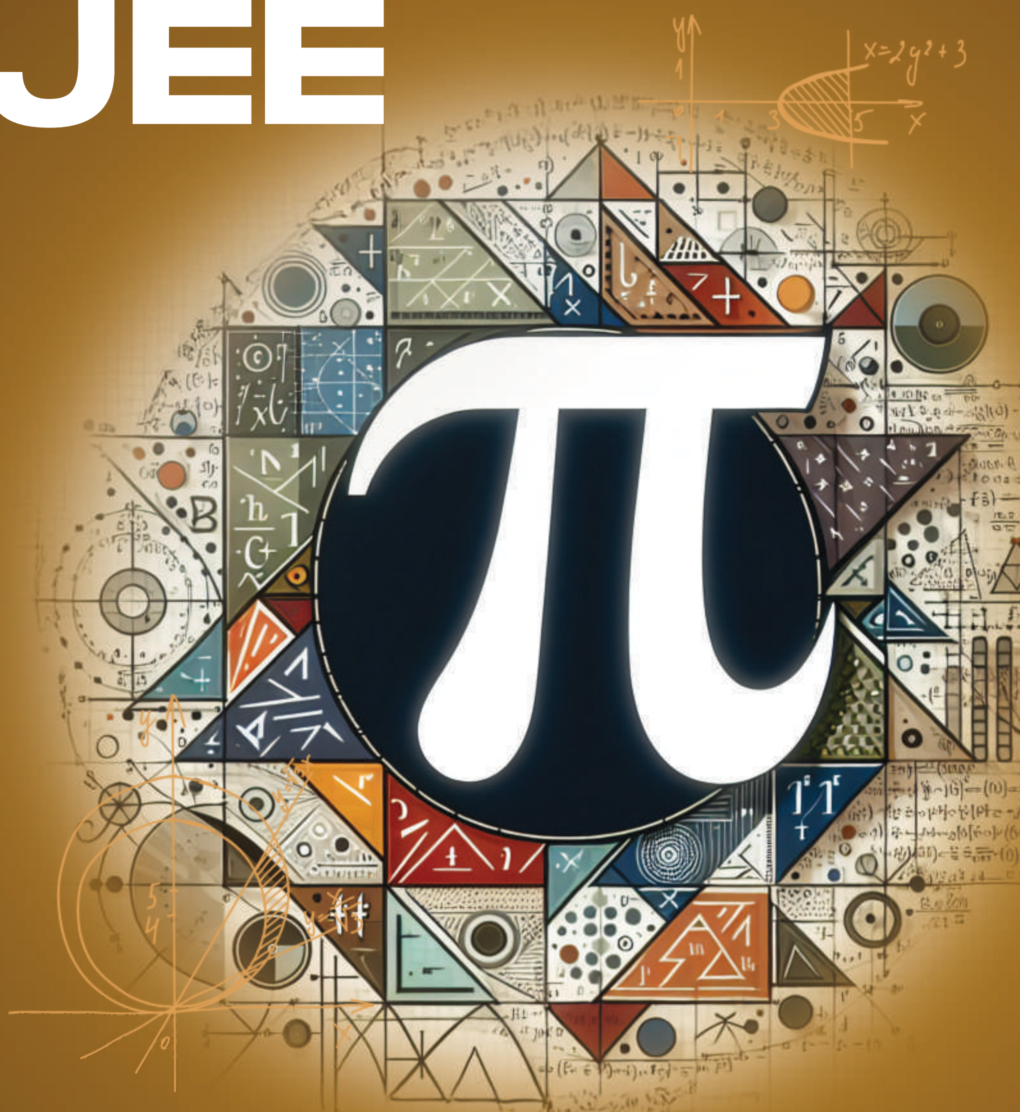
PHYSICS WALLAH

PRAYAS JEE



- Basic Maths
- Sets
- Quadratic Equations
- Sequences and Series
- Trigonometric Functions

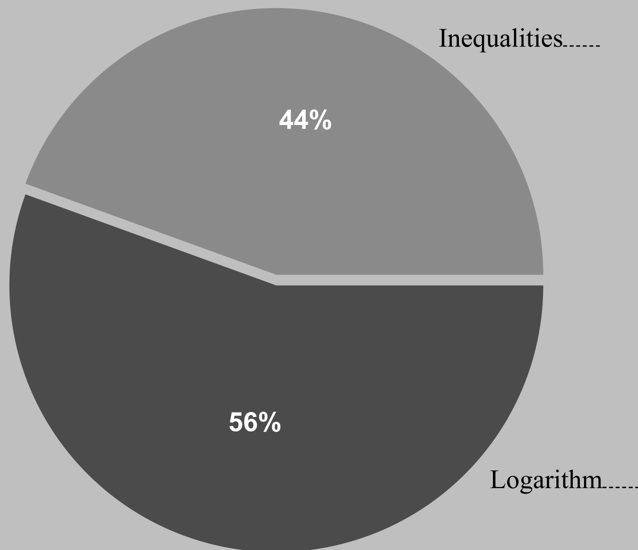
Mathematics

MODULE 1

CHAPTER
1

Basic Maths

Topicwise Weightage of JEE Main 6 Years Paper (124 Sets)



"How's the Josh?" for these Topics: *Mark your confidence level in the blank space around the topic (Low-L, Medium-M, High-H)*

NUMBER SYSTEM

(*i*) **Natural numbers:** The counting numbers 1, 2, 3, 4, ... are called Natural Numbers. The set of natural numbers is denoted by N.

Thus $N = \{1, 2, 3, 4, ...\}$.

(*ii*) **Whole numbers:** Natural numbers including zero are called whole numbers. The set of whole numbers is denoted by W.

Thus W = $\{0, 1, 2, ...\}$

(*iii*) **Integers:** The numbers ... – 3, – 2, – 1, 0, 1, 2, 3 .... are called integers and the set is denoted by I or Z. Thus I (or Z) = $\{.. -3, -2, -1, 0, 1, 2, 3...\}$

Note: (*a*) Positive integers $I^+ = \{1, 2, 3 ....\} = N$

(*b*) Negative integers $I^- = \{....., -3, -2, -1\}$.

(*c*) Non-negative integers (whole numbers) = $\{0, 1, 2, ...\}$.

(*d*) Non-positive integers = $\{......, -3, -2, -1, 0\}$.

(*iv*) **Even integers:** Integers which are divisible by 2 are called even integers.

e.g. $0, \pm 2, \pm 4$,.......

(*v*) **Odd integers:** Integers which are not divisible by 2 are called odd integers.

e.g. $\pm 1, \pm 3, \pm 5, \pm 7$......

(*vi*) **Prime numbers:** Natural numbers which are divisible by 1 and itself only are called prime numbers.

e.g. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, ...

(*vii*) **Composite number:** Let 'a' be a natural number, '*a*' is said to be composite if, it has atleast three distinct factors.

e.g. 4, 6, 8, 9, 10, 12, 14, 15 ....

Note: (*a*) 1 is neither a prime number nor a composite number.

(*b*) Numbers which are not prime are composite numbers (except 1).

(*c*) '4' is the smallest composite number.

(*d*) '2' is the only even prime number.

(*viii*) **Co-prime numbers:** Two natural numbers (not necessarily prime) are called coprime, if their H.C.F (Highest common factor) is one.

e.g. (1, 2), (1, 3), (3, 4), (3, 10), (3, 8), (5, 6), (7, 8) (15, 16) etc.

