JEE ADVANCED 48 YEARS

2025-1978

CHAPTER-WISE & TOPIC-WISE SOLVED PAPERS

WITH RESPONSE TAGGING

Analyse the question-wise difficulty level in real-time with Correct (C), Wrong (W) and Unattempted (UA) questions response tagging provided by IIT-JEE

MATHEMATICS

Answer key verified from official website of JEE Advanced

JEE ADVANCED-6 Year (2024-19) Paper Analysis

Note: Due to unavailability of 2025 paper data we are unable to incorporate the 2025 analysis. As we are coming up with this book before the 2025 result.

Explanation of ★Unique★ Feature

C – Correct, W – Wrong, UA – Unattempted, PC – Partial Correct

C-6.97 W-42.98 UA-33.15 PC-16.9 represents the % of distribution of correct, wrong, unattempted and partial correct responses by students at any specific question in real time.(Data is taken from JEE Advanced website: https://jeeadv.ac.in/reports.html)

Classification helps students understand the varying **levels of difficulty**.

Easy	Medium	Hard
C > 20%	10% < C ≤ 20%	C ≤ 10%

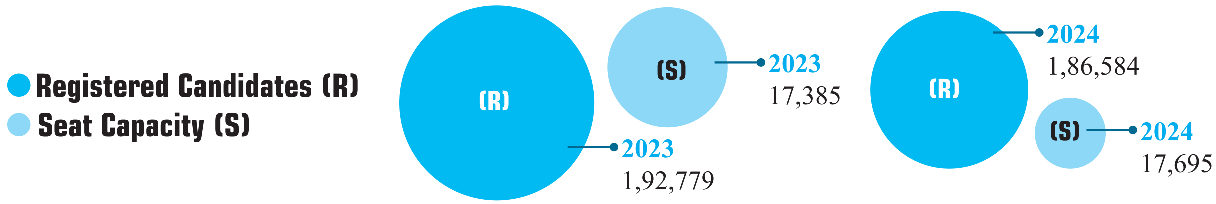
For Example

64. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix such that $PQ = kI$, where $k \in R$, $k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then **C-1.96 W-24.58 UA-66.2 PC-7.26** **[JEE Adv. 2016]**

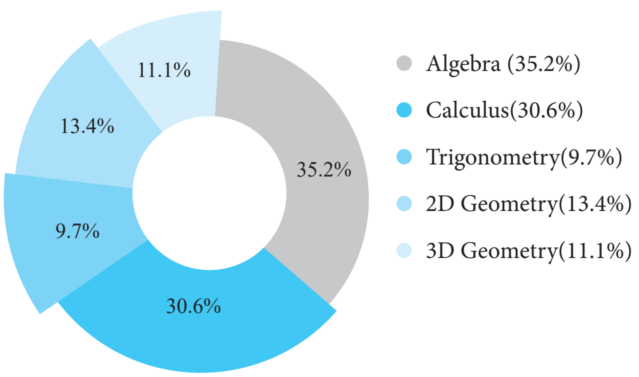
(*a*) $\alpha = 0, k = 8$ (*b*) $4\alpha - k + 8 = 0$

(*c*) $\det(P \text{ adj}(Q)) = 2^9$ (*d*) $\det(Q \text{ adj}(P)) = 2^{13}$

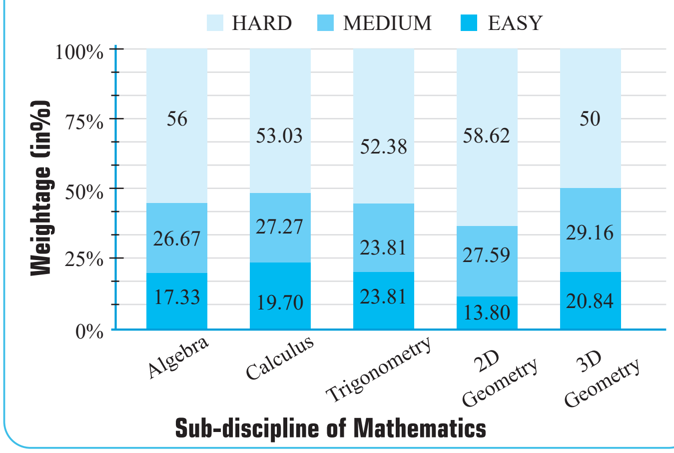
Question is considered HARD as Correct response recorded was less than **10%**



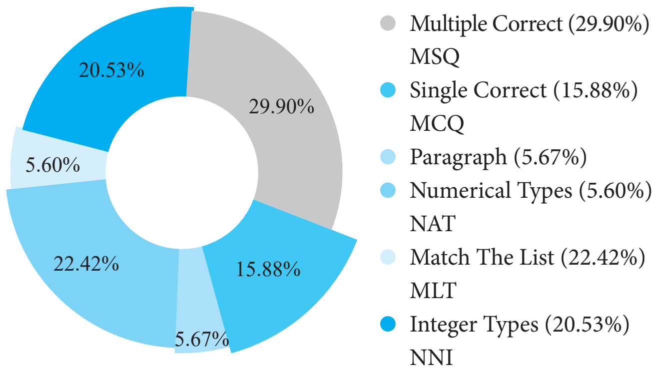
How Mathematics Marks Shape UP Across Sub-disciplines (2024-19)



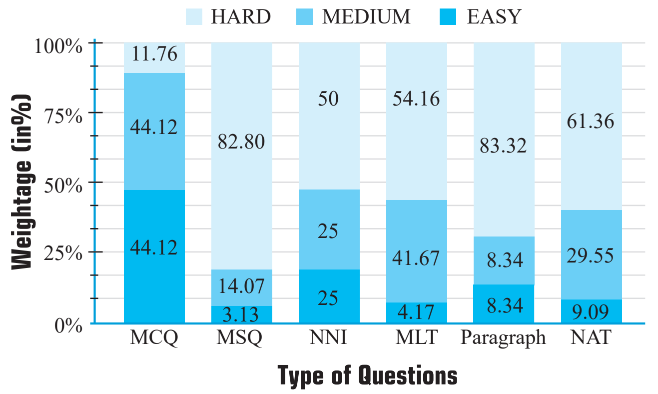
How Difficulty Level Vary in Subdiscipline (2024-19)



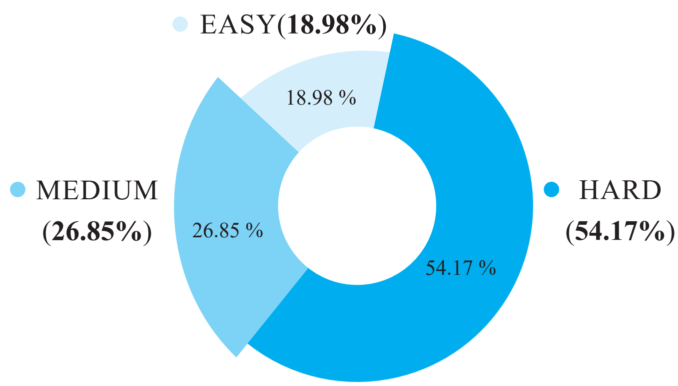
Distribution of Question Type



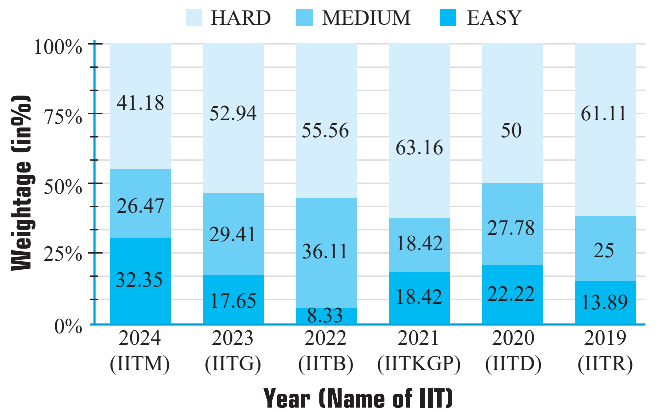
Weightage of Difficulty Level Based on Type of Questions



