(x^3+x\cos x+\tan^5 x+1)dx$
(a) $-\pi$ (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$
9. $\int_a^b\frac{\log x}{x}dx=$
(a) $\log\left(\frac{\log b}{\log a}\right)$ (b) $\log(ab)\log\left(\frac{b}{a}\right)$ (c) $\frac{1}{2}\log(ab)\log\left(\frac{b}{a}\right)$ (d) $\frac{1}{2}\log(ab)\log\left(\frac{a}{b}\right)$

Mock Test Papers: Test your preparedness with our Mock Test Papers designed to mirror the format and difficulty of real exams. Use the detailed explanations to identify areas of strength and opportunities for improvement.

Contents

Questions have been categorized according to the Bloom's Taxonomy (as per CBSE Board).
The following abbreviations have been used in the book:

(Un) - Understanding   *(Re)* - Remembering   *(Ap)* - Applying
(An) - Analysing   *(Cr)* - Creating   *(Ev)* - Evaluating

Upcoming CBSE SQPs/ APQs can be accessed through this QR

CBSE SOLVED PAPER 2025

Time allowed : 3 hours **Maximum Marks : 80**

GENERAL INSTRUCTIONS:

Read the following instructions very carefully and strictly follow them:

(i) *This Question paper contains **38** questions. **All** questions are compulsory.*

(ii) *Question paper is divided into five Sections-**Section A, B, C, D and E.***

(iii) *In **Section A Question number 1 to 18 are** Multiple Choice Questions **(MCQs) type** and question number **19 & 20** are Assertion-Reason based **questions of 1 mark each.***

(iv) *In **Section B - Question number 21 to 25 are** Very Short Answer (VSA) type **questions carrying 2 marks each.***

(v) *In **Section C - Question number 26 to 31 are** Short Answer (SA) type **questions carrying 3 marks each.***

(vi) *In **Section D - Question number 32 to 35 are** Long Answer (LA) type **questions carrying 5 marks each.***

(vii) *In **Section E - Question number 36 to 38 are** case Study based questions **carrying 4 marks each.***

(viii) ***There is no overall choice. However, an internal choice has been provided in 2 questions in Section - B, 3 questions in Section C, 2 questions in Section D and 3 questions in Section E.***

(ix) *Use of calculator is **NOT** allowed.*

SECTION – A

(This section comprise of 20 multiple choice questions (MCQs) of 1 mark each) **(20×1=20)**

1. The projection vector of vector $\vec{a}$ on vector $\vec{b}$ is *(1 M)*

(*a*) $\left(\dfrac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$ (*b*) $\dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$ (*c*) $\dfrac{\vec{a}.\vec{b}}{|\vec{a}|}$ (*d*) $\left(\dfrac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right)\vec{b}$

Ans. (*a*) The projection of vector $\vec{a}$ onto vector $\vec{b}$ is calculated using the formula, $\text{Proj}_{\vec{b}}\vec{a} = \left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$

The dot product $\vec{a}\cdot\vec{b}$ is defined as $|\vec{a}||\vec{b}|\cos\theta$, where θ is the angle between the vectors. The correct representation of the projection vector is $\left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$

Other options are incorrect because they either represent scalar projection or involve incorrect magnitudes. The correct result is, $\left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$

2. The function $f(x) = x^2 - 4x + 6$ is increasing in the interval *(1 M)*

(*a*) (0, 2) (*b*) $(-\infty\ 2]$ (*c*) [1, 2] (*d*) $[2, \infty)$

Ans. (*d*) To determine where the function $f(x) = x^2 - 4x + 6$ is increasing, we first find its derivative,

$f'(x) = 2x - 4$

Setting the derivative equal to zero to find critical points,

$2x - 4 = 0 \Rightarrow x = 2$

Next, we analyze the sign of $f'(x)$ around the critical point,

For $x < 2$ (e.g., $x = 0$),

$f'(0) = 2(0) - 4 = -4$ (Negative function is decreasing)

For $x > 2$ (e.g., $x = 3$),

$f'(3) = 2(3) - 4 = 2$ (positive, function is increasing)

Since $f'(x)$ changes from negative to positive at $x = 2$, the function is decreasing on $(-\infty, 2]$ and increasing on $[2, \infty)$.

The function is increasing in the interval $[2, \infty)$.

3. If $f(2a - x) = f(x)$, then $\int_0^{2a} f(x)\,dx$ is *(1 M)*

(*a*) $\int_0^{2a} f\left(\frac{x}{2}\right)dx$ (*b*) $\int_0^{a} f(x)\,dx$ (*c*) $2\int_a^{0} f(x)\,dx$ (*d*) $2\int_0^{a} f(x)\,dx$

Ans. (*d*) Given the symmetry property $f(2a - x) = f(x)$. Consider the integral:

$$I = \int_0^{2a} f(x)\,dx$$

Split the integral at $x = a$,

$$I = \int_0^{a} f(x)\,dx + \int_a^{2a} f(x)\,dx$$

Using the substitution $t = 2a - x$ in the second integral, $dt = -dx$.

Changing the limits, When $x = a$, $t = a$ & When $x = 2a$, $t = 0$.

Thus, $\int_a^{2a} f(x)\,dx = \int_a^{0} f(2a - t)(-dt) = \int_0^{a} f(t)\,dt$

Substituting back,

$$I = \int_0^{a} f(x)\,dx + \int_0^{a} f(x)\,dx = 2\int_0^{a} f(x)\,dx$$

Hence,The final result is $2\int_0^{a} f(x)\,dx$

4. If $A = \begin{bmatrix} 1 & 12 & 4y \\ 6x & 5 & 2x \\ 8x & 4 & 6 \end{bmatrix}$ is a symmetric matrix, then $(2x + y)$ is *(1 M)*

(*a*) -8 (*b*) 0 (*c*) 6 (*d*) 8

Ans. (*d*) Given,the symmetric matrix is,

$$A = \begin{bmatrix} 1 & 12 & 4y \\ 6x & 5 & 2x \\ 8x & 4 & 6 \end{bmatrix}$$

For a matrix to be symmetric, $A_{ij} = A_{ji}$. Equating corresponding elements,

$A_{12} = A_{21} \Rightarrow 12 = 6x \Rightarrow x = 2$

$A_{13} = A_{31} \Rightarrow 4y = 8x \Rightarrow 4y = 16 \Rightarrow y = 4$

Compute $2x + y$,

$2x + y = 2(2) + 4 = 8$

5. If $y = \sin^{-1}x$, $-1 \le x \le 0$, then the range of y is *(1 M)*

(*a*) $\left(\frac{-\pi}{2}, 0\right)$ (*b*) $\left[\frac{-\pi}{2}, 0\right]$ (*c*) $\left[\frac{-\pi}{2}, 0\right)$ (*d*) $\left(\frac{-\pi}{2}, 0\right]$

Ans. (*b*) The function $y = \sin^{-1}x$ has a domain of $[-1, 1]$ and a range of $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. For the restricted domain $-1 \le x \le 0$,

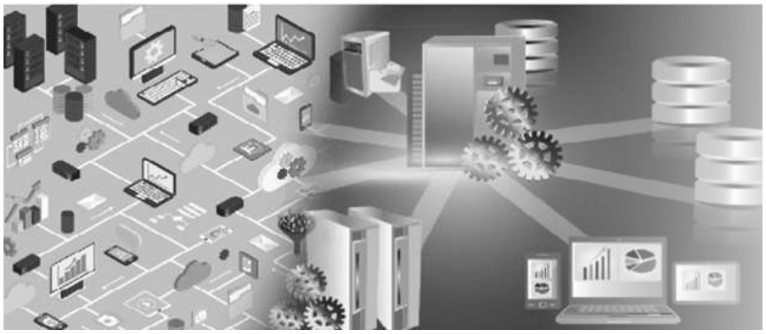
At $x = -1, y = \sin^{-1}(-1) = -\frac{\pi}{2}$.

At $x = 0$, $y = \sin^{-1}(0) = 0$.

Since $\sin^{-1}x$ is continuous and strictly increasing, the range for the given domain is $\left[-\frac{\pi}{2}, 0\right]$

RELATIONS AND FUNCTIONS

1



"In databases, relations are data tables, and functions are queries that fetch specific data, making sure each input (like an ID) gives one unique result (like a name)."

SYLLABUS & WEIGHTAGE

List of Concept Names	Years				
	2020	2022	2023	2024	2025
Relations (Type of relations: reflexive, symmetric, transitive and equivalence relations)	1 Q (2 M) 1 Q (4 M)	–	–	1 Q (4 M)	1 Q (3 M)
Functions (One to one and onto functions)	1 Q (1 M)	–	1 Q (5 M)	1 Q (1 M)	1 Q (4 M)

CONCEPT MAP

To Access Detailed Revision Notes Scan This QR Code

RELATIONS AND FUNCTIONS

Relations

Definition:

For any two non-empty sets A and B, every subset of $A \times B$ defines a relation from A to B and every relation from A to B is a subset of $A \times B$.

If $(a,b) \in R$, then $a\ R\ b$, $R \subseteq A \times B$

- If $(a, b) \in R$, then $a\ R\ b$ is read as 'a is related to b'.
- If $(a, b) \notin R$, then $a \not R\ b$ is read as 'a is not related to b'.

Type of Relations

Reflexive:

If $(a, a) \in R, \forall\ a \in A$.

i.e., $a\ R\ a, \forall\ a \in A$

Symmetric:

If $a\ R\ b \Rightarrow b\ R\ a, \forall\ a, b \in A$

i.e., if $(a, b) \in R \Rightarrow (b, a) \in R$, $\forall\ a, b \in A$

Transitive :

If $a\ R\ b$ and $b\ R\ c \Rightarrow a\ R\ c$, $\forall\ a, b, c \in A$ i.e. If $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$, $\forall\ a, b, c \in A$

Equivalence Relation:

A relation R on a non-empty set A is called an equivalence relation if and only if it is Reflexive, Symmetric as well as Transitive.

e.g.: Let T be the set of all triangles in a plane with R a relation in T given by $R=\{(T_1, T_2) : T_1$ is congruent to $T_2\}$. Show that R is an equivalence relation.

Sol: R is reflexive, since every triangle is congruent to itself.

Further, $(T_1, T_2) \in R \Rightarrow T_1$ is congruent to $T_2 \Rightarrow T_2$ is congruent to $T_1 \Rightarrow (T_2, T_1) \in R$. Hence, R is symmetric. Moreover, (T_1, T_2), $(T_2, T_3) \in R \Rightarrow T_1$ is congruent to T_2 and T_2 is congruent to $T_3 \Rightarrow T_1$ is congruent to $T_3 \Rightarrow (T_1, T_3) \in R$. Hence, R is transitive. Therefore, R is an equivalence relation.

Functions

Definition:

Let A and B are two non-empty sets. A function f from set A to set B is a rule which associated each element of A to a unique element of B, denoted by $f : A \rightarrow B$

- Set A is called domain of function 'f'
- Set B is called co-domain of function 'f'

If element x of A corresponds to $y(\in B)$ under the function f, then we say that y is the image of x and write $f(x) = y$.

Types of Functions

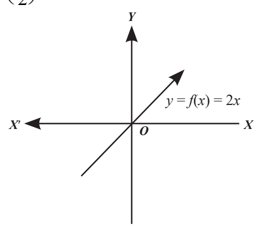
Bijective Function:

A function $f : A \rightarrow B$, is both one-one and onto function, known as bijective function.

e.g.: Prove that the function $f : R \rightarrow R$, given by $f(x) = 2x$, is one-one and onto.