Concept Application

3. If $x^{1/3} + y^{1/3} + z^{1/3} = 0$, then $(x + y + z)^3$ is equal to?

(*a*) 1 (*b*) 3

(*c*) $3xy$ (*d*) $27xyz$

4. If $a + b + c = 0$, then what is the value of

$$\frac{a^2+b^2+c^2}{(a-b)^2+(b-c)^2+(c-a)^2}$$

(*a*) 1 (*b*) 3 (*c*) $\frac{1}{3}$ (*d*) 0

5. If $x+\frac{1}{x}=p$ then $x^6+\frac{1}{x^6}$ equals to :

(*a*) $p^6 + 6p$ (*b*) $p^6 - 6p$

(*c*) $p^6 + 6p^4 + 9p^2 + 2$ (*d*) $p^6 - 6p^4 + 9p^2 - 2$

6. If $x+\frac{1}{x}=4,$ then find values of

(*i*) $x^2+\frac{1}{x^2}$ (*ii*) $x^3+\frac{1}{x^3}$

(*iii*) $x^4+\frac{1}{x^4}$

7. Prove that $(1 + x)(1 + x^2)(1 + x^4)(1 + x^8)(1 + x^{16})$ $=\frac{(1-x^{32})}{(1-x)}$

8. If x, y, z are all different real numbers, then prove that

$$\frac{1}{(x-y)^2}+\frac{1}{(y-z)^2}+\frac{1}{(z-x)^2}=\left(\frac{1}{x-y}+\frac{1}{y-z}+\frac{1}{z-x}\right)^2$$

9. If $\frac{a}{b}+\frac{b}{a}=-1,$ then find value of $a^3 - b^3$.

10. If $a - b = -8$, $ab = -12$ then $a^3 - b^3$ will be

11. The product $(x + y)(x - y)(x^2 + xy + y^2)(x^2 - xy + y^2)$ simplifies to

12. Find the real values of p, q, r satisfying $(2p - 3)^8 + (1 - q)^6 + (4 - 3r)^4 = 0$.

ADVANCED LEARNING

INDICES

If 'a' is any non zero real or imaginary number and 'm' is the positive integer, then $a^m = a \cdot a \cdot a. ... a$ (m times). Here a is called the base and m is called the index, power or exponent.

Law of indices:

1. $a^0 = 1, (a \neq 0)$
2. $a^{-m}=\frac{1}{a^m}, (a \neq 0)$
3. $a^{m+n} = a^m \cdot a^n$, where m and n are rational numbers
4. $a^{m-n}=\frac{a^m}{a^n}$, where m and n are rational numbers, $a \neq 0$
5. $(a^m)^n = a^{mn}$
6. $a^{p/q}=\sqrt[q]{a^p}$
7. $\sqrt[m]{\sqrt[n]{a}}=\sqrt[mn]{a}=\sqrt[n]{\sqrt[m]{a}}$, where $m, n \in N$ and ($m, n \geq 2$) and a is positive rational number
8. $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$, $a, b \in R$ and atleast one of a and b should be positive

SURDS

If a is a positive rational number, which is not the nth power (n is any natural number) of any rational number, then the irrational number $\pm\sqrt[n]{a}$ are called simple surds or monomial surds.

Every surd is an irrational number (but every irrational number is not a surd). So, the representation of monomial surd on a number line is same as that of irrational numbers.

Examples:

1. $\sqrt{3}$ is a surd and $\sqrt{3}$ is an irrational number.

2. $\sqrt[3]{5}$ is a surd and $\sqrt[3]{5}$ is an irrational number.

3. π is an irrational number, but it is not a surd.

4. $\sqrt[3]{3+\sqrt{2}}$ is an irrational number. It is not a surd, because $3+\sqrt{2}$ is not a rational number.

Train Your Brain

Example 6: Simplify $\left[\sqrt[3]{\sqrt[6]{a^9}}\right]^4\left[\sqrt[6]{\sqrt[3]{a^9}}\right]^4$

Sol. $a^{9(1/6)(1/3)4} \cdot a^{9(1/3)(1/6)4} = a^2 \cdot a^2 = a^4$.

Example 7: Arrange the following in ascending or descending order of magnitude:

$\sqrt[4]{6}, \sqrt[3]{7}, \sqrt{5}$

Sol. $\sqrt[4]{6}=6^{1/4}, \sqrt[3]{7}=7^{1/3}, \sqrt{5}=5^{1/2}$

LCM of the denominators of the exponents of these three terms, 4, 3 and 2 is12.

Now express the exponent of each term, as a fraction in which then denominator is 12.

$$6^{\frac{1}{4}}=6^{\frac{3}{12}}=\left(6^3\right)^{\frac{1}{12}}=\sqrt[12]{216}$$

$$7^{\frac{1}{3}}=7^{\frac{4}{12}}=\left(7^4\right)^{\frac{1}{12}}=\sqrt[12]{2401}$$

$$5^{\frac{1}{2}}=5^{\frac{6}{12}}=\left(5^6\right)^{\frac{1}{12}}=\sqrt[12]{15625}$$

Aarambh (Solved Examples)

1. The value of $81^{(1/\log_5 3)} + 27^{\log_9 36} + 3^{4/\log_7 9}$ is equal to

(*a*) 49 (*b*) 625 (*c*) 216 (*d*) 890

Sol. $81^{(1/\log_5 3)} + 27^{\log_9 36} + 3^{4/\log_7 9}$

$= 3^{4\log_3 5} + 3^{3.\frac{1}{2}\log_3 36} + 3^{4\log_9 7}$

$= 3^{\log_3 5^4} + 3^{\log_3 36^{3/2}} + 3^{\log_3 7^{4/2}}$

$= 5^4 + 36^{3/2} + 7^2 = 890$

Therefore, option (*d*) is the correct answer.

2. The largest integral value of x satisfying

$\sqrt{18^x - 5} \le \sqrt{2(18^x + 12)} - \sqrt{18^x + 5}$ is

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) no integral value of x possible

Sol. Let $18^x = p$

$\sqrt{p-5} + \sqrt{p+5} \le \sqrt{2(p+12)}$

$\Rightarrow\ p - 5 + p + 5 + 2\sqrt{p^2 - 25} \le 2p + 24$

$\Rightarrow\ \sqrt{p^2 - 25} \le 12 \Rightarrow p^2 \le 169 \Rightarrow p \le 13$

Also $p \ge 5$

Thus $5 \le p \le 13 \Rightarrow \log_{18}5 \le x \le \log_{18}13$

Therefore, option (*d*) is the correct answer.

3. Solve if $|x - 5| + |x + 4| = 9$

(*a*) $[-4, 5]$ (*b*) $(-4, 5)$ (*c*) $(-4, 5]$ (*d*) $[-4, 5)$

Sol. Given equation is of form $|a| + |b| = |a - b|$

It is true for $ab \le 0$

$(x - 5)(x + 4) \le 0$

So $x \in [-4, 5]$

Therefore, option (*a*) is the correct answer.

4. Solve $\dfrac{(e - \sin x)(x - 2)}{(x + 4)} \ge 0.$

(*a*) $(-\infty, -4) \cup [2, \infty)$ (*b*) $(-\infty, -4] \cup (2, \infty)$

(*c*) $(-\infty, -4) \cup (2, \infty)$ (*d*) None of these

Sol. Zeros $x = 2$, Pole $x \ne -4$

$e - \sin x > 0$ always positive

$\dfrac{(e - \sin x)(x - 2)}{(x + 4)} \ge 0$

Final solution $x \in (-\infty, -4) \cup [2, \infty)$

Therefore, option (*a*) is the correct answer.

5. Values of x satisfying the equation

$\log_5^2 x + \log_{5x}\left(\dfrac{5}{x}\right) = 1$ are

(*a*) 1 (*b*) 5

(*c*) $\dfrac{1}{25}$ (*d*) 3

Sol. $(\log_5 x)^2 + \log_{5x}\dfrac{5}{x} = 1$

$\Rightarrow (\log_5 x)^2 + \log_{5x}5 - \log_{5x}x = 1$

$\Rightarrow \left(\log_5 x\right)^2 + \dfrac{\log_5 5}{\log_5 5 + \log_5 x} - \dfrac{\log_5 x}{\log_5 5 + \log_5 x} = 1$

$\Rightarrow (\log_5 x)^2 + \dfrac{1}{1 + \log_5 x} - \dfrac{\log_5 x}{1 + \log_5 x} = 1$

Let $\log_5 x = t$

$\therefore\ t^2 + \dfrac{1}{1+t} - \dfrac{t}{1+t} = 1$

$\Rightarrow\ \dfrac{t^2(1+t) + 1 - t}{1+t} = 1$

$\Rightarrow t^3 + t^2 + 1 - t = 1 + t$

$t^3 + t^2 - 2t = 0$

$t(t^2 + t - 2) = 0$

$t(t - 1)(t + 2) = 0$

$t = 0, 1, -2$

$\therefore\ \log_5 x = 0, 1, -2$

$\therefore\ x = 1, 5, \dfrac{1}{25}$

Therefore, option (*a, b, c*) is the correct answers.

