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CONTENTS

CHAPTER 3

TRIGONOMETRY AND ITS APPLICATIONS

1. Trigonometric Ratios for Right-Angled Triangle

In a right-angled triangle, we define the trigonometric ratios based on the relationship between the angles and sides. Let's consider a right triangle where:

- θ is one of the non-right angles.
- The **hypotenuse** is the side opposite the right angle.
- The **opposite** side is the side opposite to angle θ.
- The **adjacent** side is the next to θ, but not the hypotenuse.

The **six trigonometric ratios** are:

1. **Sine**$(\sin\theta) = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$
2. **Cosine**$(\cos\theta) = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$
3. **Tangent**$(\tan\theta) = \dfrac{\text{Opposite}}{\text{Adjacent}}$
4. **Cosecant**$(\csc\theta) = \dfrac{\text{Hypotenuse}}{\text{Opposite}}$
5. **Secant**$(\sec\theta) = \dfrac{\text{Hypotenuse}}{\text{Adjacent}}$
6. **Cotangent**$(\cot\theta) = \dfrac{\text{Adjacent}}{\text{Opposite}}$

2. Important Trigonometric Identities

There are several important identities in trigonometry that simplify expressions and solve problems. Some of the key ones are:

Pythagoras Identities:

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \tan^2\theta = \sec^2\theta$
3. $1 + \cot^2\theta = \csc^2\theta$

Reciprocal Identities

1. $\csc\theta = \dfrac{1}{\sin\theta}$
2. $\sec\theta = \dfrac{1}{\cos\theta}$
3. $\cot\theta = \dfrac{1}{\tan\theta}$

3. Co-Function Identities

These identities relate the trigonometric functions of complementary angles (i.e. angles summing to 90° or $\frac{\pi}{2}$)

1. $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$
2. $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$
3. $\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$
4. $\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$
5. $\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$
6. $\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$

4. Double Angle Identities

The identities express the trigonometric functions of double angles (i.e., 2θ)

1. $\sin(2\theta) = 2\sin\theta\cos\theta$
2. $\cos(2\theta) = \cos^2\theta - \sin^2\theta$ (also written as $\cos(2\theta) = 2\cos^2\theta - 1$ or $\cos(2\theta) = 1 - 2\sin^2\theta$)
3. $\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$

5. Sum and Difference Identities

The identities express the trigonometric functions of sums or differences of two angles:

1. $\sin(A+B) = \sin A\cos B + \cos A\sin B$
2. $\sin(A-B) = \sin A\cos B - \cos A\sin B$
3. $\cos(A+B) = \cos A\cos B - \sin A\sin B$
4. $\cos(A-B) = \cos A\cos B + \sin A\sin B$
5. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$
6. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

6. Product-to-Sum and Sum-to-Product Identities

These identities allow conversion between products of trigonometric functions and sums or differences:

1. **Product-to-Sum:**

$$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$$

$$\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$$

$$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$$

2. **Sum-to-Product:**

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

7. Trigonometric Ratios for Specific Angles

Certain standard angles have known trigonometric values:

1. $\sin 0° = 0$, $\cos 0° = 1$, $\tan 0° = 0$
2. $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\tan 30° = \frac{1}{\sqrt{3}}$
3. $\sin 45° = \frac{\sqrt{2}}{2}$, $\cos 45° = \frac{\sqrt{2}}{2}$, $\tan 45° = 1$
4. $\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$, $\tan 60° = \sqrt{3}$
5. $\sin 90° = 1$, $\cos 90° = 0$, $\tan 90°$ is undefined.

8. Angle Measurement

Trigonometric functions are often expressed in degrees or radians. The relationship between degrees and radians is given by:

$$1 \text{ radian} = \frac{180°}{\pi}$$

1. **Sine Rule (Law of Sines)**

The Sine Rule relates the sides of a triangle to the sines of its angles. It is especially useful when we have information about angles and opposite sides.

The **Sine Rule** states:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

where:

- a, b, c are the lengths of the sides of the triangle.
- A, B, C are the angles opposite to sides a, b, c respectively.

2. **Cosine Rule (Law of Cosines)**

The Cosine Rule relates the lengths of the sides of a triangle to the cosine of one of its angles. It is useful when we have two sides and the included angle or when all three sides are known.