Distribution of Question Based on Difficulty



Distribution of Level of Questions Yearwise



Chapter wise Weightage and count of Difficulty level of JEE Advanced Questions

Chapters Name	EASY	MEDIUM	HARD	Chapterwise Weightage(in %)
Probability	3	6	14	**10.65**
Matrices And Determinants	4	2	11	**7.87**
Application of Derivatives	2	3	11	**7.41**
Integration	2	5	6	**6.02**
Circles	2	3	8	**6.02**
Complex Numbers	0	7	5	**5.56**
Three Dimensional Geometry	1	4	7	**5.56**
Vectors	4	3	5	**5.56**
Sets, Relation and Function	3	2	5	4.63
Limits of Functions	3	2	5	4.63
Permutations And Combinations	0	1	7	3.7
Solution of Triangles	1	0	7	3.7
Inverse Trigonometry Functions	1	3	3	3.24
Application of Integral	2	2	3	3.24
Sequence And Series	0	4	2	2.78
Ellipse	1	3	2	2.78

CONTENTS

HINTS AND SOLUTIONS

2025 JEE ADVANCED SOLVED PAPER

Mathematics Paper-1

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (*a*), (*b*), (*c*) and (*d*). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If **ONLY** the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases.

1. Let $\mathbb{R}$ denote the set of all real numbers. Let $a_i, b_i \in \mathbb{R}$ for $i \in \{1,2,3\}$.

Define the functions $f: \mathbb{R}\to\mathbb{R}$, $g: \mathbb{R}\to\mathbb{R}$, and $h: \mathbb{R}\to\mathbb{R}$ by:

$f(x) = a_1 + 10x + a_2x^2 + a_3x^3 + x^4$,

$g(x) = b_1 + 3x + b_2x^2 + b_3x^3 + x^4$,

$h(x) = f(x+1) - g(x+2)$.

If $f(x) \neq g(x)$ for every $x \in \mathbb{R}$, then the coefficient of x^3 in $h(x)$ is

(*a*) 8 (*b*) 2 (*c*) –4 (*d*) –6

2. Three students S_1, S_2, and S_3 are given a problem to solve. Consider the following events:

U: At least one of S_1, S_2, and S_3 can solve the problem,

V: S_1 can solve the problem, given that neither S_2 nor S_3 can solve the problem,

W: S_2 can solve the problem and S_3 cannot solve the problem,

T: S_3 can solve the problem.

For any event E, let P(*E*) denote the probability of *E*. If

$$P(U) = \frac{1}{2},\quad P(V) = \frac{1}{10},\quad and\ P(W) = \frac{1}{12}$$

then P(*T*) is equal to

(*a*) $\frac{13}{36}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{19}{60}$ (*d*) $\frac{1}{4}$

3. Let $\mathbb{R}$ denote the set of all real numbers. Define the function $f: \mathbb{R}\to\mathbb{R}$ by

$$f(x) = \begin{cases} 2 - 2x^2 - x^2 \sin\frac{1}{x} & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$$

Then which one of the following statements is TRUE?

(*a*) The function f is **NOT** differentiable at $x = 0$

(*b*) There is a positive real number δ, such that f is a decreasing function on the interval $(0, \delta)$

(*c*) For any positive real number δ, the function f is **NOT** an increasing function on the interval $(-\delta, 0)$

(*d*) $x = 0$ is a point of local minima of f

4. Consider the matrix

$$P = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Let the transpose of a matrix X be denoted by X^T. Then the number of 3×3 invertible matrices Q with integer entries, such that $Q^{-1} = Q^T$ and $PQ = QP$, is

(*a*) 32 (*b*) 8 (*c*) 16 (*d*) 24

SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (*a*), (*b*), (*c*) and (*d*). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks: –2 In all other cases.

- For example, in a question, if (*a*), (*b*) and (*d*) are the **ONLY** three options corresponding to correct answers, then
  choosing ONLY (*a*), (*b*) and (*d*) will get +4 marks;
  choosing ONLY (*a*) and (*b*) will get +2 marks;
  choosing ONLY (*a*) and (*d*) will get +2 marks;
  choosing ONLY (*b*) and (*d*) will get +2 marks;
  choosing ONLY (*a*) will get +1 mark;
  choosing ONLY (*b*) will get +1 mark;
  choosing ONLY (*d*) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –2 marks.

5. Let L_1 be the line of intersection of the planes given by the equations $2x + 3y + z = 4$ and $x + 2y + z = 5$.

Let L_2 be the line passing through the point $P(2,-1,3)$ and parallel to L_1. Let M denote the plane given by the equation

$2x + y - 2z = 6$.

Suppose that the line L_2 meets the plane M at the point Q. Let R be the foot of the perpendicular drawn from P to the plane M.

Then which of the following statements is (are) TRUE?

(a) The length of the line segment PQ is $9\sqrt{3}$

(b) The length of the line segment QR is 15

(c) The area of ΔPQR is $\frac{3}{2}\sqrt{234}$

(d) The acute angle between the line segments PQ and PR is $\cos^{-1}\left(\frac{1}{2\sqrt{3}}\right)$

6. Let $\mathbb{N}$ denote the set of all natural numbers, and $\mathbb{Z}$ denote the set of all integers. Consider the functions $f: \mathbb{N}\to\mathbb{Z}$ and $g: \mathbb{Z}\to\mathbb{N}$ defined by

$$f(n)=\begin{cases}(n+1)/2 & \text{if } n \text{ is odd,}\\ (4-n)/2 & \text{if } n \text{ is even}\end{cases} \quad \text{and}$$

$$g(n)=\begin{cases}3+2n & \text{if } n\geq 0,\\ -2n & \text{if } n<0.\end{cases}$$

Define $(g \circ f)(n) = g(f(n))$ for all $n \in \mathbb{N}$, and $(f \circ g)(n) = f(g(n))$ for all $n \in \mathbb{Z}$.

Then which of the following statements is (are) TRUE?

(a) $g \circ f$ is **NOT** one-one and $g \circ f$ is **NOT** onto

(b) $f \circ g$ is **NOT** one-one but $f \circ g$ is onto

(c) g is one-one and g is onto

(d) f is **NOT** one-one but f is onto

7. Let $\mathbb{R}$ denote the set of all real numbers. Let $z_1=1 + 2i$ and $z_2=3i$ be two complex numbers, where $i = \sqrt{-1}$. Let

$S = \{(x, y)\in \mathbb{R} \times \mathbb{R}: |x + iy - z_1| = 2|x + iy - z_2|\}$.

Then which of the following statements is (are) TRUE?