Sol: Let $f(x_1) = f(x_2) \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$ therefore $f(x)$ is one-one Also, given any real number y in R, there exists $\frac{y}{2}$ in R such that

$f\left(\frac{y}{2}\right) = 2\left(\frac{y}{2}\right) = y$. Hence, f is onto.



One-one Function or Injective Function:

A function is said to be one-one function if different elements in a domain have different images in a co-domain.

If $f(x_1) = f(x_2)$ then $x_1 = x_2$, $f(x)$ is one - one function.

Many–one Function:

A function $f : A \rightarrow B$ is said to be many one if more than one element in set A have the same image in Set B.

Into Function:

A function $f : A \rightarrow B$ is said to be into function if there exists at least one element in set B having no pre-image in set A, is known as into function.

Onto or Surjective Function:

$f : A \rightarrow B$, said to be onto function if every element in set B has a pre-image in set A such that $f(x) = y$

i.e. Range of f = co-domain of f.

1 RELATIONS

Important Terms

- **Relation:** A relation R from set X to a set Y is defined as a subset of the cartesian product $X \times Y$. We can also write it as $R \subseteq \{(x, y) \in X \times Y : xRy\}$.
- **Empty Relation:** It is the relation R in X given by $R = \phi \subset X \times X$.
- **Universal Relation:** It is the relation R in X given by $R = X \times X$.
- **Reflexive Relation:** A relation R defined on a set A is said to be reflexive, if $(x, x) \in R$, $\forall\ x \in A$ or xRx, $\forall\ x \in R$.
- **Symmetric Relation:** A relation R defined on a set A is said to be symmetric, if $(x, y) \in R \Rightarrow (y, x) \in R$, $\forall\ x, y \in A$ or $xRy \Rightarrow yRx$, $\forall\ x, y \in R$.
- **Transitive Relation:** A relation R defined on a set A is said to be transitive, if $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$, $\forall\ x, y, z \in A$ or $xRy, yRz \Rightarrow xRz$, $\forall\ x, y, z \in R$.
- **Equivalence Relation:** A relation R defined on a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

Real Life Applications

Relations such as "is a sibling of," "is a parent of," or "is a cousin of" are common in families. Students can relate these relations to their own family trees, understanding how they are related to their siblings, parents, cousins, etc.

Brothers

Brothers

Brothers

Different Problem Types

Show that the relation R in the set Z of integers given by $R = \{(a, b) : 2$ divides $a - b\}$ is an equivalence relation.

Solution:

Step I: Prove Reflexivity

R is reflexive, as 2 divides $(a - a)$ for all $a \in Z$.

Step II: Prove Symmetry

Further, if $(a, b) \in R$, then 2 divides $a - b$. Therefore,

2 divides $b - a$. Hence, $(b, a) \in R$, which shows that R is symmetric.

Step III: Prove Transitivity

Similarly, if $(a, b) \in R$ and $(b, c) \in R$, then $a - b$ and $b - c$ are divisible by 2. Now, $a - c = (a - b) + (b - c)$ is even. So, $(a - c)$ is divisible by 2. This shows that R is transitive.

Thus, R is an equivalence relation in Z.

Example 1: Let A be the set of all students of a boys school. Show that the relation R in a given by $R = \{(a, b) : a$ is sister of $b\}$ is the empty relation and $R' = \{(a, b)$: the difference between heights of a and b is less than 3 meters $\}$ is the universal relation.

Solution: Since the school is boys school, no student of the school can be sister of any student of the school.

Hence, $R = \phi$, showing that R is the empty relation. It is also obvious that the difference between heights of any two students of the school has to be less than 3 meters.

This shows that $R' = A \times A$ is the universal relation.

Example 2: If R_1 and R_2 are equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation.

Solution: Since R_1 and R_2 are equivalence relations, $(a, a) \in R_1$, and $(a, a) \in R_2$ $\forall$ $a \in A$. This implies that $(a, a) \in R_1 \cap R_2$, $\forall$ a, showing $R_1 \cap R_2$ is reflexive.

Further, $(a, b) \in R_1 \cap R_2 \Rightarrow (a, b) \in R_1$ and $(a, b) \in R_2 \Rightarrow (b, a) \in R_1$ and $(b, a) \in R_2 \Rightarrow (b, a) \in R_1 \cap R_2$, hence, $R_1 \cap R_2$ is symmetric.

Similarly, $(a, b) \in R_1 \cap R_2$ and $(b, c) \in R_1 \cap R_2 \Rightarrow (a, c) \in R_1$ and $(a, c) \in R_2 \Rightarrow (a, c) \in R_1 \cap R_2$. This shows that $R_1 \cap R_2$ is transitive.

Thus, $R_1 \cap R_2$ is an equivalence relation.

COMPETENCY BASED SOLVED EXAMPLES

Multiple Choice Questions

(1 M)

1. **Let R be the relation in the set $\{1, 2, 3, 4\}$ given by $R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$. Choose the correct answer.** *(Ev)* **(NCERT Intext)**

   (*a*) R is reflexive and symmetric but not transitive.

   (*b*) R is reflexive and transitive but not symmetric.

   (*c*) R is symmetric and transitive but not reflexive.

   (*d*) R is an equivalence relation.

Sol. Here, $R = \{(1, 2), (2, 2), (1, 1), (4, 4)\ (1, 3), (3, 3), (3, 2)\}$

Since, $(a, a) \in R$., for every $a \in \{1, 2, 3, 4\}$. Therefore, R is reflexive.

Now since $(1, 2) \in R$ but $(2, 1) \notin R$. Therefore, R is not symmetric.

Also, it is observed that $(a, b), (b, c) \in R$.

$\Rightarrow (a, c) \in R$. For all $a, b, c \in \{1, 2, 3, 4\}$.

Therefore, R is transitive. Hence, R is reflexive and transitive but not symmetric.

2. **Let set $X = (1, 2, 3)$ and a relation R is defined in X as: $R = ((1, 3), (2, 2), (3, 2))$, then minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are** *(An)* **(CBSE, 2022 Term-I)**

   (*a*) $\{(1, 1), (2, 3), (1, 2)\}$

   (*b*) $\{(3, 3), (3, 1), (1, 2)\}$

   (*c*) $\{(1, 1), (3, 3), (3, 1), (2, 3)\}$

   (*d*) $\{(1, 1), (3, 3), (3, 1), (1, 2)\}$

Sol. (*i*) R is reflexive if it contains $\{(1, 1), (2, 2)$ and $(3, 3)\}$.

Since, $(2, 2) \in R$. So, we need to add $(1, 1)$ and $(3, 3)$ to make R reflexive.

(*ii*) R is symmetric if it contains $\{(2, 2), (1, 3), (3, 1), (3, 2), (2, 3)\}$.

Since, $\{(2, 2), (1, 3), (3, 2)\} \in R$. So, we need to add $(3, 1)$ and $(2, 3)$.

Thus, minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are $\{(1, 1), (3, 3), (3, 1), (2, 3)\}$.

Nailing the Right Answer

Students should check if the relation includes all the pairs necessary for reflexivity and symmetry.

3. **Let R be a relation on the set N of natural numbers defined by nRm if n divides m. Then R is** *(Un)* **(NCERT Exemplar)**

   (*a*) Reflexive and symmetric

   (*b*) Transitive and symmetric

   (*c*) Equivalence

   (*d*) Reflexive, transitive but not symmetric

Sol. 1. **Reflexive property:** Check if every natural number n divides itself.

   - Yes, because any number divided by itself gives a quotient of 1.

2. **Transitive property:** Check if n divides m and m divides p implies n divides p.

   - Yes, if n divides m and m divides p, then n divides p (by the transitive property of divisibility).

3. **Symmetric property:** Check if n divides m implies m divides n.

   - No, this is not necessarily true. For example, 2 divides 4 ($2R4$), but 4 does not divide 2 (not $4R2$).

Hence, R is reflexive,transitive but not symmetry.

4. **Consider the non-empty set consisting of children in a family and a relation R defined as aRb if a is brother of b. Then R is** *(Ap)* **(NCERT Exemplar)**

(*a*) symmetric but not transitive

(*b*) transitive but not symmetric

(*c*) neither symmetric nor transitive

(*d*) both symmetric and transitive

Sol. Since a can't be brother of a so $(a, a) \notin R$ for any a belonging to the family.

This relation is not symmetric as if $(a, b) \in R$ then $(b, a) \notin R$ as if a is brother of b this means b can be sister of a for any a, b belonging to the family.

But this relation is also transitive as if a is brother of b and b is brother of c then certainly a is brother of c i.e. $(a, b) \in R$, $(b, c) \in R \Rightarrow (a, c) \in R$ for all a, b, c belonging to the family.

So the relation is not symmetric but transitive.

5. **Let the relation R in the set $A = \{x \in Z : 0 \le x \le 12\}$, given by $R = \{(a, b): |a - b|$ is a multiple of $4\}$. Then [1], the equivalence class containing 1, is** *(Ev)*

(*a*) $\{1, 5, 9\}$

(*b*) $\{0, 1, 2, 5\}$

(*c*) ϕ

(*d*) A

Sol. Equivalence class [1] is the set of elements related to 1 $= \{1, 5, 9\}$.

6. **Let R be the relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$, then** *(An)*

(*a*) $(2, 4) \in R$

(*b*) $(3, 8) \in R$

(*c*) $(6, 8) \in R$

(*d*) $(8, 7) \in R$

Sol. $6 = 8 - 2$, $(6, 8)$ is an element of R.

7. **The maximum number of equivalence relations on the set $A = \{1, 2, 3\}$ are** *(Un)*

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 5

Sol. Given that, $A = \{1, 2, 3\}$

Now, number of equivalence relations are as follows:

$R_1 = \{(1, 1), (2, 2), (3, 3)\}$

$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$

$R_3 = \{(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$

$R_4 = \{(1, 1), (2, 2), (3, 3), (2, 3), (3, 2)\}$

$R_5 = \{(1, 2, 3) \Leftrightarrow A \times A = A^2\}$

$\therefore$ Maximum number of equivalence relations on the set $A = \{1, 2, 3\} = 5$

Answer Key

1. (*b*) 2. (*c*) 3. (*d*) 4. (*b*) 5. (*a*) 6. (*c*) 7. (*d*)

Assertion and Reason (1 M)

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

(*a*) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

(*b*) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

(*c*) Assertion (A) is true, but Reason (R) is false.

(*d*) Assertion (A) is false, but Reason (R) is true.

1. **Assertion (A): The relation $f : \{1, 2, 3, 4\} \to \{x, y, z, p\}$ defined by $f = \{(1, x), (2, y), (3, z)\}$ is a bijective function.**

Reason (R): The function $f : \{1, 2, 3\} \to \{x, y, z, p\}$ such that $f = \{(1, x), (2, y), (3, z)\}$ is one-one.

(An) **(CBSE SQP, 2023)**

Sol. Assertion is false. As element 4 has no image under f, so relation f is not a function.

Reason is true. The given function $f : \{1, 2, 3\} \to \{x, y, z, p\}$ is one – one, as for each $a \in \{1, 2, 3\}$, there is different image in $\{x, y, z, p)$ under f.

2. **Consider the set $A = \{1, 3, 5\}$**

Assertion (A): The number of reflexive relations on set A is 2^9.

Reason (R): A relation is said to be reflexive if xRx, $\forall x \in A$. *(Cr)*

Sol. By definition, a relation in A is said to be reflexive if xRx, $\forall x \in A$. So R is true.

The number of reflexive relations on a set containing n elements is $2^{n^2 - n}$.