6. The equation $\log_{x^2} 16 + \log_{2x} 64 = 3$ has

(*a*) One irrational solution

(*b*) No prime solution

(*c*) Two real solutions

(*d*) One integral solution

Sol. $\dfrac{4}{2}\log_x 2 + \dfrac{\log_x 64}{\log_x 2x}$

$\Rightarrow\ 2\log_x 2 + \dfrac{6\log_x 2}{1 + \log_x 2} = 3$

Let $\alpha = \log_x 2$

$\therefore 2\alpha + \dfrac{6\alpha}{1+\alpha} = 3$

$\Rightarrow 2\alpha + 2\alpha^2 + 6\alpha - 3 - 3\alpha = 0$

$\Rightarrow 2\alpha^2 + 5\alpha - 3 = 0$

$\Rightarrow (\alpha + 3)(2\alpha - 1) = 0 \Rightarrow a = -3, 1/2$

$\therefore\ \log_x 2 = -3 \Rightarrow x = 2^{-1/3}$ (Irrational)

or $\log_x 2 = \dfrac{1}{2} \Rightarrow x = 4$ (Integer)

Therefore, option (*a, b, c, d*) is the correct answers.

Prarambh (Topicwise)

BASIC CONCEPTS AND NUMBER SYSTEM

1. The number of real roots of the equation $(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$ is:

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

2. If $x - a$ is a factor of $x^3 - a^2x + x + 2$, then 'a' is equal to

(*a*) 0 (*b*) 2 (*c*) –2 (*d*) 1

3. If x, y are integral solutions of $2x^2 - 3xy - 2y^2 = 7$, then value of $|x + y|$ is

(*a*) 2 (*b*) 4
(*c*) 6 (*d*) 2 or 4 or 6

4. If a, b, c are real, then $a(a-b) + b(b-c) + c(c-a) = 0$, only if

(*a*) $a + b + c = 0$ (*b*) $a = b = c$
(*c*) $a = b$ or $b = c$ or $c = a$ (*d*) $a - b - c = 0$

5. If $2x^3 - 5x^2 + x + 2 = (x-2)(ax^2 - bx - 1)$, then a & b are respectively

(*a*) 2, 1 (*b*) 2, – 1 (*c*) 1, 2 (*d*) –1, 1/2

6. If $L = \frac{1}{\sqrt{7}-\sqrt{8}} + \frac{1}{\sqrt{7}-\sqrt{6}} + \frac{1}{3-\sqrt{8}} + \frac{1}{\sqrt{5}+2} + \frac{1}{\sqrt{5}-\sqrt{6}}$
$= 1 + 2\sqrt{a} + 2\sqrt{b}$, then $a \times b$ is equal to

(*a*) 30 (*b*) 45 (*c*) 8 (*d*) 0

7. If a, b, c are real and distinct numbers, then the value of $\frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)}$ is

(*a*) 1 (*b*) $a\,b\,c$
(*c*) 2 (*d*) 3

8. The remainder obtained when the polynomial $1 + x + x^3 + x^9 + x^{27} + x^{81} + x^{243}$ is divided by $x - 1$ is

(*a*) 3 (*b*) 5 (*c*) 7 (*d*) 11

LOGARITHM AND ITS PRINCIPLE PROPERTIES

9. $\frac{1}{1+\log_b a + \log_b c} + \frac{1}{1+\log_c a + \log_c b} + \frac{1}{1+\log_a b + \log_a c}$
has the value equal to

(*a*) abc (*b*) $\frac{1}{abc}$ (*c*) 0 (*d*) 1

10. $\log_7 \log_7 \sqrt{7(\sqrt{7\sqrt{7}})} =$

(*a*) $3 \log_2 7$ (*b*) $1 - 3\log_3 7$
(*c*) $1 - 3\log_7 2$ (*d*) $1 - 10\log_2 7$

11. $\frac{1}{\log_{\sqrt{bc}} abc} + \frac{1}{\log_{\sqrt{ca}} abc} + \frac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to

(*a*) 1/2 (*b*) 1 (*c*) 2 (*d*) 4

12. If $\log_x \log_{18}(\sqrt{2}+\sqrt{8}) = \frac{1}{3}$. Then the value of $1000\,x$ is equal to

(*a*) 8 (*b*) 1/8 (*c*) 1/125 (*d*) 125

13. Number of real solutions of the equation $\sqrt{\log_{10}(-x)} = \log_{10}\sqrt{x^2}$ is:

(*a*) none (*b*) exactly 1
(*c*) exactly 2 (*d*) 4

14. Greatest integer less than or equal to the number $\log_2 15.\log_{1/6} 2.\log_3 1/6$ is

(*a*) 4 (*b*) 3 (*c*) 2 (*d*) 1

15. The ratio $\frac{2^{\log_{2^{\frac{1}{4}}} a} - 3^{\log_{27}(a^2+1)^3} - 2a}{7^{4\log_{49} a} - a - 1}$ simplifies to

(*a*) $a^2 - a - 1$ (*b*) $a^2 + a - 1$
(*c*) $a^2 - a + 1$ (*d*) $a^2 + a + 1$

16. If $3^{2\log_3 x} - 2x - 3 = 0$, then the number of values of 'x' satisfying the equation is

(*a*) zero (*b*) 1
(*c*) 2 (*d*) More than 2

17. The sum of all the solutions to the equation $2\log x - \log(2x - 75) = 2$:

(*a*) 30 (*b*) 350 (*c*) 75 (*d*) 200

INEQUALITIES

18. If the solution set of the inequality $\log_{\sqrt{0.9}} \log_5 (\sqrt{x^2+5+x})$ > 0 contains 'n' integral values, then n equals to

(*a*) 7 (*b*) 8 (*c*) 6 (*d*) 10

19. If $\log_{0.5} \log_5 (x^2 - 4) > \log_{0.5} 1$, then '$x$' lies in the interval

(*a*) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 3)$

(*b*) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 3\sqrt{5})$

(*c*) $(\sqrt{5}, 3\sqrt{5})$

(*d*) ϕ

20. Solution set of the inequality $2 - \log_2(x^2 + 3x) \geq 0$ is:

(*a*) $[-4, 1]$ (*b*) $[-4, -3) \cup (0, 1]$
(*c*) $(-\infty, -3) \cup (1, \infty)$ (*d*) $(-\infty, -4) \cup [1, \infty)$

MODULUS FUNCTION

21. Solutions of $|4x + 3| + |3x - 4| = 12$ are

(*a*) $x = -\frac{7}{3}, \frac{3}{7}$ (*b*) $x = -\frac{5}{2}, \frac{2}{5}$

(*c*) $x = -\frac{11}{7}, \frac{13}{7}$ (*d*) $x = -\frac{3}{7}, \frac{7}{5}$

22. If $|x^2-2x-8|+|x^2+x-2| = 3|x+2|$, then the set of all real values of x is

(a) $[1, 4] \cup \{-2\}$ (b) $[1, 4]$

(c) $[-2, 1] \cup [4,\infty)$ (d) $(-\infty, -2] \cup [1, 4]$

23. The complete set of real 'x' satisfying $||x-1|-1| \le 1$ is:

(a) $[0, 2]$ (b) $[-1, 3]$

(c) $[-1, 1]$ (d) $[1, 3]$

24. The number of real roots of the equation $|x|^2 - 3|x| + 2 = 0$ is

(a) 1 (b) 2

(c) 3 (d) 4

25. Number of real solution (x) of the equation $|x-3|^{3x^2-10x+3} = 1$ is

(a) exactly four (b) exactly three

(c) exactly two (d) exactly one

MISCELLANEOUS

26. Simplify: $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3}$

(a) 0 (b) 1

(c) 3 (d) 4

27. The expression $x^2 - y^2 - z^2 + 2yz + x + y - z$ has a factor

(a) $x + y + z + 1$ (b) $-x + y + z$

(c) $x + y - z + 1$ (d) $x - y + z + 1$

28. Solve the equation $\dfrac{3x^4+x^2-2x-3}{3x^4-x^2+2x+3} = \dfrac{5x^4+2x^2-7x+3}{5x^4-2x^2+7x-3}$

(a) $x = 5, 2$ (b) $x = 4, 1$

(c) $x = 3, 8$ (d) $x = 1, 5$

29. If x, y, z are positive real number and a, b, c are rational numbers, then the value of $\dfrac{1}{1+x^{b-a}+x^{c-a}} + \dfrac{1}{1+x^{a-b}+x^{c-b}} + \dfrac{1}{1+x^{b-c}+x^{a-c}}$ is