(a) S is a circle with centre $\left(-\frac{1}{3},\frac{10}{3}\right)$

(b) S is a circle with centre $\left(\frac{1}{3},\frac{8}{3}\right)$

(c) S is a circle with radius $\frac{\sqrt{2}}{3}$

(d) S is a circle with radius $\frac{2\sqrt{2}}{3}$

SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If **ONLY** the correct integer is entered;

  Zero Marks : 0 In all other cases.

8. Let the set of all relations R on the set $\{a,b,c,d,e,f\}$, such that R is reflexive and symmetric, and R contains exactly 10 elements, be denoted by S. Then the number of elements in S is ______________.

9. For any two points M and N in the XY-plane, let $\overrightarrow{MN}$ denote the vector from M to N, and $\vec{0}$ denote the zero vector. Let P, Q and R be three distinct points in the XY-plane. Let S be a point inside the triangle ΔPQR such that $\overrightarrow{SP}+5\overrightarrow{SQ}+6\overrightarrow{SR}=\vec{0}$.

Let E and F be the mid-points of the sides PR and QR, respectively. Then the value of $\frac{\text{length of the line segment } EF}{\text{length of the line segment } ES}$ is __________.

10. Let S be the set of all seven-digit numbers that can be formed using the digits 0, 1 and 2. For example, 2210222 is in S, but 0210222 is **NOT** in S.

Then the number of elements x in S such that at least one of the digits 0 and 1 appears exactly twice in x, is equal to ____________.

11. Let α and β be the real numbers such that

$$\lim_{x\to 0}\frac{1}{x^3}\left(\frac{\alpha}{2}\int_0^x \frac{1}{1-t^2}dt+\beta x\cos x\right)=2.$$

Then the value of $\alpha + \beta$ is ___________.

12. Let $\mathbb{R}$ denote the set of all real numbers. Let $f: \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) > 0$ for all $x \in \mathbb{R}$, and $f(x+y)=f(x)f(y)$ for all $x, y \in \mathbb{R}$.

Let the real numbers $a_1,a_2,\ldots,a_{50}$ be in an arithmetic progression. If $f(a_{31}) = 64 f(a_{25})$, and $\sum_{i=1}^{50} f(a_i)=3(2^{25}+1)$, then the value of $\sum_{i=6}^{30} f(a_i)$ is ___________.

13. For all $x > 0$, let $y_1(x)$, $y_2(x)$, and $y_3(x)$ be the functions satisfying

$$\frac{dy_1}{dx}-(\sin x)^2 y_1=0, y_1(1)=5,$$

$$\frac{dy_2}{dx}-(\cos x)^2 y_2=0, y_2(1)=\frac{1}{3},$$

$$\frac{dy_3}{dx}-\left(\frac{2-x^3}{x^3}\right)y_3=0, y_3(1)=\frac{3}{5e},$$

respectively. Then $\lim_{x\to 0^+}\frac{y_1(x)y_2(x)y_3(x)+2x}{e^{3x}\sin x}$ is equal to ______.

SECTION 4 (Maximum Marks: 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has Five entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : –1 In all other cases.

Mathematics Paper-2

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (*a*), (*b*), (*c*) and (*d*). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If **ONLY** the correct option is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : –1 In all other cases.

1. Let x_0 be the real number such that $e^{x_0} + x_0 = 0$. For a given real number α, define

$$g(x) = \frac{3xe^x + 3x - \alpha e^x - ax}{3(e^x + 1)} \text{ for all real numbers } x.$$

Then which one of the following statements is TRUE?

(*a*) for $\alpha = 2$, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 0$

(*b*) for $\alpha = 2$, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 1$

(*c*) for $\alpha = 3$, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 0$

(*d*) for $\alpha = 3$, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = \frac{2}{3}$

2. Let R denote the set of all real numbers. Then the area of the region

$$\left\{(x, y) \in \mathbb{R} \times \mathbb{R} : x > 0, y > \frac{1}{x}, 5x - 4y - 1 > 0, 4x + 4y - 17 < 0\right\} \text{ is}$$

(*a*) $\frac{17}{16} - \log_e 4$ (*b*) $\frac{33}{8} - \log_e 4$

(*c*) $\frac{57}{8} - \log_e 4$ (*d*) $\frac{17}{2} - \log_e 4$

3. The total number of real solutions of the equation

$$\theta = \tan^{-1}(2\tan\theta) - \frac{1}{2}\sin^{-1}\left(\frac{6\tan\theta}{9 + \tan^2\theta}\right) \text{ is}$$

(Here, the inverse trigonometric functions $\sin^{-1} x$ and $\tan^{-1} x$ assume values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ respectively.)

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

4. Let *S* denote the locus of the point of intersection of the pair of lines

$4x - 3y = 12\alpha$,

$4\alpha x + 3\alpha y = 12$,

where α varies over the set of non-zero real numbers. Let *T* be the tangent to *S* passing through the points (*p*, 0) and (0, *q*), *q* > 0, and parallel to the line $4x - \frac{3}{\sqrt{2}}y = 0$

Then the value of *pq* is

(*a*) $-6\sqrt{2}$ (*b*) $-3\sqrt{2}$ (*c*) $-9\sqrt{2}$ (*d*) $-12\sqrt{2}$

SECTION 2 (Maximum Marks: 16)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (*a*), (*b*), (*c*) and (*d*). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;

  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;

  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;

  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks: –2 In all other cases.
- For example, in a question, if (*a*), (*b*) and (*d*) are the **ONLY** three options corresponding to correct answers, then

  choosing ONLY (*a*), (*b*) and (*d*) will get +4 marks;

  choosing ONLY (*a*) and (*b*) will get +2 marks;

  choosing ONLY (*a*) and (*d*) will get +2 marks;

  choosing ONLY (*b*) and (*d*) will get +2 marks;

  choosing ONLY (*a*) will get +1 mark;

  choosing ONLY (*b*) will get +1 mark;

  choosing ONLY (*d*) will get +1 mark;

  choosing no option (i.e. the question is unanswered) will get 0 marks; and

  choosing any other combination of options will get –2 marks.

5. Let $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $P = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$. Let $Q = \begin{pmatrix} x & y \\ z & 4 \end{pmatrix}$ for some non-zero real numbers *x, y,* and *z*, for which there is a 2 × 2 matrix *R* with all entries being non-zero real numbers, such that *QR* = *RP*.

Then which of the following statements is (are) TRUE?

(*a*) The determinant of $Q - 2I$ is zero

(*b*) The determinant of $Q - 6I$ is 12

(*c*) The determinant of $Q - 3I$ is 15

(*d*) $yz = 2$

6. Let *S* denote the locus of the mid-points of those chords of the parabola $y^2 = x$, such that the area of the region enclosed between the parabola and the chord is $\frac{4}{3}$. Let *R* denote the region lying in the first quadrant, enclosed by the parabola $y^2 = x$, the curve *S*, and the lines $x = 1$ and $x = 4$.

Then which of the following statements is (are) TRUE?