Here $n = 3$

The number of reflexive relations on a set $A = 2^{9-3} = 2^6$.

Hence A is false.

Mistakes 101 : What not to do!

Students might incorrectly calculate reflexive relations by using the wrong formula, 2^{n^2-n} instead of 2^{n^2}.

Key Takeaways

For a set with n elements, the number of reflexive relations is 2^{n^2-n}.

3. **If $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and f = $\{(1, 4), (2, 5), (3, 6)\}$ is a function from A to B.**

Assertion (A): $f(x)$ is a one-one function.

Reason (R): $f(x)$ is an onto function. *(Cr)*

Sol. Given, $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f : A \to B$ is defined as $f = \{(1, 4), (2, 5), (3, 6)\}$ i.e., $f(1) = 4$, $f(2) = 5$ and $f(3) = 6$.

It can be seen that the images of distinct elements of A under f are distinct. So, f is one-one.

So, A is true.

Range of $f = \{4, 5, 6\}$.

Co-domain $= \{4, 5, 6, 7\}$.

Since co-domain $\neq$ range, $f(x)$ is not an onto function. Hence R is false.

Answer Key

1. (d) 2. (d) 3. (c)

Subjective Questions

Very Short Answer Type Questions (1 or 2 M)

1. **Show that the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ is reflexive but neither symmetric nor transitive.** *(An)* **(NCERT Intext)**

Sol. R is reflexive, since (1, 1), (2, 2) and (3, 3) lie in R. Also, R is not symmetric, as $(1, 2) \in R$ but $(2, 1) \in R$. *(1 M)*

Similarly, R is not transitive, as $(1, 2) \in R$ and $(2, 3) \in R$ but $(1, 3) \in R$. *(1 M)*

2. **Is $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ a function? If g is described by $g(x) = \alpha x + \beta$, then what value should be assigned to α and β?** *(Un)*

Sol. Given that, $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$

Here, each element of domain has unique image. So, g is a function.

Now given that,

$g(x) = \alpha x + \beta$

$g(1) = \alpha + \beta$

$\alpha + \beta = 1$ ...(*i*) *(½ M)*

$g(2) = 2\alpha + \beta$

$2\alpha + \beta = 3$ ...(*ii*) *(½ M)*

From Eqs. (*i*) and (*ii*).

$2(1 - \beta) + \beta = 3 \Rightarrow 2 - 2\beta + \beta = 3 \Rightarrow 2 - \beta = 3$

$\beta = -1$

if $\beta = -1$, then $\alpha = 2$

$\alpha = 2, \beta = -1$ *(1 M)*

Nailing the Right Answer

Students should ensere that each element in the domain has a unique image to confirm it's a function. They should use the given values to find α and β.

3. **Let $A = \{a, b, c\}$ and the relation R be defined on A as follows $R = \{(a, a), (b, c), (a, b)\}$**
Then, write minimum number of ordered pairs to be added in R to make R reflexive and transitive. *(Ev)*

Sol. Given relation, $R = \{(a, a), (b, c), (a, b)\}$.

To make R is reflexive we must add (b, b) and (c, c) to R. Also, to make R is transitive we add (a, c) to R. *(1 M)*

So, minimum number of ordered pair is to be added are $(b, b), (c, c), (a, c)$. *(1 M)*

4. **If the relation R be defined on the set $A = \{1, 2, 3, 4, 5\}$ by $R = \{(a, b) : |a^2 - b^2| < 8\}$. Then, R is given by ....** *(Ap)*

Sol. Given, $A = \{1, 2, 3, 4, 5\}$,

$R = \{(a, b) : |a^2 - b^2| < 8\}$ *(1 M)*

$R = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3), (4, 3), (3, 4), (4, 4), (5, 5)\}$ *(1 M)*

5. **Let the relation R be defined in N by aRb, if $2a + 3b = 30$. Then, R = ...** *(Ev)*

Sol. Given that, $2a + 3b = 30$

$\Rightarrow 3b = 30 - 2a \Rightarrow b = 30 - 2a/3$ *(1 M)*

For, $a = 3, b = 8$:

$a = 6, b = 6$:

$a = 9, b = 4$:

$a = 12, b = 2$:

$R = \{(3, 8), (6, 6), (9, 4), (12, 2)\}$ *(1 M)*

Short Answer Type Questions (2 or 3 M)

1. **Show that the relation R in the set Z of integers given by $R = \{(a, b) : 2 \text{ divides } a - b\}$ is an equivalence relation.** *(Un)* **(NCERT Intext)**

Sol. R is reflexive, as 2 divides $(a - a)$ for all $a \in Z$. *(½ M)*

Further, if $(a, b) \in R$, then 2 divides $a - b$. Therefore, 2 divides $b - a$. Hence, $(b, a) \in R$, which shows that R is symmetric. *(1 M)*

Similarly, if $(a, b) \in R$ and $(b, c) \in R$, then $a - b$ and $b - c$ are divisible by 2. Now, $a - c = (a - b) + (b - c)$ is even. So, $(a - c)$ is divisible by 2 . This shows that R is transitive. *(1 M)*

Thus, R is an equivalence relation in Z. *(½ M)*

2. **Check if the relation R in the set R of real numbers defined as $R = \{(a, b) : ab\}$ is (*i*) symmetric, (*ii*) transitive** *(Un)* **(CBSE,2020)**

Sol. (*i*) It is not symmetric because if $a < b$ then $b < a$ is not true. *(1 M)*

(*ii*) Here, if $a < b$ and $b < c$ then $a < c$ is also true for all a, b, c Real numbers. Therefore R is transitive. *(1 M)*

3. **Let n be a fixed positive integer. Define a relation R in Z as follows $\forall$ $a, b \in Z$, aRb if and only if $a - b$ is divisible by n. Show that R is an equivalence relation.** *(Cr)*

Sol. Given that, $\forall$ $a, b \in Z$, aRb if and only if $a - b$ is divisible by n.

Now,

Reflexive

$aRa \Rightarrow (a - a)$ is divisible by n, which is true for any integer a as '0' is divisible by n.

Hence, R is reflexive. *(1 M)*

Symmetric

aRb

$\Rightarrow a - b$ is divisible by n.

$\Rightarrow -b + a$ is divisible by n

$\Rightarrow -(b - a)$ is divisible by n.

$\Rightarrow (b - a)$ is divisible by n.

$\Rightarrow bRa$

Hence, R is symmetric *(1 M)*

Transitive

Let aRb and bRc

$\Rightarrow (a - b)$ is divisible by n and $(b - c)$ is divisible by n

$\Rightarrow (a - b) + (b - c)$ is divisible by n

$\Rightarrow (a - c)$ is divisible by n

$\Rightarrow aRc$

Hence, R is transitive.

So, R is an equivalence relation. *(1 M)*

4. **A relation R in set G = {All the countries in the world is defined as $R = \{(x, y): x$ and y share a common boundary$\}$. Determine whether R is reflexive, symmetric and transitive. Hence, conclude if R is an equivalence relation. Show your work.** *(Un) (CBSE CFPQ, 2024)*

Sol. Writes that every country shares its boundary with itself.

That is, $(x, x) \in R$, for each element $x \in G$.

Hence, R is reflexive. *(½ M)*

Writes that, whenever x shares a boundary with y, y also shares a boundary with x. That is, $(x, y) \in R \Rightarrow (y, x) \in R$. Hence, R is symmetric. *(1 M)*

Writes that, if x shares a boundary with y and y shares a boundary with z, then x need not share a boundary with z.

That is, $(x, y) \in R$, $(y, z) \in R$ need not imply $(x, z) \in R$. Hence, R is not transitive. *(1 M)*

From the above steps, concludes that R is not an equivalence relation. *(½ M)*

5. **Show that the relation R on R defined as $R = \{(a, b): a \leq b\}$, is reflexive, and transitive but not symmetric.** *(Un)*

Sol. Clearly $a \leq a \forall a \in R \Rightarrow (a, a) \in R \Rightarrow R$ is reflexive. *(1 M)*

For transitive:

Let $(a, b) \in R$ and $(b, c) \in R$, $a, b, c \in R$

$\Rightarrow a \leq b$ and $b \leq c \Rightarrow a \leq c \Rightarrow (a, c) \in R$

$\Rightarrow R$ is transitive. *(1 M)*

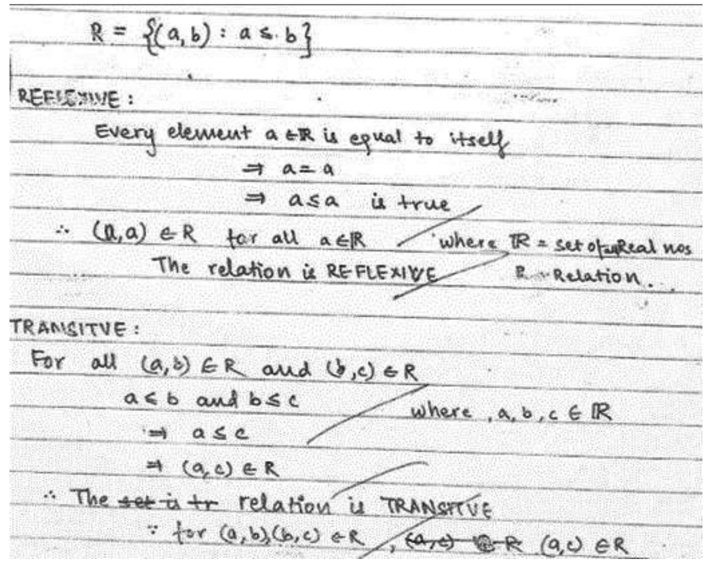
For non-symmetric:

Let $a = 1$, $b = 2$, As $1 \leq 2 \Rightarrow (1, 2) \in R$

but $2 \not\leq 1$, $\Rightarrow (2, 1) \notin$ R

$\Rightarrow R$ is non-symmetric. *(1 M)*

Topper's Explanation ***(CBSE 2019)***



R = {(a, b) : a ≤ b}

REFLEXIVE :

Every element a ∈ ℝ is equal to itself

⇒ a = a

⇒ a ≤ a is true

∴ (a, a) ∈ R for all a ∈ ℝ — where ℝ = set of Real nos, R → Relation.

The relation is REFLEXIVE

TRANSITIVE :

For all (a, b) ∈ R and (b, c) ∈ R

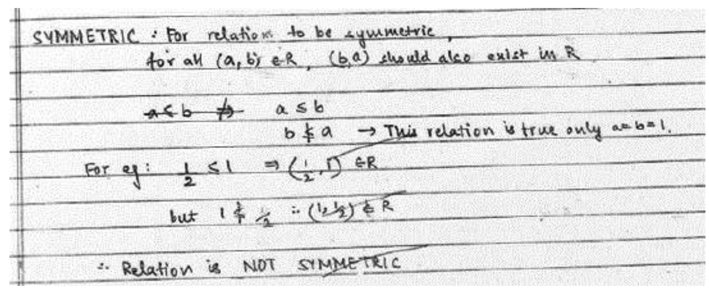
a ≤ b and b ≤ c — where, a, b, c ∈ ℝ

⇒ a ≤ c

⇒ (a, c) ∈ R

∴ The ~~set is tr~~ relation is TRANSITIVE

∵ for (a, b), (b, c) ∈ R, ~~(a, c)~~ ~~∈ R~~ (a, c) ∈ R



SYMMETRIC : For relation to be symmetric,

for all (a, b) ∈ R, (b, a) should also exist in R

~~a ≤ b ⇏~~ a ≤ b

b ≰ a → This relation is true only a = b = 1.