(a) -1 (b) 1 (c) 0 (d) 2

30. If $a^x = \sqrt{b}, b^y = \sqrt[3]{c}$ and $c^z = \sqrt{a}$ then the value of xyz is

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{3}$ (c) $\dfrac{1}{6}$ (d) $\dfrac{1}{12}$

31. If $\dfrac{\log a}{b-c} = \dfrac{\log b}{c-a} = \dfrac{\log c}{a-b}$, then $a^a \cdot b^b \cdot c^c =$

(a) 3 (b) 1

(c) 4 (d) 2

32. The number of prime numbers satisfying the inequality $\dfrac{x^2-1}{2x+5} < 3$ is

(a) 1 (b) 2 (c) 3 (d) 4

Prabal (JEE Main Level)

1. If $x^{x\sqrt[3]{x}} = \left(x.\sqrt[3]{x}\right)^x$, then $x =$

(a) 1 (b) -1

(c) 0 (d) 2

2. The equation $4^{(x^2+2)} - 9.2^{(x^2+2)} + 8 = 0$ has the solution

(a) $x = \pm 1$ (b) $x = 10$

(c) $x = \pm\sqrt{2}$ (d) $x = \sqrt{3}$

3. If $x = \log_a(bc)$, $y = \log_b(ca)$, $z = \log_c(ab)$, then which of the following is equal to 1

(a) $x + y + z$

(b) $(1+x)^{-1} + (1+y)^{-1} + (1+z)^{-1}$

(c) xyz

(d) $x + y - z$

4. The solution of the equation $\log_7 \log_5 \left(\sqrt{x^2+5+x}\right) = 0$.

(a) $x = 2$ (b) $x = 3$

(c) $x = 4$ (d) $x = -2$

5. The value of $(0.05)^{\log_{\sqrt{20}}(0.1+0.01+0.001+........)}$ is

(a) 81 (b) $\dfrac{1}{81}$ (c) 20 (d) $\dfrac{1}{20}$

6. The value of $\log_2 .\log_3 ....\log_{100} 100^{99^{98^{\cdot^{\cdot^{2^1}}}}}$ is

(a) 0 (b) 1 (c) 2 (d) 100

7. The number of solution of $\log_2(x+5) = 6 - x$ is

(a) 2 (b) 0 (c) 3 (d) 1

8. Exhaustive set of values of x satisfying $\log_{|x|}(x^2 + x + 1) \ge 0$ is

(a) $(-1, 0)$ (b) $(-\infty, -1) \cup (1, \infty)$

(c) $(-\infty, \infty) - \{-1, 0, 1\}$ (d) $(-\infty,-1)\cup(-1,0)\cup(1,\infty)$

9. The set of real values of x satisfying $\log_{1/2}\left(x^2-6x+12\right) \ge -2$ is

(a) $(-\infty, 2]$ (b) $[2, 4]$

(c) $[4, +\infty)$ (d) $[3, 8]$

10. If $\log_{0.04}(x-1) \geq \log_{0.2}(x-1)$ then x belongs to the interval

(*a*) $(1, 2]$ (*b*) $(-\infty, 2]$

(*c*) $[2, \infty)$ (*d*) $(-\infty, 2)$

11. If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then x lies in the interval

(*a*) $(2, \infty)$ (*b*) $(-2, -1)$

(*c*) $(1, 2)$ (*d*) $(-2, 2)$

12. The minimum value of $f(x) = |x-1| + |x-2| + |x-3|$ is equal to

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 0

13. The set of real value(s) of p for which the equation $|2x+3|+|2x-3| = px+6$ has more than two solutions is:

(*a*) $[0, 4)$ (*b*) $(-4, 4)$

(*c*) $R - \{4, -4, 0\}$ (*d*) $\{0\}$

14. Let $3^a = 4, 4^b = 5, 5^c = 6, 6^d = 7, 7^e = 8$ and $8^f = 9$. The value of the product $(abcdef)$ is:

(*a*) 1 (*b*) 2 (*c*) $\sqrt{6}$ (*d*) 3

15. There are two positive solutions to the equation

$\log_{2x}2 + \log_4 2x = -\dfrac{3}{2}$. The product of these two solution is:

(*a*) $\dfrac{1}{32}$ (*b*) $\dfrac{1}{8}$ (*c*) $\dfrac{1}{2}$ (*d*) $\dfrac{1}{21}$

16. Number of values of x satisfying the equation

$x^4 = |x|^{\log_2(x^2+12)}$ is:

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

17. If $a, b, c \in R$ and $a, b, c \neq 0$ such that $\dfrac{a}{b}+\dfrac{b}{c}+\dfrac{c}{a} = 6$ and $\dfrac{b}{a}+\dfrac{c}{b}+\dfrac{a}{c} = 8$ then $\dfrac{a^3}{b^3}+\dfrac{b^3}{c^3}+\dfrac{c^3}{a^3}-3$ is equal to

(*a*) 81 (*b*) 48 (*c*) 72 (*d*) 84

18. The value of $x + y + z$ satisfying the system of equations

$\log_2 x + \log_4 y + \log_4 z = 2$ is

$\log_3 y + \log_9 z + \log_9 x = 2$

$\log_4 z + \log_{16} x + \log_{16} y = 2$

(*a*) $\dfrac{175}{12}$ (*b*) $\dfrac{349}{24}$ (*c*) $\dfrac{353}{24}$ (*d*) $\dfrac{112}{3}$

19. The smallest integral value of x such that $\sqrt{x+2}-\sqrt{x-2} < \dfrac{1}{10}$ is

(*a*) 400 (*b*) 20 (*c*) 401 (*d*) 399

20. $10^{\log_p(\log_q(\log_r x))} = 1$ and $\log_q(\log_r(\log_p x)) = 0$ then 'p' equals

(*a*) $r^{q/r}$ (*b*) rq (*c*) 1 (*d*) $r^{r/q}$

21. The set of all solutions of the inequality $(1/2)^{x^2-2x} < 1/4$ contains the set

(*a*) $(-\infty, 0)$ (*b*) $(-\infty, 1)$ (*c*) $(1, \infty)$ (*d*) $(3, \infty)$

22. The value of b satisfying the equation,
$\log_e 2 \cdot \log_b 625 = \log_{10} 16 \cdot \log_e 10$ is

(*a*) 5 (*b*) 7 (*c*) 9 (*d*) 10

23. The solution set of the system of equation

$\log_3 x + \log_3 y = 2 + \log_3 2$ and $\log_{27}(x+y) = \dfrac{2}{3}$ is:

(*a*) $\{6, 3\}$ (*b*) $\{9, 6\}$

(*c*) $\{6, 12\}$ (*d*) $\{12, 6\}$

24. Which of the following statements are true?

(*a*) $\log_2 3 < \log_{12} 10$ (*b*) $\log_6 5 < \log_7 8$

(*c*) $\log_3 26 > \log_2 9$ (*d*) $\log_{16} 15 > \log_{10} 11 > \log_7 6$

25. The number of real solution/s of the equation
$9^{\log_3(\log_e x)} = \log_e x - (\log_e x)^2 + 1$ is:

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

26. The set of all the solutions of the inequality $\log_{1-x}(x-2) \geq -1$ is

(*a*) $(-\infty, 0)$ (*b*) $(2, \infty)$

(*c*) $(-\infty, 1)$ (*d*) ϕ

27. The complete solution of $\dfrac{x^2-1}{x+3} \geq 0$ & $x^2-5x+2 \leq 0$ is:

(*a*) $x \in \left[\dfrac{5-\sqrt{17}}{2}, \dfrac{5+\sqrt{17}}{2}\right]$ (*b*) $x \in \left[1, \dfrac{5+\sqrt{17}}{2}\right]$

(*c*) $x \in (-3, -1]$ (*d*) $(-3, -1) \cup [1, \infty)$

28. The number of the integral solutions of
$x^2 + 9 < (x+3)^2 < 8x + 25$ is:

(*a*) 1 (*b*) 3 (*c*) 4 (*d*) 5

29. Number of non-negative integral values of x satisfying the inequality $\dfrac{2}{x^2-x+1}-\dfrac{1}{x+1}-\dfrac{2x-1}{x^3+1} \geq 0$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