(*a*) $(4, \sqrt{3}) \in S$ (*b*) $(5, \sqrt{2}) \in S$

(*c*) Area of $\mathscr{R}$ is $\frac{14}{3} - 2\sqrt{3}$ (*d*) Area of $\mathscr{R}$ is $\frac{14}{3} - \sqrt{3}$

Solutions Paper-1

1. (*b*) $f(x+1) = a_1 + 10(x+1) + a_2(x+1)^2 + a^3(x+1)^3 + (x+1)^4$

Coefficient of x^3 in $f(x+1) = a_3 + 4$

$g(x+2) = b_1 + 3(x+2) + b_2(x+2)^2 + b_3(x+2)^3 + (x+2)^4$

Coefficient of x^3 in $g(x+2) = b_3 + 8$

$\Rightarrow$ Coefficient of x^3 in $h(x)$ = coefficient of x^3 in $f(x+1)$ – coefficient of x^3 in $g(x+2)$

$= (a_3 + 4) - (b_3 + 8) = a_3 + 4 - b_3 - 8$

$= a_3 - b_3 - 4$

But $f(x) \neq g(x)\ \forall\ x$

$\Rightarrow f(x) - g(x) \neq 0$

$\Rightarrow f(x) - g(x) = 0$ have no real roots

$(a_1 - b_1) + 7x + (a_2 - b_2)x^2 + (a_3 - b_3)x^3 = 0$ have no real roots

Cubic equation will become zero at least one value of x.

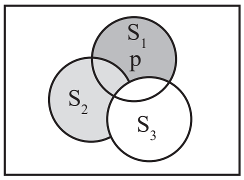
So, it will be quadratic.

$\Rightarrow a_3 - b_3 = 0$

$\Rightarrow$ Coefficient of x^3 in $h(x) = -4$

Option (*c*) is correct.

2.



$U = S_1 \cup S_2 \cup S_3$,

$$P(U) = \frac{1}{2}$$

$$P(V) = P\left(\frac{S_1}{(S_2 \cup S_3)'}\right) = \frac{1}{10}$$

Let $P\left(S \cap (S_2 \cup S_3)'\right) = p$

$$P(S_2 \cup S_3) = \frac{1}{2} - p$$

$$\left(\because P(S_1 \cup S_2 \cup S_3) = \frac{1}{2}\right)$$

$$\Rightarrow P(S_2 \cup S_3)' = 1 - \left(\frac{1}{2} - p\right) = \frac{1}{2} + p$$

$$P(V) = \frac{P\left(S_1 \cap (S_2 \cup S_3)'\right)}{P(S_2 \cup S_3)'} = \frac{p}{\frac{1}{2} + p} = \frac{1}{10}$$

$$\Rightarrow p = \frac{1}{18}$$

$$P(W) = P(S_2 \cap S_3') = \frac{1}{12}$$

$$P(T) = P(S_3) = \frac{1}{2} - \frac{1}{18} - \frac{1}{12}$$

$$= \frac{18 - 2 - 3}{36} = \frac{13}{36}.$$

3. $$f(x) = \begin{cases} 2 - 2x^2 - x^2 \sin\frac{1}{x} & x \neq 0 \\ 2 & x = 0 \end{cases}$$

$\lim\limits_{x \to 0} f(x) = 2 = f(0)$ $\quad\therefore f(x)$ is continous

Now, $$f'(0) = \lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

$$\lim_{h \to 0} \frac{2 - 2h^2 - h^2 \sin\left(\frac{1}{h}\right) - 2}{h}$$

$$= \lim_{h \to 0} \frac{-2h - h\sin\left(\frac{1}{h}\right)}{1} = 0$$

$$\left(\because \left(\sin\frac{1}{h}\right) \text{ oscillate between } -1 \text{ and } 1\right)$$

$\therefore f'(0) = 0$

$\therefore f(x)$ is differentiable at $x = 0$

$$RHD = \lim_{h \to 0^+} -h\left(2 + \sin\left(\frac{1}{h}\right)\right) < 0$$

$\Rightarrow f(x)$ is decreasing for a interval $(0, \delta)$, where $\delta > 0$

$$LHD = \lim_{h \to 0^-} -h\left(2 + \sin\left(\frac{1}{h}\right)\right) > 0$$

$\Rightarrow f(x)$ is increasing for a interval $(-\delta, 0)$ where $\delta > 0$

$\therefore x = 0$ is a point of maxima

option (b) is correct.

4. As $PQ = QP$

$$\therefore \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2a & 2b & 2c \\ 2d & 2e & 2f \\ 3g & 3h & 3i \end{bmatrix} = \begin{bmatrix} 2a & 2b & 3c \\ 2d & 2e & 3f \\ 2g & 2h & 3i \end{bmatrix}$$

$\Rightarrow c = 0, f = 0, h = 0, g = 0$

$$Q = \begin{bmatrix} a & b & 0 \\ d & e & 0 \\ 0 & 0 & i \end{bmatrix}$$

Given $QQ^T = I$

$$\Rightarrow \begin{bmatrix} a & b & 0 \\ d & e & 0 \\ 0 & 0 & i \end{bmatrix}\begin{bmatrix} a & d & i \\ b & e & 0 \\ 0 & 0 & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2 + b^2 & ad + be & 0 \\ ad + be & d^2 + e^2 & 0 \\ 0 & 0 & i^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow a^2 + b^2 = 1, ad + be = 0, d^2 + e^2 = 1, i^2 = 1.$

a	*b*	*d*	*e*	*i*
1	0	0	1, –1	1, –1
–1	0	0	1, –1	1, –1
0	1	1, –1	0	1, –1
0	–1	1, –1	0	1, –1

Total 16 matrices.

5. (*a, c*) $L_1 : 2x + 3y + z = 4$

$x + 2y + z = 5$

Let, line L_1 in standard form is $\vec{r} = \vec{a} + t\vec{b}$

The point on L_1 is $(-7, 6, 0)$

$$\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 2 & 1 \end{vmatrix} = \hat{i} - \hat{j} + \hat{k}$$

$$L_1 : \vec{r} = \left(-7\hat{i} + 6\hat{j} + 0\hat{k}\right) + t\left(\hat{i} - \hat{j} + \hat{k}\right).$$

or, $$L_1 : \frac{x+7}{1} = \frac{y-6}{-1} = \frac{z}{1}.$$

And equation of line L_2 is

$$\frac{x-2}{1} = \frac{y+1}{-1} = \frac{z-3}{1} = \lambda$$

Equation of plane $M : 2x + y - 2z = 6$

Let coordinate of $Q = (\lambda + 2, -\lambda - 1, \lambda + 3)$.

$\because Q$ lies on plane M

$2(\lambda + 2) + (-\lambda - 1) - 2(\lambda + 3) = 6$

$\Rightarrow -\lambda - 3 = 6$

$\Rightarrow \lambda = -9$

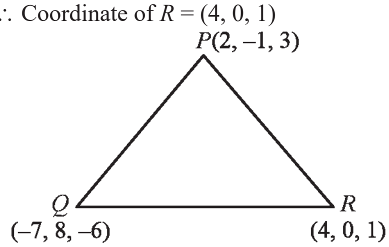
$\therefore$ coordinate of Q = $(-7, 8, -6)$.