For eg: 1/2 ≤ 1 ⇒ (1/2, 1) ∈ R

but 1 ≰ 1/2 ∴ (1, 1/2) ∉ R

∴ Relation is NOT SYMMETRIC

Long Answer Type Questions (4 or 5 M)

1. **Let R be a relation defined on the set of natural numbers N as follows: $R = \{(x, y); x \in N, y \in N, 2x + y = 41\}$. Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive.** *(Un)*

Sol. Given that,

$R = \{(1, 39), (2, 37), (3, 35)\ldots(19, 3), (20, 1)\}$

Domain = $\{1, 2, 3, \ldots, 20\}$ *(1 M)*

Range = $\{1, 3, 5, 7 \ldots, 39\}$ *(1 M)*

R is not reflexive as $(2, 2) \notin R$ as $2 \times 2 + 2 \neq 41$.

R is not symmetric as $(1, 39) \in R$ but $(39, 1) \notin R$. *(1 M)*

R is not transitive as $(11, 19) \in R$, $(19, 3) \in R$

But $(11, 3) \notin R$. *(1 M)*

Hence, R is neither reflexive, nor symmetric and nor transitive. *(1 M)*

2. **Let $\mathbb{N}$ be the set of all natural numbers and R be a relation on $\mathbb{N} \times \mathbb{N}$ defined by $(a, b)\ R\ (c, d) \Leftrightarrow ad = bc$ for all $(a, b), (c, d) \in \mathbb{N} \times \mathbb{N}$. Show that R is an equivalence relation on $\mathbb{N} \times \mathbb{N}$. Also, find the equivalence class of $(2, 6)$, i.e., $[(2, 6)]$.** *(Cr) (CBSE SQP, 2023)*

Sol. Let (a, b) be an arbitrary element of $\mathbb{N}\times\mathbb{N}$. Then, $(a, b) \in \mathbb{N}\times\mathbb{N}$ and $a, b \in \mathbb{N}$.

We have, $ab = ba$; (As $a,b \in \mathbb{N}$ and multiplication is commutative on $\mathbb{N}$) *(1 M)*

$\Rightarrow$ $(a, b) R (a, b)$, according to the definition of the relation R on $\mathbb{N}\times\mathbb{N}$

Thus $(a, b) R (a, b)$, $\forall (a, b) \in \mathbb{N}\times\mathbb{N}$.

So, R is reflexive relation on $\mathbb{N}\times\mathbb{N}$. *(½ M)*

Let (a, b), (c, d) be arbitrary elements of $\mathbb{N}\times\mathbb{N}$ such that $(a, b) R (c, d)$.

Then, $(a, b) R (c, d) \Rightarrow ad = bc \Rightarrow bc = ad$; (changing LHS and RHS)

$\Rightarrow$ $cb = da$; (As $a, b, c, d \in \mathbb{N}$ and multiplication is commutative on $\mathbb{N}$)

$\Rightarrow$ $(c, d) R (a, b)$; according to the definition of the relation R on $\mathbb{N}\times\mathbb{N}$

Thus $(a, b) R (c, d) \Rightarrow (c, d) R (a, b)$

So, R is symmetric relation on $\mathbb{N}\times\mathbb{N}$. *(1 M)*

Let (a, b), (c, d), (e, f) be arbitrary elements of $\mathbb{N}\times\mathbb{N}$ such that $(a, b) R (c, d)$ and $(c, d) R (e, f)$.

Then $\left.\begin{array}{l}(a, b)R(c, d) \Rightarrow ad = bc \\ (c, d)R(e, f) \Rightarrow cf = de\end{array}\right\} \Rightarrow (ad)(cf) = (bc)(de)$

$\Rightarrow af = be$ *(1 M)*

$\Rightarrow$ $(a, b) R (e, f)$; (according to the definition of the relation R on $\mathbb{N}\times\mathbb{N}$)

Thus $(a, b) R (c, d)$ and $(c, d) R (e, f) \Rightarrow (a, b) R (e, f)$

So, R is transitive relation on $\mathbb{N}\times\mathbb{N}$.

As the relation R is reflexive, symmetric and transitive so, it is equivalence relation on $\mathbb{N}\times\mathbb{N}$. *(1 M)*

$[(2, 6)] = \{(x, y) \in \mathbb{N}\times\mathbb{N} : (x, y) R (2, 6)\}$

$= \{(x, y) \in \mathbb{N}\times\mathbb{N} : 3x = y\}$

$= \{(x, 3x): x \in \mathbb{N}\} = \{(1, 3), (2, 6), (3, 9), ...\}$ *(½ M)*

3. If Z is the set of all integers and R is the relation on Z defined as $R = \{(a, b): a, b \in Z$ and $a - b$ is divisible by 5$\}$. Prove that R is an equivalence relation. *(Ap)*

Sol. The given relation is $R = \{(a, b): a, b \in Z$ and $a - b$ is divisible by 5$\}$:

Reflexive:

As for any $x \in Z$, we have $x - x = 0$, which is divisible by 5.

$\Rightarrow$ $(x - x)$ is divisible by 5.

$\Rightarrow$ $(x, x) \in R$, $\forall x \in Z$ Therefore, R is reflexive. *(1 M)*

Symmetric:

Let $(x, y) \in R$, where $x, y, \in$ Z.

$\Rightarrow$ $(x - y)$ is divisible by 5. [by definition of R] *(½ M)*

$\Rightarrow$ $x - y = 5A$ for some $A \in Z$.

$\Rightarrow$ $x - y = 5(-A) \Rightarrow (y - x)$ is also divisible by 5.

$\Rightarrow$ $(y, x) \in R$.

Therefore, R is symmetric. *(1 M)*

Transitive:

Let $(x, y) \in R$, where $x, y, \in Z$.

$\Rightarrow$ $(x - y)$ is divisible by 5.

$\Rightarrow$ $x - y = 5A$ for some $A \in Z$ Again, let $(y, z) \in R$, where $y, z \in Z$.

$\Rightarrow$ $(y - z)$ is divisible by 5.

$\Rightarrow$ $y - z = 5B$ for some $B \in Z$. *(1 M)*

Now, $(x - y) + (y - z) = 5A + 5B$

$\Rightarrow$ $x - z = 5(A + B)$

$\Rightarrow$ $(x - z)$ is divisible by 5 for some $(A + B) \in Z$

$\Rightarrow$ $(x, z) \in R$

Therefore, R is transitive. *(1 M)*

Thus, R is reflexive, symmetric and transitive. Hence, it is an equivalence relation. *(½ M)*

Mistakes 101 : What not to do!

Students might forget to verify all three properties (reflexivity, symmetry, and transitivity) or misunderstand the condition for divisibility by 5.

Key Takeaways

For a relation to be an equivalence relation, it must be reflexive, symmetric and transitive. Check each property individually:

- Reflexivity: (a, a) is in R for all $a \in Z$.
- Symmetric: if (a, b) is in R, then (b, a) must also be in R.
- Transitivity: if (a, b) and (b, c) are in R, then (a, c) must be in R.

4. If A = {1, 2, 3, 4}, define relations on A which have properties of being

(*i*) reflexive, transitive but not symmetric.

(*ii*) symmetric but neither reflexive nor transitive.

(*iii*) reflexive, symmetric and transitive. *(Un)*

Sol. Given that, $A = \{1, 2, 3, 4\}$

(*i*) Let $R_1 = \{(1, 1), (1, 2), (2, 3), (2, 2), (1, 3), (3, 3)\}$

R_1 is reflexive, since, (1, 1) (2, 2) (3, 3) lie in R_1.

Now, $(1, 2) \in R_1$, $(2, 3) \in R_1 \Rightarrow (1, 3) \in R_1$. *(1 M)*

Hence, R_1 is also transitive but $(1, 2) \in R_1$

$\Rightarrow (2, 1) \notin R_1$.

So, it is not symmetric. *(1 M)*

(*ii*) Let $R_2 = \{(1, 2), (2, 1)\}$

Now, $(1, 2) \in R_2$, $(2, 1) \in R_2$

So, it is symmetric. *(1 M)*

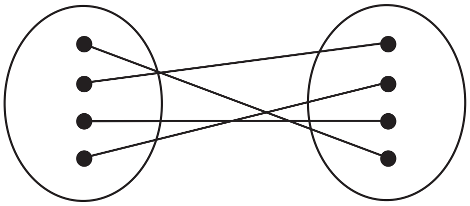
(*iii*) Let $R_3 = \{(1, 2), (2, 1), (1, 1), (2, 2), (3, 3), (1, 3), (3, 1), (2, 3)\}$ *(1 M)*

Hence, R_3 is reflexive, symmetric and transitive. *(1 M)*

2 FUNCTIONS

Important Terms

- **Functions:** A function f from a set A to a set B is a rule which associates each element of set A to a unique element of set B.



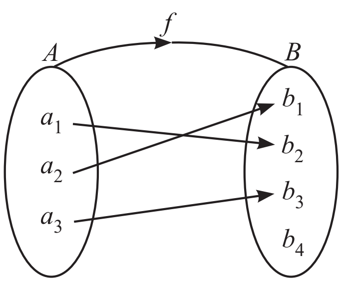
Set A is domain and set B is co-domain of the function

- **Range:** Range is the set of all possible resulting value given by the function. For example: x^2 is a function where values of x will be the domain and value given by x^3 is range.
- **Types of Functions:** As we know that if $f : A \rightarrow B$ is a function, then f associates all the elements of set A to the elements in set B such that an element of set A is associated to a unique element of set B. But there are some more possibilities, which may occur in a function, such as
  - (*i*) more than one elements of A may have same image in
  - (*ii*) each elements of B is image of some elements of A.
  - (*iii*) there may be some elements in B, which are not the images of any element of A.

Important Concepts

- **One-One (Injective) Function:** A function $f : A \rightarrow B$ is called a one-one or injective function, if distinct elements of A have distinct images in B, i.e. for every $a_1, a_2 \in A$, if $a_1 \neq a_2$, then $f(a_1) \neq f(a_2)$ and if $f(a_1) = f(a_2)$, then $a_1 = a_2$.

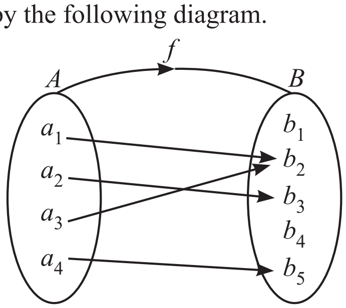
  e.g. Let $f : A \rightarrow B$ be a function represented by the following diagram.



Here, f is a one-one function, because each element have distinct image.

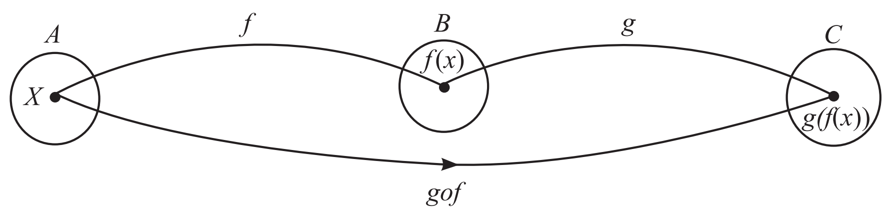
- **Many-One Function**: A function $f : A \rightarrow B$ is called a many-one function, if there exist atleast two distinct elements in A, whose images are same in B, i.e. if there exist $a_1, a_2 \in A$, such that $a_1 \neq a_2$ and $f(a_1) = f(a_2)$, then f is many-one. In other words, a function $f : A \rightarrow B$ is called a many-one function, if it is not one-one.

  e.g. Let $f : A \rightarrow B$ be a function represented by the following diagram.