30. The solution set of inequality $\log_{(3x^2+1)} 2 < \dfrac{1}{2}$

(*a*) $|x| > 1$ (*b*) $|x| < 1$

(*c*) ϕ (*d*) None of these

31. The solution set of $\log 4 + \left(1+\dfrac{1}{2x}\right)\log 3 = \log\left((3)^{1/x}+27\right)$

(*a*) $(1/4, 1/2)$ (*b*) $\{1/4, 1/2\}$

(*c*) $(1/4, 1/2]$ (*d*) None of these

32. The solution set of $\log_7\left(\dfrac{2x-6}{2x-1}\right) > 0$

(*a*) $\left(-\infty, \dfrac{1}{2}\right)$ (*b*) $\left(-\infty, \dfrac{1}{2}\right]$

(*c*) $\left(-\dfrac{1}{2}, \dfrac{1}{2}\right]$ (*d*) None of these

33. If $\log_{x-3}(2x-3)$ is a meaningful quantity then find the interval in which x must lie.

(*a*) $x \in (3, 4] \cup (4, \infty)$ (*b*) $x \in [3, 4) \cup (4, \infty)$

(*c*) $x \in (3, 4) \cup (4, \infty)$ (*d*) None of these

INTEGER TYPE QUESTIONS

34. If $\log_2(\log_3(\log_4(x))) = 0$ and $\log_3(\log_4(\log_2(y))) = 0$ and $\log_4(\log_2(\log_3(z))) = 0$ then the sum of x, y and z is

35. Let $\log_2 x + \log_4 x + \log_8 x = \log_k x$ for all $x \in R^+$. If $k = \sqrt[b]{a}$ where $a, b \in N$ then find the smallest positive value of $(a + b)$.

36. Find the value of the expression

$$6\left(\frac{2}{\log_4(2000)^6} + \frac{3}{\log_5(2000)^6}\right).$$

37. If $N = 7^{\log_{49} 900}, A = 2^{\log_2 4} + 3^{\log_2 4} + 4^{\log_2 2} - 4^{\log_2 3}$

$D = (\log_5 49)(\log_7 125)$

Find $P = \log_{\left(A - \frac{N}{10}\right)} |N + A + D + 6| - \log_5 2,$

38. If $a + b + c = 1$, $a^2 + b^2 + c^2 = 9$, $a^3 + b^3 + c^3 = 1$, then find value of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$.

39. If $a + b + c = 10$ and $ab + bc + ac = 20$ then find the value of $a^3 + b^3 + c^3 - 3abc$

40. If $(a - b)^3 + (b - c)^3 + (c - a)^3 = p(a - b)(b - c)(c - a)$, then $p =$

41. The value of $\sqrt{5\sqrt{5\sqrt{5\sqrt{5......}}}}$ is

42. If $x = \sqrt[3]{7 + 5\sqrt{2}} - \frac{1}{\sqrt[3]{7 + 5\sqrt{2}}}$, then the value of $x^3 + 3x - 14$ is equal to

43. If $\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{-1/3}\right]^{1/4} = 7^x$ then $-3x =$

44. Number of cyphers after decimal before a significant figure starts in $\left(\frac{5}{4}\right)^{-100}$ is equal to [**Use:** $\log_{10}2 = 0.3010$]

45. Number of real solution of $\log_5 [2 + \log_3 (x + 3)] = 0$ is

46. If $4^A + 9^B = 10^C$, where $A = \log_{16}4$, $B = \log_3 9$ & $C = \log_x 83$, then find x.

Parikshit (JEE Advanced Level)

SINGLE CORRECT TYPE QUESTIONS

1. The solution set of the inequality $\log_{\sin\left(\frac{\pi}{3}\right)}(x^2 - 3x + 2) \geq 2$ is

(*a*) $\left(\frac{1}{2}, 2\right)$ (*b*) $\left(1, \frac{5}{2}\right)$

(*c*) $\left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$ (*d*) $(1, 2)$

2. If $\sqrt{\log_4\{\log_3\{\log_2(x^2 - 2x + a)\}\}}$ is defined $\forall\, x \in R$, then the set of values of 'a' is

(*a*) $[9, \infty)$ (*b*) $[10, \infty)$ (*c*) $[15, \infty)$ (*d*) $[2, \infty)$

3. Number of integers, which satisfy the inequality, $\frac{(16)^{1/x}}{(2^{x+3})} > 1$ is equal to:

(*a*) Infinite (*b*) Zero (*c*) 1 (*d*) 4

4. The solution set of inequality $\frac{(3^x - 4^x) \cdot \ln(x + 2)}{x^2 - 3x - 4} \leq 0$ is

(*a*) $(-\infty, 0] \cup (4, \infty)$ (*b*) $(-2, 0] \cup (4, \infty)$

(*c*) $(-1, 0] \cup (4, \infty)$ (*d*) $(-2, -1) \cup (-1, 0] \cup (4, \infty)$

5. Number of values of x in the interval $(0, 5)$ satisfying the equation $\frac{\ln(\sqrt{\sqrt{x^2+1}+x}) + \ln(\sqrt{\sqrt{x^2+1}-x})}{\ln x} = x$, is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 0

6. The number of positive integers not satisfying the inequality $\log_2(4^x - 2.2^x + 17) > 5$.

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 1

7. If a, b, c are positive real numbers such that

$a^{\log_3 7} = 27; b^{\log_7 11} = 49$ and $c^{\log_{11} 25} = \sqrt{11}$.

The value of $\left(a^{(\log_3 7)^2} + b^{(\log_7 11)^2} + c^{(\log_{11} 25)^2}\right)$ equals

(*a*) 489 (*b*) 469 (*c*) 464 (*d*) 400

8. Find the value of x that satisfies the equation $\log\left(\frac{x^{1/x}}{x^{1/(x+1)}}\right) = \frac{1}{5050}$

(*a*) 1 (*b*) 10 (*c*) 100 (*d*) 1000

9. Sum of all the real solutions of the inequality

$\frac{(x^2+2)(\sqrt{x^2-16})}{(x^4+2)(x^2-9)} \leq 0$ is

(*a*) 5 (*b*) 4 (*c*) 8 (*d*) 0

10. The set of values of x satisfying simultaneously the inequalities $\frac{\sqrt{(x-8)(2-x)}}{\log_{0.3}\left(\frac{10}{7}(\log_2 5 - 1)\right)} \geq 0$ and $2^{x-3} - 31 > 0$ is:

(*a*) singleton set

(*b*) an empty set

(*c*) an infinite set

(*d*) a set consisting of exactly two elements.

11. If $|x - 3|^2 - 6\,|x - 3| - 23 < 4$, then

(*a*) $x \in (-12, 6)$

(*b*) 17 integers satisfy the inequality

(*c*) 11 non-negative integers satisfy the inequality

(*d*) 6 negative integers satisfy the inequality

MULTIPLE CORRECT TYPE QUESTIONS

12. Indicate all correct alternatives, where base of the log is 2.

The equation $x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2}$ has:

(*a*) At least one real solution

(*b*) Exactly three real solutions

(*c*) Exactly one irrational solution

(*d*) Imaginary roots

13. The equation $x^{\left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right]} = 3\sqrt{3}$ has

(*a*) Exactly three real solution

(*b*) At least one real solution

(*c*) Exactly one irrational solution

(*d*) Complex roots

14. Solution set of the inequality

$$(\log_2 x)^4 - \left(\log_{1/2}\frac{x^3}{8}\right)^2 + 9\log_2\left(\frac{32}{x^2}\right) < 4\left(\log_{1/2} x\right)^2 \text{ is}$$

$(a, b) \cup (c, d)$ then the correct statement is

(*a*) $a = 2b$ and $d = 2c$

(*b*) $b = 2a$ and $d = 2c$

(*c*) $\log_c d = \log_b a$

(*d*) there are 4 integers in (c, d)

15. Choose the correct from the following

(*a*) $\dfrac{81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409}\left((\sqrt{7})^{\frac{2}{\log_{25} 7}} - (125)^{\log_{25} 6}\right) = 1$

(*b*) $5^{\log_{1/5}\frac{1}{2}} + \log_{\sqrt{2}}\left(\dfrac{4}{\sqrt{7}+\sqrt{3}}\right) + \log_{1/2}\left(\dfrac{1}{(10+2\sqrt{21})}\right) = 6$

(*c*) $\sqrt{10^{2+\frac{1}{2}\log(16)}} = 20$

(*d*) None of these

16. Choose the correct from the following

(*a*) $\log_2(\log_{1/2}(x)) < 2$, for all $x \in \left(\frac{1}{16}, 1\right)$

(*b*) $\log_{1/2}(\log_3(x)) > 3$, for all $x \in (1, 3^{1/8})$

(*c*) $(\log_2(x) - 1)(\log_3(x) - 2) \le 0$, for all $x \in [2, 9]$

(*d*) $(\log_2(x) - 1)(\log_{1/2}(x) - 2) \le 0$, for all $x \in \left(0, \frac{1}{4}\right] \cup [2, \infty)$

17. The solution set of the system of equations $\log_3 x + \log_3 y = 2 + \log_3 2$ and $\log_{27}(x + y) = \frac{2}{3}$ is

(*a*) (6, 3) (*b*) (3, 6) (*c*) (6, 12) (*d*) (12, 6)

18. Consider the quadratic equation, $(\log_{10}8)x^2 - (\log_{10}5)x = 2(\log_2 10)^{-1} - x$. Which of the following quantities are irrational.