For foot of perpendicular $R(x_1, y_1, z_1)$

$$\frac{x_1 - 2}{2} = \frac{y_1 + 1}{1} = \frac{z_1 - 3}{-2} = \frac{-(4 - 1 - 6 - 6)}{9}$$

$x_1 = 4, y_1 = 0, z_1 = 1.$

$\therefore$ Coordinate of $R = (4, 0, 1)$



$$PQ = \sqrt{9^2 + (-9)^2 + (9)^2} = 9\sqrt{3} \text{ units}$$

Area (ΔPQR)

$$= \frac{1}{2}\left|\overline{PQ} \times \overline{PR}\right| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -9 & 9 & -9 \\ 2 & 1 & -2 \end{vmatrix}$$

$$= \frac{3}{2}\sqrt{234}, \text{ sq units}$$

Let θ be acute angle between PQ and PR, then

$\Rightarrow \vec{w}\left(|\vec{v}|^2-1\right)-\vec{v}\left(\vec{v}\cdot\vec{w}\right)=\vec{0}$

$\Rightarrow |\vec{v}|^2=1$ and $\vec{v}\cdot\vec{w}=0$ (P) → (2)

$|\vec{u}||\vec{v}|=|\vec{\omega}| \Rightarrow |\vec{u}|\times 1=\sqrt{1+1+4}$

$\Rightarrow |\vec{u}|=\sqrt{6}$

$\vec{u}\cdot\vec{w}=0 \quad \Rightarrow \alpha+\beta-2\gamma=0$

and $-t\alpha+\beta+\gamma=0$ ... (*i*)

$\alpha-t\beta+\gamma=0$ ... (*ii*)

$\alpha+\beta-t\gamma=0$ ... (*iii*)

α, β, γ can not be simultaneously zero.

$$\Rightarrow \begin{vmatrix} -t & 1 & 1 \\ 1 & -t & 1 \\ 1 & 1 & -t \end{vmatrix}=0 \quad \Rightarrow t=-1, 2$$

When $t=-1, \quad \Rightarrow \alpha+\beta+\gamma=0.$

When $t=2, \Rightarrow -2\alpha+\beta+\gamma=0, \alpha-2\beta+\gamma=0$

$\Rightarrow \alpha=\dfrac{\beta+\gamma}{2},$

$\Rightarrow \dfrac{\beta+\gamma}{2}-2\beta+\gamma=0 \quad \Rightarrow \beta=\gamma$

$\Rightarrow \alpha=\beta=\gamma$

$\because |\vec{u}|=6 \quad \Rightarrow \sqrt{\alpha^2+\alpha^2+\alpha^2}=\sqrt{6}$

$\Rightarrow \alpha=\beta=\gamma=\sqrt{2}.$

Since, $\alpha=\sqrt{3}$

$\Rightarrow t=-1$

$\Rightarrow \alpha+\beta+\gamma=0$

and $\alpha+\beta-2\gamma=0$

$\Rightarrow \gamma=0$ $(Q)\to(1)$

$\alpha+\beta+\gamma=0$

$\Rightarrow (\beta+\gamma)=-\alpha \Rightarrow (\beta+\gamma)^2=\alpha^2=3,$

$(R)\to(4)$

If $\alpha=\sqrt{2} \quad \Rightarrow t=2$

$\Rightarrow t+3=(2)+3=5, (S)\to(5)$

Solutions Paper-2

1. **(*c*)** The given equation is $e^{x_0}+x_0=0$

For $\alpha=2$ the function $g(x)$ is defined as

$$g(x)=\frac{3x(e^x+1)-2(e^x+x)}{3(e^x+1)}$$

$$\Rightarrow g(x)=x-\frac{2}{3}\left(\frac{e^x+x}{e^x+1}\right)$$

The limit l_1 is evaluated as

$$\Rightarrow l_1=\lim_{x\to x_0}\left|\frac{g(x)+e^{x_0}}{x-x_0}\right|$$

$$=\lim_{x\to x_0}\left|\frac{x-\frac{2}{3}\left(\frac{e^x+x}{e^x+1}\right)+e^{x_0}}{x-x_0}\right| \left[\because e^{x_0}=-x_0\right]$$

$$l_1=\lim_{x\to x_0}\left|\frac{(x-x_0)-\frac{2}{3}\left(\frac{e^x+x}{e^x+1}\right)}{x-x_0}\right|$$

$$=\lim_{x\to x_0}\left|1-\frac{2}{3}\left(\frac{e^x+x}{e^x+1}\right).\frac{1}{x-x_0}\right| \quad ...(i)$$

$$\text{Let, } l_2=\lim_{x\to x_0}\frac{\left(\frac{e^x+x}{e^x+1}\right)}{x-x_0},\left(\frac{0}{0}\text{form}\right)$$

Using L'Hospital rule we get

$$\Rightarrow l_2=\lim_{x\to x_0}\frac{(e^x+1)(e^x+1)-(e^x+x)(e^x)}{(e^x+1)^2}=1$$

From equation (*i*)

$$\Rightarrow l_1=\left|1-\frac{2}{3}\right|=\frac{1}{3}$$

For the case whose $\alpha=3$, the function $g(x)$ is given by

$$g(x)=\frac{3x(e^x+1)-3(e^x+x)}{3(e^x+1)}$$

$$\Rightarrow g(x)=x-\frac{e^x+x}{e^x+1}$$

The limit l is defined as

$$l=\lim_{x\to x_0}\left|\frac{g(x)+e^{x_0}}{x-x_0}\right|$$

Substituting the expression for $g(x)$ and using the condition $e^{x_0}=-x_0$, the limit l is.

$$\Rightarrow l=\lim_{x\to x_0}\left|\frac{x-\left(\frac{e^x+x}{e^x+1}\right)+e^{x_0}}{x-x_0}\right|$$

$$l=\lim_{x\to x_0}\left|\frac{(x-x_0)-\frac{2}{3}\left(\frac{e^x+x}{e^x+1}\right)}{x-x_0}\right|$$

$$l=\lim_{x\to x_0}\left|1-\frac{\left(\frac{e^x+x}{e^x+1}\right)}{x-x_0}\right| \quad ...(ii)$$

$$\text{Let, } l_2=\lim_{x\to x_0}\left|\frac{\left(\frac{e^x+x}{e^x+1}\right)}{x-x_0}\right|,\left(\frac{0}{0}\text{form}\right)$$

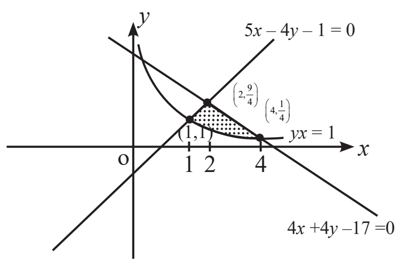
Using L'Hospital Rule

$\Rightarrow l_2=1$

From equation (*ii*)

$\Rightarrow l=|1-1|=0$

2. **(*b*)**



Required Area =

$$\int_1^2\left(\frac{5x-1}{4}-\frac{1}{x}\right)dx+\int_2^4\left(\frac{17-4x}{4}-\frac{1}{x}\right)dx$$

$$=\left[\frac{5x^2}{8}-\frac{x}{4}-\ln x\right]_1^2+\left[\frac{17x}{4}-\frac{x^2}{2}-\ln x\right]_2^4$$

$$=\left[\left(\frac{5(4)}{8}-\frac{2}{4}-\ln 2\right)-\left(\frac{5}{8}-\frac{1}{4}-\ln 1\right)\right]$$

$$+\left[(17-8-\ln 4)-\left(\frac{17}{2}-2-\ln 2\right)\right]$$

$$=\frac{33}{8}-\ln 4$$

3. **(*c*)** Let $\alpha=\dfrac{1}{2}\sin^{-1}\left(\dfrac{6\tan\theta}{9+\tan^2\theta}\right)$

$\Rightarrow \theta=\tan^{-1}(2\tan\theta)-\alpha$

$\Rightarrow \theta+\alpha=\tan^{-1}(2\tan\theta)$

$\Rightarrow \tan(\theta+\alpha)=2\tan\theta$

$$\Rightarrow \frac{\tan\theta+\tan\alpha}{1-\tan\alpha\tan\theta}=2\tan\theta \quad ......(i)$$

$$\Rightarrow \sin 2\alpha=\frac{6\tan\theta}{9+\tan^2\theta}=\frac{2\tan\alpha}{1+\tan^2\alpha}$$