Here, f is many-one function, because a_1 and a_3 have same image b_2.

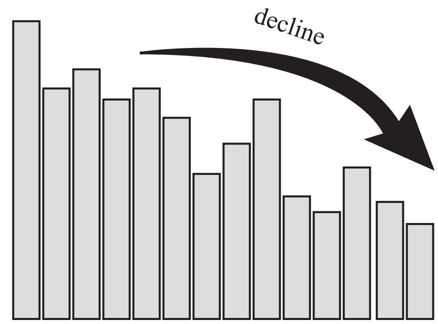
- **Onto Function:** A function *f* from set *A* to set *B* is called onto function if each element of set *B* has a preimage in set *A* or range of function *f* is equal to the codomain i.e., set *B*.
  Onto function is also called surjective function.
  For example: If a function *f* from a set of natural number to a set of *n* Natural number, then $f(x) = x - 1$ is onto function.
- **Bijective Function:** A function *f* from set *A* to set *B* is called bijective function if it is both one-one function and onto function.
  For example: If a function *f* from a set of real number to a set of real number, then $f(x) = 2x$ is one-one function and onto function.
- **Composition of Functions:** Let $f : A \to B$ and $g : B \to C$ be two functions. Then the composition of *f* and *g*, denoted by *gof*, is defined as the function $gof: A \to C$ given by $gof(x) = g(f(x)), \forall x \in A$.



- **Invertible Function:** A function $f : X \to Y$ is defined to be invertible, if there exists a function $g : Y \to X$ such that $gof = I_X$ and $fog = I_Y$. The function *g* is called the inverse of *f* and is denoted by f^{-1}. Thus, if *f* is invertible, then *f* must be one-one and onto and conversely, if *f* is one-one and onto, then f must be invertible.

Real Life Applications

Functions, have real-world applications in economics (profit functions), physics (distance-time graphs), temperature conversions, population growth modeling, financial calculations (loan amortization), signal processing (telecommunications), and statistical analysis (probability distributions, regression analysis).



Different Problem Types

To show that *f* is one-to-one, we'll prove that if $f(x_1) = f(x_2)$, then $x_1 = x_2$ for all x_1, x_2 in the domain *Z*.

Solution:

Step I: Assumption: Assume $f(x_1) = f(x_2)$.

Using the function $f(x) = x^2$: $x_1^2 = x_2^2$

Step II: Taking the square root of both sides (considering the absolute value since we're dealing with integers):

$|x_1| = |x_2|$

Step III: Implication: Since the absolute value of a number is always non-negative, this implies either $x_1 = x_2$ or $x_1 = -x_2$.

Step IV: Conclusion: Since $x_1 = -x_2$ contradicts the definition of the domain (integers), we conclude that $x_1 = x_2$.

Thus, for any two integers x_1 and x_2, if $f(x_1) = f(x_2)$, then $x_1 = x_2$. Therefore, *f* is one-to-one (injective).

Example 1: Show that $f : N \to N$, given by $f(x) = \begin{cases} x+1 & \text{if } x \text{ is odd} \\ x-1 & \text{if } x \text{ is even} \end{cases}$ **is both one-one and onto.**

Solution: Suppose $f(x_1) = f(x_2)$. Note that if x_1 is odd and x_2 is even, then we will have $x_1 + 1 = x_2 - 1$, i.e., $x_2 - x_1 = 2$ which is impossible. Similarly, the possibility of x_1 being even and x_2 being odd can also be ruled out, using the similar argument. Therefore, both x_1 and x_2 must be either odd or even.

Suppose both x_1 and x_2 are odd. Then $f(x_1) = f(x_2)$
$\Rightarrow x_1 + 1 = x_2 + 1 \Rightarrow x_1 = x_2$.

Similarly, if both x_1 and x_2 are even, then also $f(x_1) = f(x_2) \Rightarrow x_1 - 1 = x_2 - 1 \Rightarrow x_1 = x_2$. Thus, *f* is one-one.

Also, any odd number $2r + 1$ in the co-domain *N* is the image of $2r + 2$ in the domain *N* and any even number $2r$ in the co-domain *N* is the image of $2r - 1$ in the domain *N*. Thus, *f* is onto.

Example 2: Show that a one-one function $f : \{1, 2, 3\} \to \{1, 2, 3\}$ must be onto.

Solution: Since *f* is one-one, three elements of {1, 2, 3} must be taken to 3 different elements of the co-domain {1, 2, 3} under *f*. Hence, f has to be onto.

COMPETENCY BASED SOLVED EXAMPLES

Multiple Choice Questions (1 M)

1. Let $f : R \to R$ be defined by $f(x) = \frac{1}{x}$ $\forall x \in$ R. . Then f is

(a) one-one (b) onto

(c) bijective (d) f is not defined

(Un) (NCERT Exemplar)

Sol. We have, $f(x) = \frac{1}{x}, \forall x \in R$

For $x = 0$, $f(x)$ is not defined.

Hence, $f(x)$ is a not defined function.

2. Let $X = \{x^2 : x \in N\}$ and the function $f : N \to X$ is defined by $f(x) = x^2, x \in N$. Then this function is

(An) (CBSE 2022, Term-I)

(a) injective only (b) not bijective

(c) surjective only (d) bijective

Sol. Let $x_1, x_2 \in N$

$f(x_1) = f(x_2) \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1^2 - x_2^2 = 0$

$\Rightarrow (x_1 + x_2)(x_1 - x_2) = 0 \Rightarrow x_1 = x_2$ $\{x_1 + x_2 \neq 0$ as $x_1, x_2 \in N\}$

Hence, $f(x)$ is injective.

Also, the elements like 2 and 3 have no pre-image in N. Thus, $f(x)$ is not surjective.

3. Let $f : [2, \infty) \to R$ be the function defined by $f(x) = x^2 - 4x + 5$, then the range of f is (Ev) (NCERT Exemplar)

(a) R (b) $[1, \infty)$

(c) $[4, \infty)$ (d) $[5, \infty)$

Sol. Given that, $f(x) = x^2 - 4x + 5$

Let $y = x^2 - 4x + 5 \Rightarrow y = x^2 - 4x + 4 + 1 = (x - 2)^2 + 1$

$\Rightarrow (x - 2)^2 = y - 1 \quad \Rightarrow x - 2 = \sqrt{y-1} \Rightarrow x = 2 + \sqrt{y-1}$

$\therefore y - 1 \geq 0, y \geq 1$

Range $= [1, \infty]$

4. Let $f : R \to R$ be defined as $f(x) = x^4$. Choose the correct answer. (Ap) (NCERT Intext)

(a) f is one-one onto

(b) f is many-one onto

(c) f is one-one but not onto

(d) f is neither one-one nor onto.

Sol. A function $f : X \to Y$ is called an onto function if the range of f is Y.

In other words, if each $y \in Y$ there exists at least one $x \in X$ such that $f(X) = Y$,

then f is an onto function.

$f : R \to R$ defined as $f(x) = x^4$

$x, y \in R$ such that $f(x) = f(y)$

$\Rightarrow x^4 = y^4 \Rightarrow x = \pm y$

Therefore,

$f(x) = f(y)$ does not imply that $x = y$.

For example $f(1) = f(-1) = 1 \Rightarrow f$ is not one-one.

Consider an element 2 in codomain R there does not exist any x in domain R such that

$f(x) = 2$.

Therefore, f is not onto.

Function f is neither one-one nor onto.

Mistakes 101 : What not to do!

Students might forget to differentiate between one-one (injective) and onto (surjective) properties or overlook negative values in the range.

Key Takeaways

- One-One (Injective): Each element of the domain maps to a unique element in the codomain.
- Onto (Surjective): Every element of the codomain is mapped by some element of the domain.

5. Let $f : A \to B$ and $g : B \to C$ be the $B \to C$ be the bijective functions. Then $(g\ o\ f)^{-1}$ is (Ap) (NCERT Exemplar)

(a) f^{-1} o g^{-1} (b) $f\ o\ g$

(c) $g^{-1}\ o\ f^{-1}$ (d) $g\ o\ f$

Sol. Given that, $f : A \to B$ and $g : B \to C$ be the bijective functions.

$(f^{-1}\ o\ g^{-1})\ o\ (g\ o\ f) = f^{-1}\ o\ (g^{-1}\ o\ g\ o\ f) = f^{-1}\ o\ (g^{-1}\ o\ g)\ o\ f$

(As composition of functions is associative)

$= (f^{-1}\ o\ I_B\ o\ f)$ (where I_B is identity function on B)

$= (f^{-1}\ o.\ I_B)\ o\ f = f^{-1}\ o\ f = I_A$

Thus $(g\ o\ f)^{-1} = f^{-1}\ o\ g^{-1}$

6. Let $f : [3, \infty)$ R be the function defined by $f(x) = x^2 - 6x + 10$, then the range of f is (Ap) (NCERT Exemplar)

(a) R (b) $[1, \infty)$

(c) $[4, \infty)$ (d) $[5, \infty)$

Sol. Given that, $f(x) = x^2 - 6x + 10$

Let $y = x^2 - 6x + 10$

$y = x^2 - 6x + 9 + 1 = (x - 3)^2 + 1$

$(x - 3)^2 = y - 1 \Rightarrow x - 3 = \sqrt{y-1} \Rightarrow x = 3 + \sqrt{y-1}$

$\therefore y - 1 \geq 0, y \geq 1$

Range $= [1, \infty)$

7. **Let $f : R \to R$ be defined as $f(x) = x^3$. Choose the correct answer** *(Ev)*

(*a*) f is one-one and onto

(*b*) f is many one onto

(*c*) f is one-one but not onto

(*d*) f is neither one-one nor onto.

Sol. $f(a) = f(b) \Rightarrow a^3 = b^3 \Rightarrow a = b$ (Injective or one-one)

$f(x) = y \Rightarrow x^3 = y \Rightarrow x = (y)^{1/3}$

For any $y \in R$, $(y)^{1/3}$ is real.

Hence x^3 is onto.

8. **The domain of the function $f(x) = \log_{3+x}(x^2 - 1)$ is** *(Cr)*

(*a*) $(-3, -1) \cup (1, \infty)$

(*b*) $[-3, -1) \cup [1, \infty)$

(*c*) $(-3, -2) \cup (-2, -1) \cup (1, \infty)$

(*d*) $[-3, -2) \cup (-2, -1) \cup [1, \infty)$

Sol. Given function is $f(x) = \log_{3+x}(x^2 - 1)$

It is obvious that $f(x)$ is defined when $x^2 - 1 > 0$, $3 + x > 0$ and $3 + x \neq 1$.

Now, $x^2 - 1 > 0$ $x^2 > 1$

$x < -1$ or $x > 1$

$3 + x > 0 \Rightarrow x > -3$

$3 + x \neq 1 \Rightarrow x \neq -2$

Therefore, domain of the function $f(x) = (-3, -2) \cup (-2, -1) \cup (1, \infty)$

Answer Key

1. (*d*)	2. (*a*)	3. (*b*)	4. (*d*)	5. (*a*)
6. (*b*)	7. (*a*)	8. (*c*)		

Assertion and Reason **(1 M)**

Direction: The following questions consist of two statements – Assertion (A) and Reason (R). Answer these questions by selecting the appropriate option given below:

(*a*) Both A and R are true, and R is the correct explanation of A.

(*b*) Both A and R are true, but R is not the correct explanation of A.