(*a*) sum of the roots

(*b*) product of the roots

(*c*) sum of the coefficients

(*d*) discriminant

COMPREHENSION BASED QUESTIONS

Comprehension (Q. No. 19 to 21): Let α and β are the solutions of the equation $\left(\sqrt{x}\right)^{\log_5 x - 1} = 5$ where $\alpha \in I$ and $\beta \in Q$ Then

[**Use:** $\log_{10}2 = 0.3010$, $\log_{10}3 = 0.4771$]

19. The number of significant digits before decimal in $(\alpha)^{10}$ is

(*a*) 13 (*b*) 14

(*c*) 15 (*d*) None of these

20. Number of zeros after decimal before a significant digit in $(\beta)^{10}$ is

(*a*) 5 (*b*) 7 (*c*) 8 (*d*) 6

21. The value of $(\beta)^{\log_{25} 9}$ is

(*a*) $\frac{1}{3}$ (*b*) 5 (*c*) $\frac{1}{5}$ (*d*) 9

MATCH THE COLUMN TYPE QUESTIONS

22. Match the Column:

	Column-I		Column-II
A.	The value(s) of x, which does not satisfy the equation $\log_2^2(x^2 - x) - 4\log_2(x - 1)\log_2 x = 1$, is (are)	p.	2
B.	The value of x satisfying the equation $2^{\log_2 e^{\ln 5^{\log_5 7^{\log_7 10^{\log_{10}(8x-3)}}}}} = 13$, is	q.	3
C.	The number $N = \left(\frac{1}{\log_2 \pi} + \frac{1}{\log_6 \pi}\right)$ is less than	r.	4
D.	Let $l = (\log_3 4 + \log_2 9)^2 - (\log_3 4 - \log_2 9)^2$ and $m = (0.8)\left(1 + 9^{\log_3 8}\right)^{\log_{65} 5}$ then $(l + m)$ is divisible by	s.	5
		t.	6

(*a*) A → r, t, s; B → q; C → r, s, q; D → q, r

(*b*) A → q, r, s, t; B → p; C → q, r; D → r, s

(*c*) A → q, r, s, t; B → p; C → q, r, s, t; D → p, r, s

(*d*) A → t; B → s; C → q, t; D → r, s

23. Match the column:

	Column-I		Column-II
A.	$\log_{\sin 30°}(\cos 60°) + 1$	p.	3
B.	$\log_{4/3}(1.\overline{3}) + 3$	q.	5
C.	$\log_{2-\sqrt{3}}(2+\sqrt{3}) + 6$	r.	4
D.	$\log_{\tan 20°} \tan 70° + 4$	s.	2
E.	$\log_{\cot 40°} \tan 50°$	t.	0
F.	$\log_{0.125}(8) + 8$	u.	–1
G.	$\log_{1.5}(0.\overline{6}) + 9$	v.	8
H.	$\log_{2.25}(0.\overline{4})$	w.	7
I.	$\log_{10}(0.\overline{9})$	x.	1

35. If the solution set of $\log_3 \frac{|x^2-4x|+3}{x^2+|x-5|} \geq 0$ is $\left(-\infty, -\frac{\alpha}{\beta}\right] \cup \left[\frac{1}{\alpha}, \alpha\right]$, then $\alpha\beta =$

36. For the equation

$(0.4)^{\log^2 x+1} = (6.25)^{2-p\log x}$

(base 10)

If $p = 2$, let number of real roots be m,

If $p = 3$, let number of real roots be n,

Then $m + n =$

37. If p is the smallest value of x satisfying the equation $2^x + \frac{15}{2^x} = 8$ then the value of 4^p is equal to

38. Positive numbers x, y and z satisfy $xyz = 10^{81}$ and $(\log_{10}x)(\log_{10}yz) + (\log_{10}y)(\log_{10}z) = 468$.

Find the value of

$\sqrt{(\log_{10} x)^2 + (\log_{10} y)^2 + (\log_{10} z)^2}$

39. If (x_1, y_1) and (x_2, y_2) are the solution of the system of equation

$\log_{225}(x) + \log_{64}(y) = 4$

$\log_x(225) - \log_y(64) = 1$,

then find the value of $\log_{30}(x_1y_1x_2y_2)$.

40. Suppose n be an integer greater than 1. Let $a_n = \frac{1}{\log_n 2002}$.

Suppose $b = a_2 + a_3 + a_4 + a_5$ and $c = a_{10} + a_{11} + a_{12} + a_{13} + a_{14}$. Then find the value of $(c - b)$.

41. If $\log_b a . \log_c a + \log_a b . \log_c b + \log_a c . \log_b c = 3$ (where a, b, c are different positive real numbers $\neq 1$), then find the value of $a\,b\,c$.

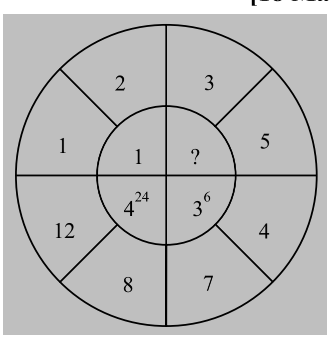
42. If the product of alll solutions of the equation $\frac{(2009)x}{2010} = (2009)^{\log_x(2010)}$ can be expressed in the lowest form as $\frac{m}{n}$ then the value of $(m - n)$ is

PYQ's (Past Year Questions)

INEQUALITIES

1. Let the point $(p, p + 1)$ lie inside the region $E = \left\{(x, y): 3 - x \leq y \leq \sqrt{9-x^2}, 0 \leq x \leq 3\right\}$. If the set of all values of p is the interval (a, b). then $b^2 + b - a^2$ is equal to _________ **[6 April, 2023 (Shift-I)]**

2. The missing value in the following figure is **[18 Mar, 2021 (Shift-I)]**



Use the logic which gives answer in single digit.

3. The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is **[10 April, 2019 (Shift-II)]**

(*a*) 3 (*b*) 2 (*c*) 4 (*d*) 1

LOGARITHM

4. The number of integral solutions x of $\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$ is **[11 April, 2023 (Shift-I)]**

(*a*) 6 (*b*) 8 (*c*) 5 (*d*) 7

5. If the sum of all the roots of the equation $e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$ is $\log_e p$, then p is equal to _________ .

[27 June, 2022 (Shift-I)]

6. The number of solutions of the equation $\log_4(x - 1) = \log_2(x - 3)$ is **[26 Feb, 2021 (Shift-I)]**

7. If for $x \in \left(0, \frac{\pi}{2}\right)$ $\log_{10} \sin x + \log_{10} \cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1), n > 0$, then the value of n is equal to: **[16 March, 2021 (Shift-I)]**

(*a*) 16 (*b*) 12 (*c*) 9 (*d*) 20

8. The inverse of $y = 5^{\log x}$ is: **[17 March, 2021 (Shift-I)]**

(*a*) $x = y^{\log 5}$ (*b*) $x = y^{\frac{1}{\log 5}}$

(*c*) $x = e^{\log_5 y}$ (*d*) $x = 5^{\frac{1}{\log y}}$

9. The sum of the roots of the equation,

[31 Aug, 2021 [Shift-II]

$x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$, is:

(*a*) $\log_2 12$ (*b*) $\log_2 13$

(*c*) $\log_2 11$ (*d*) $\log_2 14$

10. The number of solutions of the equation $\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x+1)^2 - 4 = 0$, $x > 0$, is.