$\Rightarrow 3\tan\theta+3\tan\theta\tan^2\alpha=9\tan\alpha+\tan\alpha\tan^2\theta$

$\Rightarrow 3(\tan\theta-3\tan\alpha)=\tan\alpha\tan\theta(\tan\theta-3\tan\alpha)$

$$\Rightarrow \tan\theta=\frac{3}{\tan\alpha}\text{ or } \tan\theta=3\tan\alpha$$

Case-I: $\tan\theta=3\tan\alpha$

1 QUADRATIC EQUATIONS

JEE-Advanced

Nature of Roots, Relation Between Roots and Coefficients

Single Correct

1. Suppose a, b denote the distinct real roots of the quadratic polynomial $x^2 + 20x - 2020 = 0$ and suppose c, d denote the distinct complex roots of the quadratic polynomial $x^2 - 20x + 2020 = 0$. Then the value of $ac(a-c) + ad(a-d) + bc(b-c) + bd(b-d)$ is
C-41.95 W-32.92 UA-25.13 **[JEE Adv. 2020]**
(*a*) 0 (*b*) 8000
(*c*) 8080 (*d*) 16000

2. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x\sec\theta + 1 = 0$, and α_2 and β_2 are the roots of the equation $x^2 + 2x\tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals
C-18.36 W-48.75 UA-32.89 **[JEE Adv. 2016]**
(*a*) $2(\sec\theta - \tan\theta)$ (*b*) $2\sec\theta$
(*c*) $-2\tan\theta$ (*d*) 0

3. In the quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then, the equation $p[p(x)] = 0$ has
C-14.2 W-56.37 UA-29.43 **[JEE Adv. 2014]**
(*a*) Only purely imaginary roots
(*b*) All real roots
(*c*) Two real and two purely imaginary roots
(*d*) Neither real nor purely imaginary roots

4. Let p and q be real numbers such that $p \neq 0, p^3 \neq q$ and $p^3 \neq -q$. If α and β are non-zero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is **[IIT-JEE 2010]**
(*a*) $(p^3+q)x^2 - (p^3+2q)x + (p^3+q) = 0$
(*b*) $(p^3+q)x^2 - (p^3-2q)x + (p^3+q) = 0$
(*c*) $(p^3-q)x^2 - (5p^3-2q)x + (p^3-q) = 0$
(*d*) $(p^3-q)x^2 - (5p^3+2q)x + (p^3-q) = 0$

5. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 - qx + r = 0$. Then, the value of r is **[IIT-JEE 2007]**
(*a*) $\frac{2}{9}(p-q)(2q-p)$ (*b*) $\frac{2}{9}(q-p)(2p-q)$
(*c*) $\frac{2}{9}(q-2p)(2q-p)$ (*d*) $\frac{2}{9}(2p-q)(2q-p)$

6. If a, b, c are the sides of a triangle ABC such that $x^2 - 2(a+b+c)x + 3\lambda(ab+bc+ca) = 0$ has real roots, then **[IIT-JEE 2006]**
(*a*) $\lambda < \frac{4}{3}$ (*b*) $\lambda > \frac{5}{3}$
(*c*) $\lambda \in \left(\frac{4}{3}, \frac{5}{3}\right)$ (*d*) $\lambda \in \left(\frac{1}{3}, \frac{5}{3}\right)$

7. If one root is square of the other root of the equation $x^2 + px + q = 0$, then the relation between p and q is **[IIT-JEE 2004]**
(*a*) $p^3 - q(3p-1) + q^2 = 0$ (*b*) $p^3 - q(3p+1) + q^2 = 0$
(*c*) $p^3 + q(3p-1) + q^2 = 0$ (*d*) $p^3 + q(3p+1) + q^2 = 0$

8. The set of all real numbers x for which $x^2 - |x+2| + x > 0$ **[IIT-JEE 2002]**
(*a*) $(-\infty, -2) \cup (2, \infty)$ (*b*) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$
(*c*) $(-\infty, -1) \cup (1, \infty)$ (*d*) $(\sqrt{2}, \infty)$

9. The number of solutions of $\log_4(x-1) = \log_2(x-3)$ is **[IIT-JEE 2001]**
(*a*) 3 (*b*) 1 (*c*) 2 (*d*) 0

10. For the equation $3x^2 + px + 3 = 0, p > 0$, if one of the root is square of the other, then p is equal to **[IIT-JEE 2000]**
(*a*) 1/3 (*b*) 1 (*c*) 3 (*d*) 2

11. If α and $\beta (\alpha < \beta)$ are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then **[IIT-JEE 2000]**
(*a*) $0 < \alpha < \beta$ (*b*) $\alpha < 0 < \beta < |\alpha|$
(*c*) $\alpha < \beta < 0$ (*d*) $\alpha < 0 < |\alpha| < \beta$

12. The equation $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$ has **[IIT-JEE 1997]**
(*a*) no solution (*b*) one solution
(*c*) two solutions (*d*) more than two solutions

13. The equation $x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2}$ has **[IIT-JEE 1989]**
(*a*) at least one real solution
(*b*) exactly three real solutions
(*c*) exactly one irrational solution
(*d*) complex roots

14. If α and β are the roots of $x^2+px+q=0$ and α^4,β^4 are the roots of $x^2-rx+s=0$, then the equation $x^2-4qx+2q^2-r=0$ has always **[IIT-JEE 1989]**

(a) two real roots

(b) two positive roots

(c) two negative roots

(d) one positive and one negative root

15. The equation $x-\dfrac{2}{x-1}=1-\dfrac{2}{x-1}$ has **[IIT-JEE 1984]**

(a) one solution (b) no solution

(c) two equal roots (d) infinitely many roots

16. For real x, the function $\dfrac{(x-a)(x-b)}{(x-c)}$ will assume all real values provided **[IIT-JEE 1984]**

(a) $a>b>c$ (b) $a<b<c$

(c) $a>c<b$ (d) $a\le c\le b$

17. The number of real solutions of the equation $|x|^2-3|x|+2=0$ is **[IIT-JEE 1981]**

(a) 4 (b) 1 (c) 3 (d) 2

18. Both the roots of the equation

$(x-b)(x-c)+(x-a)(x-c)+(x-a)(x-b)=0$ are always **[IIT-JEE 1980]**

(a) positive (b) negative

(c) Real (d) None of the above

19. Let $a>0, b>0$ and $c>0$. Then, both the roots of the equation $ax^2+bx+c=0$ **[IIT-JEE 1979]**

(a) are real and negative (b) have negative real parts

(c) have positive real parts (d) None of the above

Comprehension/Passage Based

Let p, q be integers and let α, β be the roots of the equation, $x^2-x-1=0$ where $\alpha\ne\beta$. For $n=0, 1, 2,.....$, let $a_n=p\alpha^n+q\beta^n$.

FACT: If a and b are rational numbers and $a+b\sqrt{5}=0$, then $a=0=b$.

20. a_{12}= C-28.59 W-67.14 UA-4.27 **[JEE Adv. 2017]**

(a) $a_{11}+2a_{10}$ (b) $2a_{11}+a_{10}$

(c) $a_{11}-a_{10}$ (d) $a_{11}+a_{10}$

FACT: If a and b are rational numbers and $a+b\sqrt{5}=0$ then $a=0=b$.