(*c*) A is true, but R is false

(*d*) A is false, but R is true.

1. **Assertion (A): Let $f(x) = x^2$, $g(x) = \cos x$. Then $fog \neq gof$**

Reason (R): $(fog)(x) = f(x)g(x)$ *(An)*

Sol. We have, $f(x) = x^2$, $g(x) = \cos x$

$fog(x) = f(g(x)) = f(\cos x) = (\cos x)^2$

$gof(x) = g(f(x)) = g(x^2) = \cos x^2$

$fog \neq gof$

Hence, Assertion is true.

Reason (R): Given, $fog(x) = f(x)\, g(x)$

Reason is false because we know $fog(x) = f(g(x))$

2. **Consider the function $f : R \to R$ defined as $f(x) = \dfrac{x}{1 + x^2}$**

Assertion (A): $f(x)$ is not one-one.

Reason (R): $f(x)$ is not onto *(Ap)*

Sol. Given, $f: R \to R$;

$$f(x) = \frac{x}{1 + x^2}$$

Taking $x_1 = 4, x_2 = \frac{1}{4} \in R$

$$f(x_1) = f(4) = \frac{4}{17}$$

$$f(x_2) = f\left(\frac{1}{4}\right) = \frac{4}{17} \quad (x_1 \neq x_2) \qquad (x_1 \neq x_2)$$

$\therefore f$ is not one-one.

A is true.

Let $y \in R$ (co-domain)

$f(x) = y$

$$\Rightarrow \frac{x}{1 + x^2} = y \Rightarrow y.(1 + x^2) = x \Rightarrow yx^2 + y - x = 0$$

$$\Rightarrow x = \frac{1 \pm \sqrt{1 - 4y^2}}{2y}$$

since, $x \in R$

$\therefore 1 - 4y^2 \geq 0$

$$\Rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2}$$

So Range $(f) \neq R$ (Co-domain)

$\therefore f$ is not onto.

R is true.

R is not the correct explanation for A.

3. **Assertion (A): The function $f(x) = x^2$ is one-to-one (injective).**

Reason (R): If $f(a) = f(b)$ for $f(x) = x^2$, then $a = b$ or $a = -b$ *(Ev)*

Sol. The function $f(x) = x^2$ is not injective because different inputs can yield the same output (e.g., $f(2) = 4$ and $f(-2) = 4$).

The reason correctly identifies that $f(a) = f(b)$ implies $a = b$ or $a = -b$.

Answer Key

1. (*c*)	2. (*b*)	3. (*d*)

Subjective Questions

Very Short Answer Type Questions **(1 or 2 M)**

1. **Let $A \in [-1, 1]$. Then discuss whether $h(x) = x|x|$ functions defined on A are one-one, onto or bijective.** *(An)*

Sol. $h(x) = x|x|$

Let $h(x_1) = h(x_2)$

$\Rightarrow h(x_1) = h(x_2) \quad \Rightarrow x_1|x_1| = x_2|x_2|$

If $x_1, x_2 > 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2) = 0$ *(½ M)*

$\Rightarrow x_1 = x_2$ (as $x_1 + x_2 \neq 0$)

Similarly for $x_1, x_2 < 0$ we get $x_1 = x_2$

Clearly for x_1 and x_2 of opposite sign,

$x_1 \neq x_2$

Thus, $f(x)$ is one-one. *(½ M)*

For $x \in [0, 1], f(x) = x^2 \in [0, 1]$

For $x < 0, f(x) = -x^2 \in [-1, 0]$

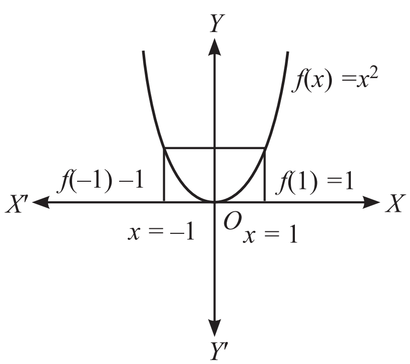
Thus range is $[-1, 1]$

So, $(x(x))$ is onto.

Hence $h(x)$ is bijective. *(1 M)*

2. Show that the function $f : R \to R$, defined as $f(x) = x^2$, is neither one-one nor onto. ***(Un)* (NCERT Intext)**

Sol. Since $f(-1) = 1 = f(1)$, f is not one- one. Also, the element -2 in the co-domain R is not image of any element x in the domain R. Therefore f is not onto. *(1 M)*



(½ M)

The image of 1 and –1 under f is 1. *(½ M)*

3. Check whether the function $y = \dfrac{x}{1+|x|}$, $x \in R, y \in R$ is one-one and onto. ***(Un)***

Sol. Let $f(x) = y = \dfrac{x}{1+|x|} \forall x \in R,\ y \in R$

$\therefore f(x) = \dfrac{x}{1+x}$ or $\dfrac{x}{1-x}$ is one-one. *(½ M)*

Here range of $f(x)$ is $R - \{-1, 1\}$. *(½ M)*

But y can not have any of the values $-1, 1$ for some x.

$\therefore f(x)$ is not an onto function. *(1 M)*

4. Let the function $f : R \to R$ be defined by $f(x) = \cos x, \forall x \in R$. Show that f is neither one-one nor onto. ***(An)***

Sol. Given function, $f(x) = \cos x, \forall x \in R$

Now, $f\left(\dfrac{\pi}{2}\right) = \cos\dfrac{\pi}{2} = 0$

$\Rightarrow f\left(\dfrac{-\pi}{2}\right) = \cos\dfrac{\pi}{2} = 0 \Rightarrow f\left(\dfrac{\pi}{2}\right) = f\left(\dfrac{-\pi}{2}\right)$

But $\dfrac{\pi}{2} \neq \dfrac{-\pi}{2}$ *(1 M)*

So, $f(x)$ is not one-one.

Now, $f(x) = \cos x$, $\forall x \in R$ is not onto as there is no pre-image for any real number. Which does not belonging to the intervals $[-1, 1]$, the range of $\cos x$. *(1 M)*

5. Let $f : A \to B$ and $g : B \to C$ be two functions.

(*i*) If f is not one-one, can *gof* be one-one?

(*ii*) If f is not onto, can *gof* be onto?

Justify your answer. ***(Un)* (CBSE CFPQ, 2024)**

Sol. (*i*) Writes that *gof* cannot be one-one, since *gof* is one-one implies f is one-one. *(1 M)*

(*ii*) Writes that *gof* can be onto, since *gof* is onto implies g is onto and three is no restriction on f to be one-one or onto. *(1 M)*

Nailing the Right Answer

Students should remember that the composition of functions depends on the properties of both f and g.

Key Takeaways

For *gof* to be one-one, both f and g must be one-one; for onto, only g needs to be onto.

Short Answer Type Questions (2 or 3 M)

1. Show that an onto function $f : \{1, 2, 3\} \to \{1, 2, 3\}$ is always one-one. ***(Ap)* (NCERT Intext)**

Sol. Suppose f is not one-one. Then there exists two elements, say 1 and 2 in the domain whose image in the co-domain is same. Also, the image of 3 under f can be only one element. *(1½ M)*

Therefore, the range set can have at the most two elements of the co-domain $\{1, 2, 3\}$, showing that is not onto, a contradiction. Hence, f must be one-one. *(1½ M)*

2. Graph of a certain function $f : R \to R$ is a straight line parallel to x-axis, where R is the set of real numbers.

(*i*) Is the function one-one?

(*ii*) Is the function onto?

Justify your answer. ***(Ap)* (NCERT Intext)**

Sol. (*i*) Writes that the function must be of the form $f(x) = k$, where 'k' is a real number. *(1 M)*

Writes that f is not one-one.

Justifies by giving an example as follows:

$f(1) = k = f(2)$, but $1 \neq 2$. *(1 M)*

(*ii*) Writes that f is not onto.

Justifies by giving an example as follows:

Consider $\beta(\neq k) \in R$ (co-domain) there is no element $x \in R$ (domain) such that $f(x) = \beta$. *(1 M)*

Nailing the Right Answer

Students should remember that a horizontal line function is neither one-one nor onto. They should check specific values to prove both properties.

3. *f* and *g* are real functions such that *f* is bijective and $g(x) = 3x + 4$, for all $x \in R$.
Is (*gof*) invertible? justify your answer.
***(Un)* (CBSE CFPQ, 2024)**

Sol. Writes that $g(x)$ is one-one as:

$g(x_1) = g(x_2)$

$3x_1 + 4 = 3x_2 + 4 \Rightarrow 3x_1 = 3x_2 \Rightarrow x_1 = x_2$. *(1 M)*

Writes that for any real value of y in R, there exists y–4/3 in R such that:

$g(y - 4/3) = 3 \times y - 4 + 4 = y - 4 + 4 = 2$

Thus, $g(x)$ is onto.

Concludes that $g(x)$ is bijective therefore invertible. *(1 M)*

Writes that $f(x)$ is bijective and therefore invertible.

Uses the theorem that if *f* and g are invertible, then (*gof*) is also invertible it conclude that (*gof*) is invertible. *(1 M)*

Long Answer Type Questions (4 or 5 M)

1. Show that the function $f: R - \{3\} \to R - \{1\}$, given by $f(x) = \frac{x-2}{x-3}$ is a bijection. ***(Cr)***

Sol. Given, $A = R - \{3\}$, $B = R - \{1\}$

$f: A \to B$ is defined as $f(x) = \left(\frac{x-2}{x-3}\right)$.

Let $x, y \in A$ such that $f(x) = f(y)$.

$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$ *(1 M)*

$\Rightarrow (x-2)(y-3) = (y-2)(x-3)$

$\Rightarrow xy - 3x - 2y + 6 = xy - 3y - 2x + 6$

$\Rightarrow -3x - 2y = -3y - 2x \Rightarrow 3x - 2x = 3y - 2y$

$\Rightarrow x = y$

Therefore, *f* is one-one. *(1 M)*

Let $y \in B = R - \{1\}$

Then, $y \neq 1$.

The function *f* is onto if there exists $x \in A$ such that $f(x) = y$.

Now, $f(x) = y$

$\Rightarrow \frac{x-2}{x-3} = y \Rightarrow x - 2 = xy - 3y$

$\Rightarrow x(1 - y) = -3y + 2$

$\Rightarrow x = \frac{2-3y}{1-y} \in A \quad (y \neq 1)$ *(1 M)*

Thus, for any $y \in B$, there exists $\frac{2-3y}{1-y} \in A$ such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right) - 2}{\left(\frac{2-3y}{1-y}\right) - 3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y$$ *(1 M)*

$\therefore$ *f* is onto.

Hence, function *f* is one-one and onto. *(1 M)*

Mistakes 101 : What not to do!

Students might forget to check one-one and onto properties or make mistakes with domain and range exclusions.