[20 July, 2021 (Shift-II)]

11. The number of distinct solutions of the equation, $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|$ in the interval $[0, 2\pi]$, is __________ **[9 Jan, 2020 (Shift-I)]**

12. Let m be the minimum possible value of $\log_3(3^{y_1} + 3^{y_2} + 3^{y_3})$, where y_1, y_2, y_3 are real numbers for which $y_1 + y_2 + y_3 = 9$. Let M be the maximum possible value of $(\log_3 x_1 + \log_3 x_2 + \log_3 x_3)$, where x_1, x_2, x_3 are positive real numbers for which $x_1 + x_2 + x_3 = 9$. Then the value of $\log_2(m^3) + \log_3(M^2)$ is _____. **[JEE Adv, 2020]**

PW Challengers

1. If $\log_4(x + 2y) + \log_4(x - 2y) = 1$, then the minimum value of $|x| - |y|$ is ____.

2. Let a,b,c,d be positive integers and $\log_a b = \frac{3}{2}, \log_c d = \frac{5}{4}$. If $a - c = 9$, then $b - d =$

3. Let $x \in N$ such that $2^{1+[\log_2(x-2)]} - x = 20$. ([·] is G.I.F.) The smallest value of x, is

4. If $\sqrt{4+\sqrt{8-\sqrt{32+\sqrt{768}}}} = a\sqrt{2}\cos\left(\frac{11\pi}{b}\right)$, where a and b are natural numbers then find $a + b$.

5. Let $r_1, r_2, r_3 \ldots r_n$ be n positive integers, not necessarily distinct, such that $(x + r_1)(x + r_2)(x + r_1)\ldots(x + r_n) = x^n + 56x^{n-1} \ldots + 2009$ then the value of n is equal to

6. If $(a + 1)(b + 1)(c + 1)(d + 1) = 1$

$(a + 2)(b + 2)(c + 2)(d + 2) = 2$

$(a + 3)(b + 3)(c + 3)(d + 3) = 3$

$(a + 4)(b + 4)(c + 4)(d + 4) = 4$

Then the value of $(a + 5)(b + 5)(c + 5)(d + 5)$ is equal to.

7. Find sum of all possible natural numbers 'n' for which $\frac{5n^2 - 7n + 84}{n}$ is divisible by 5.

8. The value of

$$\left[2008 + \log_{\left(\frac{6561}{256}\right)}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots}}}\right)\right] \text{ is}$$

(where $[\cdot]$ is G.I.F.)

9. Let a, b and c be distinct non zero real numbers such that $\frac{1-a^3}{a} = \frac{1-b^3}{b} = \frac{1-c^3}{c}$. The value of $10(a^3 + b^3 + c^3)$, is

10. Match the Column:

	Column-I		Column-II
A.	Number of integral pair of the form (x, y) satisfying $\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}} = \frac{1}{\sqrt{20}}$ is/are equal to	p.	16
B.	Number of positive integral solutions of the equation $3x + 5y = 1008$ is/are equal to	q.	2
C.	Number of integers n such that $\sqrt{\frac{3n-5}{n+1}}$ is also an integer, is/are equal to	r.	0
D.	Number of integers n (positive, negative or 0) such that $n^2 + 73$ is divisible by $(n + 73)$, is/are equal to	s.	67
		t.	3

(*a*) A → t; B → s; C → q; D → p
(*b*) A → r; B → p; C → q; D → s
(*c*) A → q; B → p; C → r; D → s
(*d*) A → s; B → p; C → q; D → p

11. $\sqrt[3]{20+14\sqrt{2}} + \sqrt[3]{20-14\sqrt{2}} = a$ then find the absolute value of $a - 2023$.

12. If the sum of all real numbers x and y such that the following system of inequalities holds:

$$\left\{\begin{array}{l} 4^{-x} + 27^{-y} = \frac{5}{6} \\ \log_{27} y - \log_4 x \geq \frac{1}{6} \\ 27^y - 4^x \leq 1 \end{array}\right\}$$ is k then find 6 k.

13. Solve the inequality $|\log_2 x - 3| + |2^x - 8| \geq 9$.

(*a*) $x \in (0,1] \cup [4, \infty]$ (*b*) $x \in (0,2] \cup [4, \infty]$
(*c*) $x \in (0,1] \cup [3, \infty]$ (*d*) $x \in (0,1] \cup [2, \infty]$

14. Solve the inequality

$$\log_2(x^{12} + 3x^{10} + 5x^8 + 3x^6 + 1) < 1 + \log_2(x^4 + 1)$$

(*a*) $x \in \left(-\sqrt{\frac{-1+\sqrt{6}}{2}}, \sqrt{\frac{-1+\sqrt{5}}{2}}\right)$

(*b*) $x \in \left(-\sqrt{\frac{-1+\sqrt{5}}{2}}, \sqrt{\frac{-1+\sqrt{5}}{2}}\right)$

(*c*) $x \in \left(-\sqrt{\frac{-1+\sqrt{6}}{2}}, \sqrt{\frac{-1+\sqrt{6}}{2}}\right)$

(*d*) $x \in \left(-\sqrt{\frac{-1+\sqrt{7}}{2}}, \sqrt{\frac{-1+\sqrt{7}}{2}}\right)$

15. For what values of a, the inequality (for x)

$$\log_{\frac{1}{a}}\left(\sqrt{x^2 + ax + 5} + 1\right) \cdot \log_5\left(x^2 + ax + 6\right) + \log_a 3 \geq 0$$

has exactly one solution?

ANSWER KEY

CONCEPT APPLICATION

1. (*a*) **2.** (*a*) **3.** (*d*) **4.** (*c*) **5.** (*d*) **6.** (*i*) 14 (*ii*) 52 (*iii*) 194 **9.** [0] **10.** [–224]

11. $x^6 - y^6$ **12.** $p = 3/2, q = 1, r = 4/3$ **13.** (*d*) **14.** (*c*) **15.** (*b*) **16.** [60] **17.** [99] **18.** [4]

19. (*d*) **20.** [2*ab*] **21.** ϕ **22.** $x = \pm\sqrt{\frac{3}{5}}$ **23.** (*c*) **24.** $x \in (-\infty, -3) \cup [-2, 0] \cup [6, \infty)$

25. $x \in (-6, 0] \cup [2, 3] \cup (6, \infty) \cup \{4\}$ **26.** [6] **27.** (*d*) **28.** [1, –1] **29.** [$x \in (\sin 4, \sin 3) \cup [\sin 1, \sin 2]$]

30. $x \in [1, 2]$ **31.** {9} **32.** $x \in (2, 3]$

33. $\left(\frac{1}{5}, \frac{2}{5}\right)$ **34.** (243) **35.** [–5] **36.** [18]

37. [–1] **38.** (*i*) $(1, \infty)$ (*ii*) $[1, \infty)$ (*iii*) $(0, 1)$ (*iv*) $(0, 1]$ (*v*) $(0, 1)$ (*vi*) $(0, 1]$ (*vii*) $(1, \infty)$ (*viii*) $[1, \infty)$ (*ix*) $(3, \infty)$ (*x*) $[5/2, \infty)$

39. (*i*) $x \in \left(\frac{3}{2}, \frac{19}{2}\right)$ (*ii*) $x \in \left(\frac{2}{3}, \frac{17}{24}\right]$ (*iii*) $x \in (4^{16}, \infty)$ (*iv*) $x \in \left(0, \frac{1}{2}\right)$ **40.** {3, –3} **41.** {–2, 2} **43.** $\frac{12 - 4a}{3 + a}$

44. (*i*) $\left\{2^{\pm\sqrt{2}}\right\}$ (*ii*) $x = a^{-\log_5 2}$ (*iii*) {1/32, 2} (*iv*) {1} (*v*) {1}

47. [–1] **48.** [44] **49.** (*d*) **50.** (*b*, *c*)

51. (*i*) {–1, 5} (*ii*) {–3, –1, 7, 9} (*iii*) (14, –4, 0, 10, 2, 8) **52.** (–6, 8) **53.** (*d*) **54.** $(0, 2) \cup (4, \infty)$ **55.** $x \geq 1$

56. (*i*) $x \in (-\infty, 1) \cup (1, \infty)$ (*ii*) $x \in (-\infty, -5] \cup [5, \infty)$ (*iii*) $x \in (-7, 7)$ (*iv*) $x \in [-10, 10]$ (*v*) $x \in R$ (*vi*) $x \in \phi$ (*vii*) $x \in R$ (*viii*) $x \in R$ (*ix*) $x \in \phi$