21. If $a_4=28$ then $p+2q=$ C-28.59 W-67.14 UA-4.27 **[JEE Adv. 2017]**

(a) 14 (b) 7 (c) 21 (d) 12

Assertion Reason/Statement Based

22. Let a, b, c, p, q be the real numbers. Suppose α, β are the roots of the equation $x^2+2px+q=0$ and $\alpha,\dfrac{1}{\beta}$ are the roots of the equation $ax^2+2bx+c=0$, where $\beta^2\notin\{-1,0,1\}$.

Statement-I $(p^2-q)(b^2-ac)\ge 0$

Statement-II $b\ne pa$ or $c\ne qa$ **[IIT-JEE 2008]**

(a) **Statement-I** is true, Statement-II is also true; Statement-II is the correct explanation of Statement-I

(b) **Statement-I** is true, Statement-II is also true; Statement-II is not the correct explanation of Statement-I

(c) **Statement-I** is true; Statement-II is false

(d) **Statement-I** is false; Statement-II is true.

Fill in the Blanks

23. The sum of all the real roots of the equation $|x-2|^2+|x-2|-2=0$ is...... . **[IIT-JEE 1997]**

24. If the products of the roots of the equation $x^2-3kx+2e^{2\ln k}-1=0$ is 7, then the roots are real for $k=$ .... **[IIT-JEE 1984]**

25. If $2+i\sqrt{3}$ is a root of the equation $x^2+px+q=0$, where p and q are real, then $(p,q)=(\ldots,\ldots)$. **[IIT-JEE 1982]**

26. The coefficient of x^{99} in the polynomial $(x-1)(x-2)\ldots(x-100)$ is.... **[IIT-JEE 1982]**

True/False

27. If $P(x)=ax^2+bx+c$ and $Q(x)=-ax^2+dx+c$, where $ac\ne 0$, then $P(x)\,Q(x)$ has atleast two real roots. **[IIT-JEE 1985]**

28. There exists a value of θ between 0 and 2π that satisfies the equation $\sin^4\theta-2\sin^2\theta+1=2$. **[IIT-JEE 1984]**

29. The equation $2x^2+3x+1=0$ has an irrational root. **[IIT-JEE 1983]**

Numerical/Integer Types

30. Let $f(x)=x^4+ax^3+bx^2+c$ be a polynomial with real coefficients such that $f(1)=-9$. Suppose that $i\sqrt{3}$ is a root of the equation $4x^3+3ax^2+2bx=0$, where $i=\sqrt{-1}$. If $\alpha_1,\alpha_2,\alpha_3$, and α_4 are all the roots of the equation $f(x)=0$, then $|\alpha_1|^2+|\alpha_2|^2+|\alpha_3|^2+|\alpha_4|^2$ is equal ________. C-9.3 W-60.69 UA-30.01 **[JEE Adv. 2024]**

Subjective

31. If $x^2-10ax-11b=0$ have roots c and d. $x^2-10cx-11d=0$ have roots a and b, then find $a+b+c+d$. **[IIT-JEE 2006]**

32. If $x^2+(a-b)x+(1-a-b)=0$ where $a,b\in R$, then find the values of a for which equation has unequal real roots for all values of b. **[IIT-JEE 2003]**

33. If α, β are the roots of $ax^2+bx+c=0,(a\ne 0)$ and $\alpha+\delta,\beta+\delta$ are the roots of $Ax^2+Bx+C=0,(A\ne 0)$ for some constant δ, then prove that **[IIT-JEE 2000]**

$$\frac{b^2-4ac}{a^2}=\frac{B^2-4AC}{A^2}$$

34. Let $f(x)=Ax^2+Bx+C$ where A, B, C are real numbers. Prove that if $f(x)$ is an integer whenever x is an integer, then the numbers 2A, A + B and C are all integers. Conversely, prove that if the numbers 2A, A + B and C are all integers, then $f(x)$ is an integer whenever x is an integer. **[IIT-JEE 1998]**

35. Find the set of all solutions of the equation $2^{|y|}-\left|2^{y-1}-1\right|=2^{y-1}+1$ **[IIT-JEE 1997]**

36. Find the set of all x for which $\dfrac{2x}{2x^2+5x+2}>\dfrac{1}{x+1}$ **[IIT-JEE 1987]**

37. Solve $|x^2+4x+3|+2x+5=0$ **[IIT-JEE 1987]**

38. Solve for $x:(5+2\sqrt{6})^{x^2-3}+(5-2\sqrt{6})^{x^2-3}=10$ **[IIT-JEE 1985]**

57. If $a+b+c=0$, then the quadratic equation $3ax^2+2bx+c=0$ has **[IIT-JEE 1983]**

(*a*) at least one root in (0, 1)

(*b*) one root in (2, 3)and the other in (–2, –1)

(*c*) imaginary roots

(*d*) None of the above

58. The largest interval for which $x^{12}-x^9+x^4-x+1>0$ is **[IIT-JEE 1982]**

(*a*) $-4<x\le 0$ (*b*) $0<x<1$

(*c*) $-100<x<100$ (*d*) $-\infty<x<\infty$

Multiple Correct

59. Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following interval(s) is/are a subset of S? **[JEE Adv. 2015]**

(*a*) $\left(-\frac{1}{2},-\frac{1}{\sqrt{5}}\right)$ (*b*) $\left(-\frac{1}{\sqrt{5}},0\right)$

(*c*) $\left(0,\frac{1}{\sqrt{5}}\right)$ (*d*) $\left(\frac{1}{\sqrt{5}},\frac{1}{2}\right)$

60. Let $a \in R$ and $f: R \to R$ be given by $f(x) = x^5 - 5x + a$.

Then, C-32.05 W-56.96 UA-10.99 **[JEE Adv. 2014]**

(*a*) $f(x)$ has three real roots, if $a > 4$

(*b*) $f(x)$ has only one real root, if $a > 4$

(*c*) $f(x)$ has three real roots, if $a < -4$

(*d*) $f(x)$ has three real roots, if $-4 < a < 4$

Comprehension/Passage Based

Passage 1

Consider the polynomial $f(x) = 1 + 2x + 3x^2 + 4x^3$. Let s be the sum of all distinct real roots of $f(x)$ and let $t = |s|$. **[IIT-JEE 2010]**

61. The real numbers s lies in the interval

(*a*) $\left(-\frac{1}{4},0\right)$ (*b*) $\left(-11,-\frac{3}{4}\right)$ (*c*) $\left(-\frac{3}{4},-\frac{1}{2}\right)$ (*d*) $\left(0,\frac{1}{4}\right)$

62. The area bounded by the curve $y = f(x)$ and the lines $x = 0$, $y = 0$ and $x = t$, lies in the interval **[IIT-JEE 2010]**

(*a*) $\left(\frac{3}{4},3\right)$ (*b*) $\left(\frac{21}{64},\frac{11}{16}\right)$

(*c*) (9, 10) (*d*) $\left(0,\frac{21}{64}\right)$

63. The function $f'(x)$ is **[IIT-JEE 2010]**

(*a*) increasing in $\left(-t,-\frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4},t\right)$

(*b*) decreasing in $\left(-t,-\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4},t\right)$

(*c*) increasing in $(-t, t)$

(*d*) decreasing in $(-t, t)$

Passage 2

If a continuous function f defined on the real line R, assumes positive and negative values in R, then the equation $f(x) = 0$ has a root in R. For example, if it is known that a continuous function f on R is positive at some point and its minimum values is negative, then the equation $f(x) = 0$ has a root in R, Consider $f(x) = ke^x - x$ for all real x where k is real constant.