2. Show that the function $f: \mathbb{R} \to \{x \in \mathbb{R} : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in \mathbb{R}$ is one-one and onto function. ***(Un)* (CBSE SQP, 2023)**

Sol. We have, $f(x) = \begin{cases} \frac{x}{1+x}, & \text{if } x \geq 0 \\ \frac{x}{1-x}, & \text{if } x < 0 \end{cases}$

Now, we consider the following cases

Case I: when $x \geq 0$, we have $f(x) = \frac{x}{1+x}$

Injectivity: let $x, y \in \mathbb{R}^+ \cup \{0\}$ such that $f(x) = f(y)$, then

$\Rightarrow \frac{x}{1+x} = \frac{y}{1+y} \Rightarrow x + xy = y + xy \Rightarrow x = y$

So, *f* is injective function. *(1 M)*

Surjectivity: when $x \geq 0$, we have $f(x) = \frac{x}{1+x} \geq 0$ and $f(x)$

$= 1 - \frac{1}{1+x} < 1$, as $x \geq 0$

Let $y \in [0,1)$, thus for each $y \in [0,1)$ there exists $x = \frac{y}{1-y}$

≥ 0 such that $f(x) = \frac{\frac{y}{1-y}}{1 + \frac{y}{1-y}} = y$

So, *f* is onto function on $[0, \infty)$ to $[0, 1)$. *(1 M)*

Case II: when $x < 0$, we have $f(x) = \frac{x}{1-x}$

Injectivity: Let $x, y \in \mathbb{R}^-$ i.e., $x, y < 0$, such that $f(x) = f(y)$, then

$\Rightarrow \frac{x}{1-x} = \frac{y}{1-y} = x - xy = y - xy \Rightarrow x = y$

So, *f* is injective function. *(1 M)*

Surjectivity: $x < 0$, we have $f(x) = \frac{x}{1-x} < 0$ also,

$f(x) = \frac{x}{1-x} = -1 + \frac{1}{1-x} > -1$

$\Rightarrow -1 < f(x) < 0$

Let $y \in (-1, 0)$ be an arbitrary real number and there exists $x = \frac{y}{1-y} < 0$ such that,

$$f(x) = f\left(\frac{y}{1+y}\right) = \frac{\frac{y}{1+y}}{1-\frac{y}{1+y}} = y$$

So, for $y \in (-1, 0)$, there exists $x = \frac{y}{1+y} < 0$ such that $f(x) = y$.

Hence, f is onto function on $(-\infty, 0)$ to $(-1, 0)$. *(1 M)*

Case III:

Injectivity: Let $x > 0$ and $y < 0$ such that $f(x) = f(y)$

$$\Rightarrow \frac{x}{1+x} = \frac{y}{1-y}$$

$\Rightarrow x - xy = y + xy \Rightarrow x - y = 2xy$, here LHS > 0 but RHS < 0, which is inadmissible.

Hence, $f(x) \neq f(y)$ when $x \neq y$.

Hence f is one-one and onto function. *(1 M)*

3. If $f: R \to R$ is the function defined by $f(x) = 4x^3 + 7$, then show that f is a bijection ***(Un)***

Sol. Injectivity: To prove that f is injective, we need to show that if $f(x_1) = f(x_2)$, then $x_1 = x_2$.

Let $f(x_1) = f(x_2)$:

$\Rightarrow 4{x_1}^3 + 7 = 4{x_2}^3 + 7$ *(1 M)*

$\Rightarrow 4x_1^3 = 4x_2^3 \Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2$

Thus, f is injective *(1 M)*

Surjectivity

Let $y \in \mathbb{R}$. We need to find x such that: $f(x) = y$

$\Rightarrow 4x^3 + 7 = y$ *(1 M)*

$\Rightarrow 4x^3 = y - 7$

$$\Rightarrow x^3 = \frac{y-7}{4}$$

$$\Rightarrow x = \sqrt[3]{\frac{y-7}{4}} \quad (1\ M)$$

Since y is any real number, $x = \sqrt[3]{\frac{y-7}{4}}$ is a real number because the cube root of any real number is real. Thus, for every $y \in \mathbb{R}$, there exists an $y \in \mathbb{R}$ such that $f(x) = y$

Therefore, f is surjective. *(1 M)*

MISCELLANEOUS EXERCISE

Multiple Choice Questions **(1 M)**

1. The maximum number of equivalence relations on the set $A = \{1, 2\}$ are **(NCERT Exemplar)**
   (*a*) 1 (*b*) 2
   (*c*) 3 (*d*) 5

2. Let R be the relation in the set **N** given by $R = \{(a, b): a = b - 4, b > 6\}$. Choose the correct answer. **(NCERT Intext)**
   (*a*) $(2, 4) \in R$ (*b*) $(3, 8) \in R$
   (*c*) $(4, 8) \in R$ (*d*) $(8, 7) \in R$

3. A function $f: R \to R$ defined by $f(x) = 2 + x^2$ is **(CBSE 2022, Term-1)**
   (*a*) not one-one
   (*b*) one-one
   (*c*) not onto
   (*d*) neither one-one nor onto

4. If a relation R on the set $\{1, 2, 3\}$ be defined by $R = \{(1, 2)\}$, then R is **(NCERT Exemplar)**
   (*a*) reflexive (*b*) transitive
   (*c*) symmetric (*d*) none of these

5. Let $f: \mathbf{N} \to \mathbf{R}$ be the function defined by $f(x) = \frac{2x-1}{2}$ and $g: \mathbf{Q} \to \mathbf{R}$ be another function defined by $g(x) = x + 2$. Then $(g\,o\,f)\,\frac{3}{2}$ is **(NCERT Exemplar)**
   (*a*) 1 (*b*) 1
   (*c*) $\frac{7}{2}$ (*d*) none of these

6. Let $f: \mathbf{R} \to \mathbf{R}$ be defined as $f(x) = 3x$. Choose the correct answer. **(NCERT Intext)**
   (*a*) f is one-one onto
   (*b*) f is many-one onto
   (*c*) f is one-one but not onto
   (*d*) f is neither one-one nor onto.

7. Let $A = \{3, 5\}$. Then number of reflexive relations on A is
   (*a*) 2
   (*b*) 4
   (*c*) 0
   (*d*) 8

Long Answer Type Questions **(4 or 5 M)**

1. The Earth has 24 time zones, defined by dividing the Earth into 24 equal longitudinal segments. These are the regions on Earth that have the same standard time. For example, USA and India fall in different time zones, but Sri Lanka and India are in the same time zone.
   A relation R is defined on the set U = {All people on the Earth} such that $R = \{(x, y)|$ the time difference between the time zones x and y reside in is 6 hours}.
   (*i*) Check whether the relation R is reflexive, symmetric and transitive.
   (*ii*) Is relation R an equivalence relation?
   Show your work. **(CBSE APQ, 2023)**

2. Let T be the set of all triangles in a plane with R as a relation in T given by $R = \{(T_1, T_2) : (T_1) \cong T_2\}$. Show that R is an equivalence relation.

3. Check whether the relation R defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive. **(CBSE, 2019)**

4. If $A = \{1, 2, 3, 4\}$, define relations on A which have properties of being:
   (*i*) reflexive, transitive but not symmetric
   (*ii*) symmetric but neither reflexive nor transitive
   (*iii*) reflexive, symmetric and transitive.

5. Each of the following defines a relation on N:
   (*i*) x is greater than y, $xy \in N$
   (*ii*) $x + y = 10$, $x, y \in N$
   (*iii*) xy is square of an integer $x, y \in N$
   (*iv*) $x + 4y = 10x, y \in n$
   Determine which of the above relations are reflective, symmetric and transitive.

6. A function $f : R - 1, 1\} \rightarrow R$ is defined by: $f(x) = \dfrac{x}{x^2 - 1}$
   (*i*) Check if f is one-one.
   (*ii*) Check if f is onto.
   Show your work. **(CBSE APQ, 2023)**

7. Let $A = [-1, 1]$. Then, discuss whether the following functions defined on A are one-one, onto or bijective:
   (*i*) $f(x) = \dfrac{x}{2}$ (*ii*) $g(x) = |x|$
   (*iii*) $h(x) = x|x|$ (*iv*) $k(x) = x^2$

Case Based Questions

Case Based-I

An organization conducted bike race under two different categories- Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project. Let $B = (b_1, b_2, b_2)$ and $G = (g_1, g_2)$, where B represents the set of Boys selected and G the set of Girls selected for the final race. **[CBSE, 2023]**

(*i*) How many relations are possible from B to G?

(*ii*) Among all the possible relations from B to G, how many functions can be formed from B to G?

(*iii*) Let R: B→ B be defined by $R = \{(x, y)$: x and y are students of the same sex). Check if R is an equivalence relation in B or not.

OR

A function f: $B \rightarrow G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_3)\}$. Check if f is bijective, Justify your answer.

Case Based-II

Dhanush wants take a test of his son Amit is a student of class XII. Dhanush said to Amit, the two functions $f(x)$ and $g(x)$ carefully f: $\mathbb{R} \rightarrow \mathbb{R}$, g: $\mathbb{R} \rightarrow \mathbb{R}$ such that $f(x) = x$, $g(x) = x^2$

The Dhanush asked some questions related to $f(x)$ and $g(x)$ and Amit answered correctly. Write the correct response given by Amit of the following questions.

(*i*) Check whether $f(x)$ is bijective or not.

(*ii*) Check whether $g(x)$ is bijective or not

Case Based-III

A general election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote and voter turnout was about 67%, the highest ever.

Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2019. A relation 'R' is defined on I as follows:

$R = \{(V_1, V_2)$: $V_1, V_2 \in$ I and both use their voting right in general election - 2019).

(*i*) Two neighbours X and Y $\in I$. X exercised his voting right while Y did not cast her vote in general election - 2019. Is XRY? Give reason.

(*ii*) Mr. 'X' and his wife 'W' both exercised their voting right in general election -2019. Is it true that XRY and YRX? Give reason.

(*iii*) Three friends F_1, F_2 and F_3 exercised their voting right in general election- 2019. Is it true that F_1RF_2, F_2RF_3 F_1RF_3? Give reason.

OR

Mr. Shyam exercised his voting right in General Election 2019, then find the equivalence class of Mr. Shyam.

Case Based-IV

Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set (1,2,3,4,5,6). Let A be the set of players while B be the set of all possible outcomes. **[CBSE QB]**

A = $\{S, D\}$, $B = \{1, 2, 3, 4, 5, 6\}$

(i) Let $R : B \to B$ be defined by $R = \{(x, y) : y$ is divisible by $x)$. Verify that whether R is reflexive, symmetric and transitive.

(ii) Raji wants to know the number of functions from A to B. Find the number of all possible functions.

(iii) Let R be a relation on B defined by $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$ Then R is which kind of relation?

OR

Raji wants to know the number of relations possible from A to B. Find the number of possible relations.

Case Based-V

Students of Grade 9, planned to plant saplings along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the rows of the saplings along the line $y = x - 4$. Let L be the set of all lines which are parallel on the ground and R be a relation on L. **[CBSE QB]**

(i) Let relation R be defined by $R = \{(L_1, L_2): L_1 \parallel L_2$ where $L_1, L_2 \in L)$. What is the type of relation R?

(ii) Let $R = ((L_1, L_2): L_1 \perp L_2$ where $L_1, L_2 \in L)$. What is the type of relation R?

(iii) Check whether the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x + 4$ is bijective or not.

OR

Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x + 4$. Find the range of $f(x)$.

ANSWER KEYS

Multiple Choice Questions

1. (*c*) **2.** (*c*) **3.** (*d*) **4.** (*d*) **5.** (*d*) **6.** (*a*) **7.** (*b*) **8.** (*a*) **9.** (*b*) **10.** (*d*)
11. (*c*)

Assertion and Reason

1. (*c*) **2.** (*d*) **3.** (*a*) **4.** (*b*) **5.** (*c*)

HINTS & EXPLANATIONS

Multiple Choice Questions

1. (*c*) Determine the possible partitions of the set $A = \{1, 2\}$ into equivalence classes:

- All elements in one class: $\{\{1,2\}\}$
- Each element in its own class: $\{\{1\}, \{2\}\}$

Count the number of distinct partitions:

- One equivalence class: 1 way
- Two equivalence classes: 2 ways (as listed above)

Add up the counts: $1 + 2 = 3$.