57. (*i*) $x \in (-\infty, 0) \cup (2, \infty)$ (*ii*) $1 < x < 3$ (*iii*) $x \in (-2, -1) \cup (0, 1)$ (*iv*) $x \in [-1, 0] \cup [1, 2]$ (*v*) $-4/3 \leq x \leq 2$ (*vi*) $x \in \phi$

58. (*i*) $-1 \leq x \leq 5$ (*ii*) $x \in (-\infty, -3] \cup [9, \infty) \cup [-1, 7]$ (*iii*) $x \in [-4, 0] \cup [2, 8] \cup [10, 14]$

59. (*i*) $x \in \left[\frac{1}{5}, \frac{1}{4}\right] \cup \left[\frac{3}{4}, \frac{4}{5}\right]$ (*ii*) $x \in \left[\frac{2}{3}, 2\right]$ (*iii*) $x \in \left[\frac{2}{15}, \frac{2}{5}\right] \cup \left[\frac{6}{5}, \frac{22}{15}\right]$ (*iv*) $x \in (-3, -2) \cup (2, 3)$

60. (*i*) $x \in \{-11, -5, -1\}$ (*ii*) $x \in \{-8, -6, -2, -4, 0, 2, 4, 6, 8, 10, 12, 14, 16, 18\}$

61. (*i*) $x \in [1, 13]$ (*ii*) $x \in [-8, -6] \cup [-4, 0] \cup [2, 4] \cup [6, 8] \cup [10, 14] \cup [16, 18]$

62. (*i*) $x \in (-2, -1] \cup [1, 2)$ (*ii*) $x \in (-\infty, -3] \cup (-2, -1] \cup [3, \infty) \cup [1, 2)$ (*iii*) $x \in (-\infty, -3] \cup [3, \infty)$
(*iv*) $x \in (-\infty, -4] \cup [-1, 5] \cup [6, \infty)$ (*v*) $x \in (-\infty, -4] \cup [6, \infty] \cup \{2\}$

63. (*i*) $x \in [1, \infty]$ (*ii*) $x \in (0, \infty) \cup \{-1\}$ (*iii*) $x \in (-1, 0) \cup (0, 3)$ (*iv*) $x \in \phi$ (*v*) $x \in (2, 6)$

64. $[-6, \infty)$

PRARAMBH (TOPICWISE)

1. (*a*) **2.** (*c*) **3.** (*b*) **4.** (*b*) **5.** (*a*) **6.** (*d*) **7.** (*d*) **8.** (*c*) **9.** (*d*) **10.** (*c*)
11. (*b*) **12.** (*d*) **13.** (*c*) **14.** (*c*) **15.** (*d*) **16.** (*b*) **17.** (*d*) **18.** (*b*) **19.** (*a*) **20.** (*b*)
21. (*c*) **22.** (*a*) **23.** (*b*) **24.** (*d*) **25.** (*b*) **26.** (*a*) **27.** (*d*) **28.** (*c*) **29.** (*b*) **30.** (*d*)
31. (*b*) **32.** (*d*)

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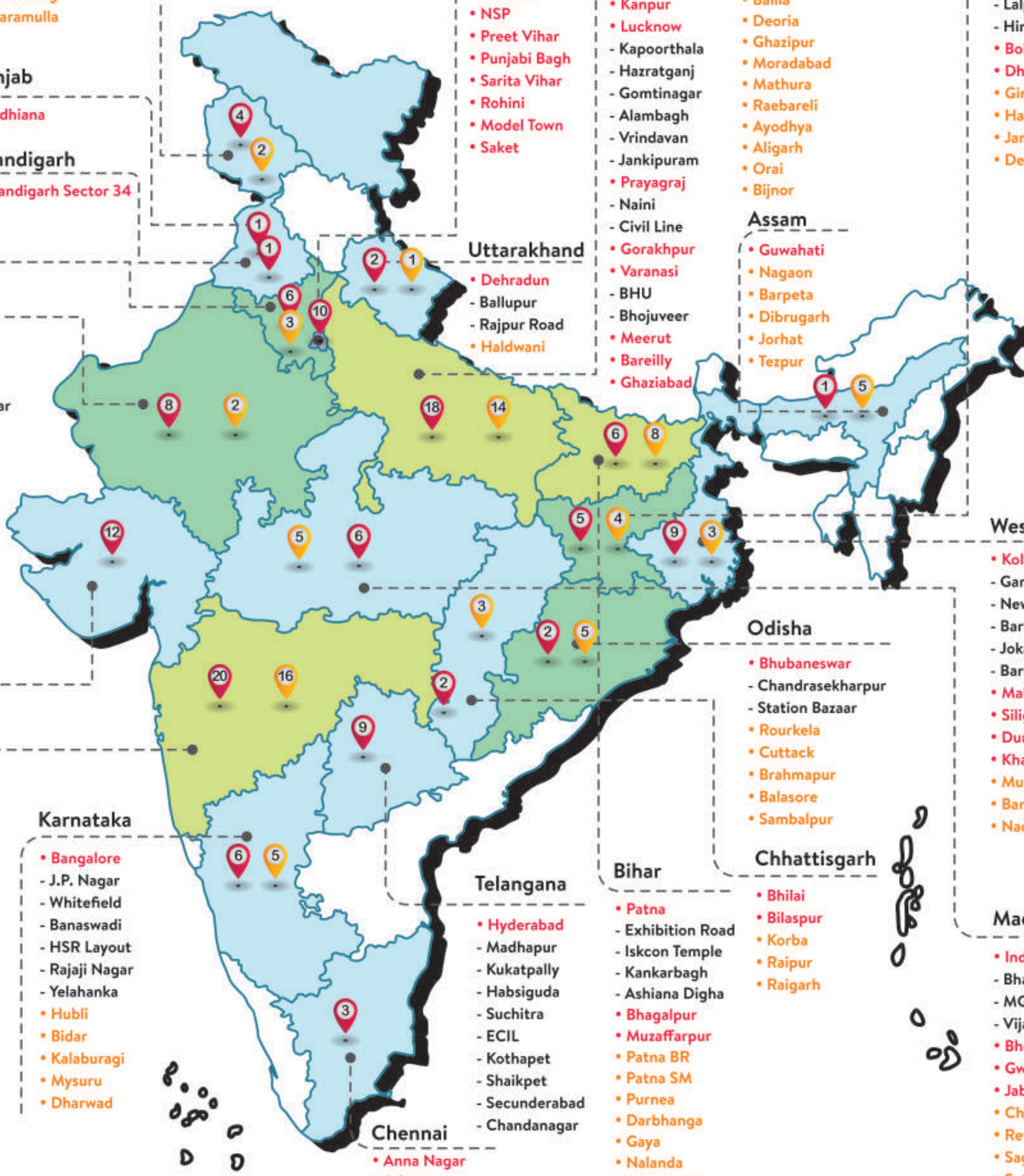

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  - Rajbagh
  - Barzulla
- Jammu
  - Paloura
  - Channi Rama
- Anantnag
- Baramulla

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- Faridabad
- Panchkula
- Hisar
- Rohtak
- Gurugram
  - South City
  - Sector 16
- Narnaul
- Sonipat
- Sirsa

Punjab
- Ludhiana

Chandigarh
- Chandigarh Sector 34

Delhi
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- Kalusarai
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- NSP
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- Punjabi Bagh
- Sarita Vihar
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  - Lalpur
  - Hinoo
- Bokaro
- Dhanbad
- Giridih
- Hazaribagh
- Jamshedpur
- Deoghar

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  - Ballupur
  - Rajpur Road
- Haldwani

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- Barpeta
- Dibrugarh
- Jorhat
- Tezpur

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  - Adajan
  - Vesu
- Bothra
  - Dahod
  - Satellite
- Bharuch
- Gandhinagar
- Ahmedabad
  - Satellite
  - Maninagar
  - Nikol
  - Gota
- Vadodara
  - Manjalpur
  - Vadiwadi

Rajasthan
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  - Tonk Road
  - RS Circle
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  - Habsiguda
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  - Shaikpet
  - Secunderabad
  - Chandanagar

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- Adyar
- Medvakkam

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  - Exhibition Road
  - Iskcon Temple
  - Kankarbagh
  - Ashiana Digha
- Bhagalpur
- Muzaffarpur
- Patna BR
- Patna SM
- Purnea
- Darbhanga
- Gaya
- Nalanda
- Samastipur
- Motihari

Chhattisgarh
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- Bilaspur
- Korba
- Raipur
- Raigarh

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