64. The line $y = x$ meets $y = ke^x$ for $k \le 0$ at **[IIT-JEE 2007]**

(*a*) no point (*b*) one point

(*c*) two points (*d*) more than two points

65. The positive value of k for which $ke^x - x = 0$ has only one root is **[IIT-JEE 2007]**

(*a*) $\frac{1}{e}$ (*b*) 1 (*c*) e (*d*) $\log_e 2$

66. For $k > 0$ the set of all values of k for which $ke^x - x = 0$ **[IIT-JEE 2007]**

(*a*) $\left(0,\frac{1}{e}\right)$ (*b*) $\left(\frac{1}{e},1\right)$ (*c*) $\left(\frac{1}{e},\infty\right)$ (*d*) (0, 1)

Numerical/Integer Types

67. For $x \in R$, the number of real roots of the equation

$3x^2 - 4\left|x^2-1\right| + x - 1 = 0$ is ________ .

C-38.81 W-55.58 UA-5.61 **[JEE Adv. 2021]**

True/False

68. If $a < b < c < d$, then the roots of the equation $(x - a)(x - c) + 2(x - b)(x - d) = 0$ are real and distinct. **[IIT-JEE 1984]**

Subjective

69. Let a, b and c be real numbers with $a \ne 0$ and let α, β be the roots of the equation $ax^2+bx+c=0$. Express the roots of $a^3x^2+abcx+c^3=0$ in terms of α, β **[IIT-JEE 2001]**

70. Let $-1 \le p \le 1$. Show that the equation $4x^3 - 3x - p = 0$ has a unique root in the interval [1/2, 1] and identify it. **[IIT-JEE 2001]**

ANSWER KEY

1. (*d*)	2. (*c*)	3. (*d*)	4. (*b*)	5. (*d*)	6. (*a*)	7. (*a*)	8. (*b*)	9. (*b*)	10. (*c*)
11. (*b*)	12. (*a*)	13. (*b*)	14. (*a*)	15. (*b*)	16. (*d*)	17. (*a*)	18. (*c*)	19. (*b*)	20. (*d*)
21. (*d*)	22. (*b*)	23. [4]	24. [2]	25. [{–4, 7}]	26. [–5050]	27. [True]	28. [False]	29. [False]	30. [20]
43. (*b*)	44. [–1]	45. [False]	46. (*b*)	47. (*d*)	48. (*a*)	49. (*b*)	50. (*b*, *c*, *d*)	51. [2]	55. (*c*)
56. (*d*)	57. (*a*)	58. (*d*)	59. (*a*, *d*)	60. (*b*, *d*)	61. (*c*)	62. (*a*)	63. (*b*)	64. (*b*)	65. (*a*)
66. (*a*)	67. [4]	68. [True]							

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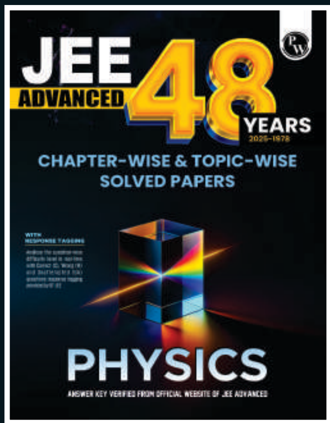

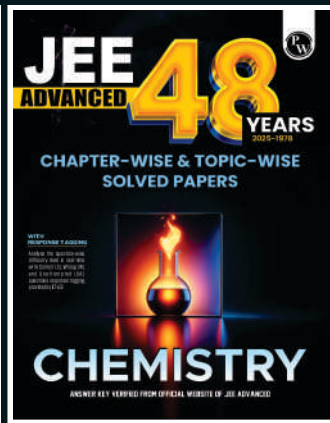

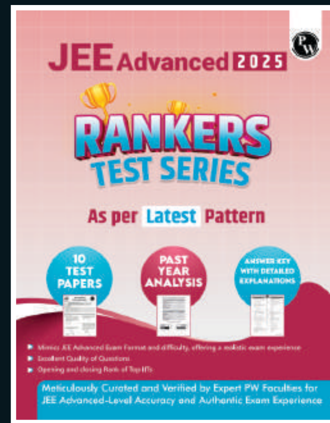

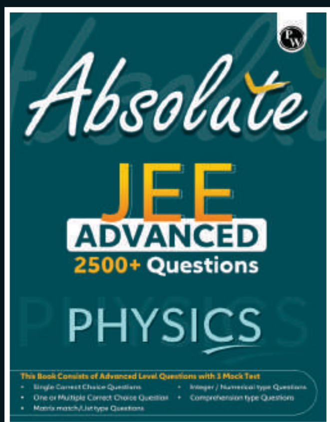

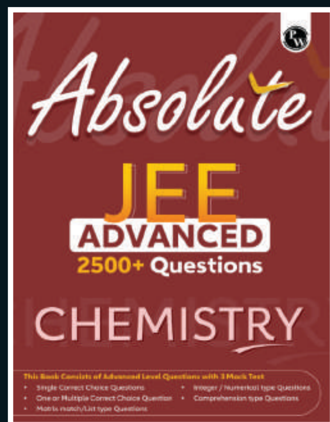

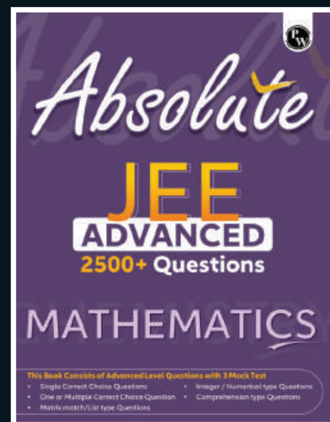

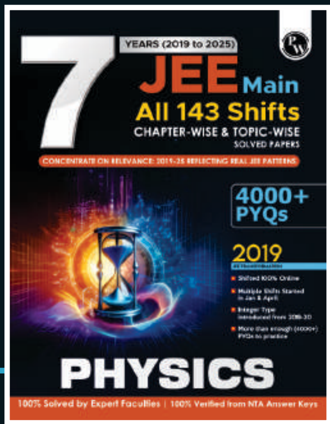

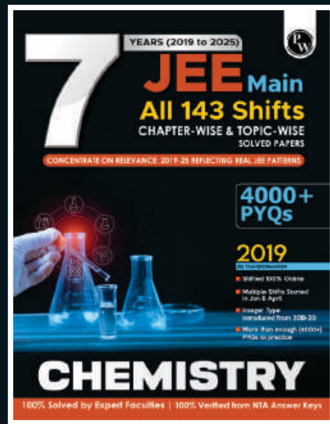

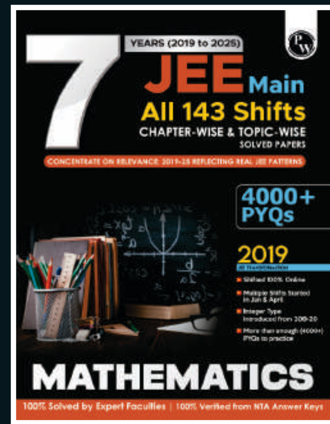

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