Therefore, the maximum number of equivalence relations on the set A $\{1, 2\}$ is 3.

2. (*c*) $R = (a, b) : a = b - 4, b > 6$

Now, since $b > 6, (2, 4) \notin R$

Also, as $3 \neq 8 - 4$, $(3, 8) \notin R$

And, as $8 \neq 7 - 4$, $(8, 7) \notin R$

Now, consider (6, 8). We have $8 > 4$ and also, $4 = 8 - 4$.

$\therefore (4, 8) \in R$

MOCK TEST-2

Time allowed : 3 hours | Maximum Marks : 80

GENERAL INSTRUCTIONS:

(i) *This question paper contains **38** questions. **All** question are **compulsory**.*

(ii) *This question paper is divided into **five** sections - **A**, **B**, **C**, **D** and **E***

(iii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(iv) *Use of calculators is **not** allowed.*

SECTION - A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}, n \in N,$ then A^6 equals

(a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

2. $\int \frac{x^9}{(4x^2+1)^6} dx$ is equal to:

(a) $\frac{1}{5x}\left(4+\frac{1}{x^2}\right)^{-5}+c$ (b) $\frac{1}{5}\left(4+\frac{1}{x^2}\right)^{-5}+c$ (c) $\frac{1}{10x}\left(4+\frac{1}{x^2}\right)^{-5}+c$ (d) $\frac{1}{10}\left(4+\frac{1}{x^2}\right)^{-5}+c$

3. Area bounded by the curve $y = \log x$, x-axis and the ordinates $x = 1, x = 2$ is

(a) log 4 sq. units (b) (log 4 + 1) sq. units (c) (log 4 – 1) sq. units (d) None of these

4. If $x^p y^p = (x + y)^{(p+q)}$, then $\frac{dy}{dx} =$

(a) $\frac{x}{y}$ (b) $\frac{y}{x}$ (c) $\frac{x}{x+y}$ (d) $\frac{y}{y+x}$

5. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$ is:

(a) 1 (b) 0 (c) –1 (d) $\frac{\pi}{4}$

6. The solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is

(a) $1 + xy + c(y + x) = 0$ (b) $x + y = c(1 - xy)$ (c) $y - x = c(1 + xy)$ (d) $1 + xy = c(x + y)$

7. The value of the objective function is maximum under linear constraints

(a) At the centre of feasible region (b) At (0, 0)

(c) At any vertex of feasible region (d) The vertex which is at maximum distance from (0, 0)

8. The image of (1, 6, 3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ is:

(a) (0, 5, 1) (b) (1, 0, 7) (c) (5, 2, 1) (d) (3, 2, 1)

9. $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx =$

(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{16}$ (c) $\frac{\pi^2}{4}$ (d) $\frac{\pi^2}{32}$

10. If $A=\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then the value of |adj A|, is

(a) a^9 (b) a^{27} (c) a^6 (d) a^2

11. The corner points of the feasible region determined by the system of linear constraints are: (20, 160), (40, 80), (20, 280). Let $Z = lx + my$, where $l, m > 0$. Condition on l on m, so that the maximum Z occurs at two points (20, 160) and (40, 80) is:

(a) $l = m$ (b) $3l = 4m$ (c) $l = 4m$ (d) $3l = m$

12. The vectors $\hat{i}+2\hat{j}+3\hat{k},\ \lambda\hat{i}+4\hat{j}+7\hat{k},\ -3\hat{i}-2\hat{j}-5\hat{k}$ are collinear, if λ equals

(a) 3 (b) 4 (c) 5 (d) 6

13. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix}=\begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to

(a) –6 (b) ±6 (c) 6 (d) 0

14. There are two children in a family. The probability that both of them are boys is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) None of these

15. The degree of the differential equation $\frac{d^2y}{dx^2}-\sqrt{\frac{dy}{dx}-3}=x$ is

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) 3

16. If $\vec{a}$ is any vector in space, then

(a) $\vec{a}=(\vec{a}\cdot\hat{i})\hat{i}+(\vec{a}\cdot\hat{j})\hat{j}+(\vec{a}\cdot\hat{k})\hat{k}$ (b) $\vec{a}=(\vec{a}\times\hat{i})+(\vec{a}\times\hat{j})+(\vec{a}\times\hat{k})$

(c) $\vec{a}=\hat{j}(\vec{a}\cdot\hat{i})+\hat{k}(\vec{a}\cdot\hat{j})+\hat{i}(\vec{a}\cdot\hat{k})$ (d) $\vec{a}=(\vec{a}\times\hat{i})\times\hat{i}+(\vec{a}\times\hat{j})\times\hat{j}+(\vec{a}\times\hat{k})\times\hat{k}$

17. The number of points at which the function $f(x)=\frac{1}{x-[x]}$ is not continuous are

(a) 1 (b) 2 (c) 3 (d) For every integer

18. The direction ratios of the perpendicular to the lines $\frac{x-7}{2}=\frac{y+17}{-3}=\frac{z-6}{1}$ and $\frac{x+5}{1}=\frac{y+3}{2}=\frac{z-4}{-2}$ are proportional to

(a) 4, 5, 7 (b) 4, –5, 7 (c) 4, –5, –7 (d) –4, 5, 7

DIRECTIONS: In the question number **19** and **20**, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option out of the following:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

2026
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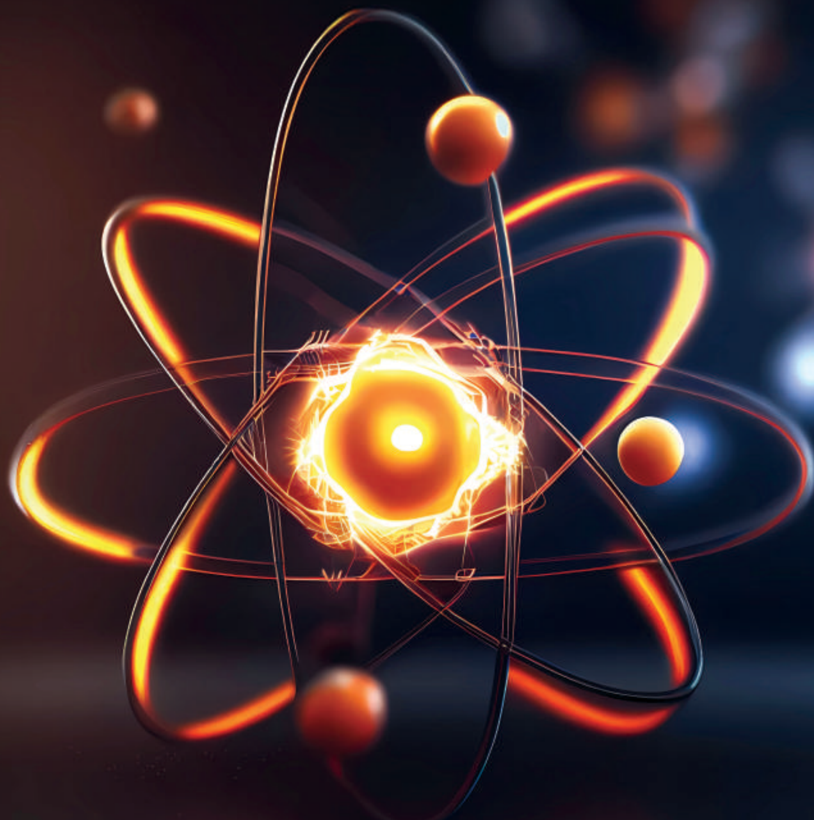

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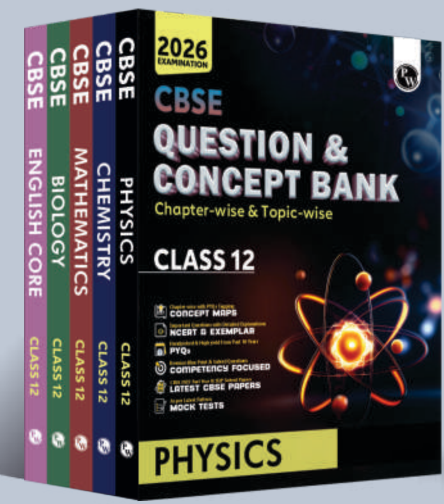

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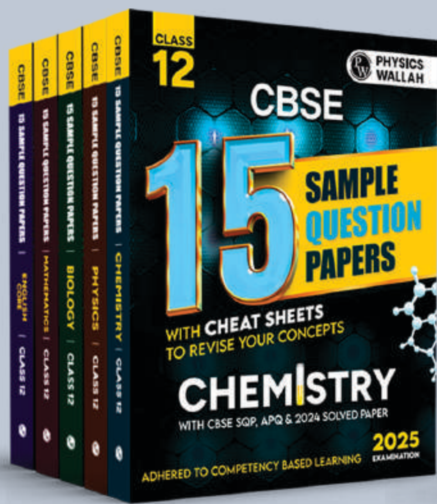

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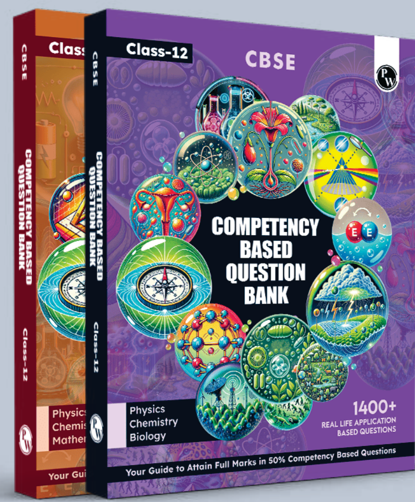

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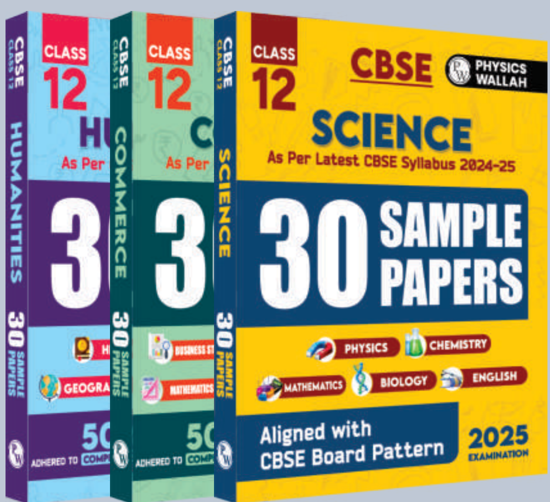

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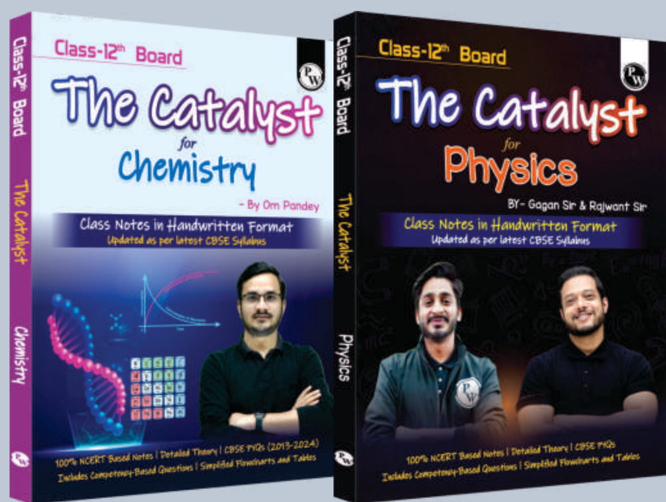

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