The **Cosine Rule** states:

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

where:

- a, b, c are the lengths of the sides of the triangle.
- C is the angle opposite to side c.

Similarly, for the other sides, we have:

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

Solutions of a Trigonometric Equation

A **trigonometric equation** is an equation involving trigonometric functions like $\sin x$, $\cos x$, $\tan x$, etc., where the goal is to find the values of x that satisfy the equation.

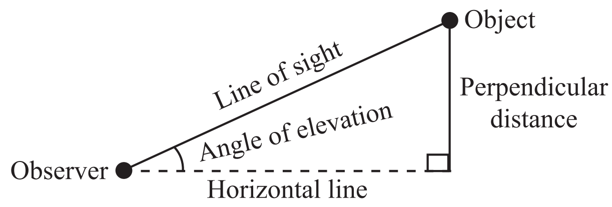
General Solutions – Trigonometric Equations

1. $\sin x = a$ $\quad x = n\pi + (-1)^n \sin^{-1} a, \quad n \in \mathbb{Z}$
2. $\cos x = a$ $\quad x = 2n\pi \pm \cos^{-1} a, \quad n \in \mathbb{Z}$
3. $\tan x = a$ $\quad x = n\pi + \tan^{-1} a, \quad n \in \mathbb{Z}$
4. $\cot x = a$ $\quad x = n\pi + \cot^{-1} a, \quad n \in \mathbb{Z}$
5. $\sec x = a$ $\quad x = 2n\pi \pm \sec^{-1} a, \quad n \in \mathbb{Z}$
6. $\csc x = a$ $\quad x = n\pi + (-1)^n \csc^{-1} a, \quad n \in \mathbb{Z}$

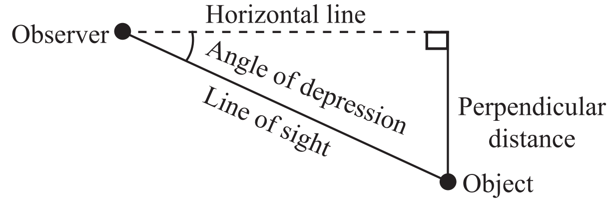
Heights and Distances – Key Concepts

1. **Angle of Elevation:** The angle formed by the line of sight of an observer looking up at an object.
2. **Angle of Depression:** The angle formed by the line of sight of an observer looking down at an object.
3. **Line of Sight:** The straight line that an observer would naturally follow to see the object in question.
4. **Horizontal Line:** The reference line from which the angles of elevation and depression are measured. It is typically assumed to be parallel to the ground.

Right Triangle for an Angle of Elevation



Right Triangle for an Angle of Depression



PRACTICE EXERCISE

1. Value of cos 510° cos 330° + sin 390° cos 120° is

(*a*) 0 (*b*) 1

(*c*) –1 (*d*) 2

2. If $\sin\theta = \dfrac{24}{25}$ and θ lies in the second quadrant, then sec θ + tan θ =

(*a*) –3 (*b*) –5

(*c*) –7 (*d*) –9

3. If $\sin A = \dfrac{3}{5}$ and $\cos B = \dfrac{-12}{13}$, where *A* and *B* both lie in second quadrant, then find the value of sin (*A* + *B*)

(*a*) $\dfrac{65}{56}$ (*b*) $-\dfrac{65}{56}$

(*c*) $-\dfrac{56}{65}$ (*d*) $\dfrac{56}{65}$

4. If $\cos\left(\dfrac{3\pi}{4}+x\right)-\cos\left(\dfrac{3\pi}{4}-x\right)=-\sqrt{m}\sin x$, then the value of *m* is _______

(*a*) 4 (*b*) 2

(*c*) 16 (*d*) 25

5. If $\operatorname{cosec}\theta - \cot\theta = \dfrac{1}{\sqrt{3}}$ where, $\theta \neq 0$, then what is the value of cos θ?

(*a*) 0 (*b*) $\dfrac{\sqrt{3}}{2}$

(*c*) $\dfrac{1}{2}$ (*d*) $\dfrac{1}{\sqrt{2}}$

6. What is the value of $\dfrac{\sin\theta+\cos\theta-\tan\theta}{\sec\theta+\operatorname{cosec}\theta-\cot\theta}$ when $\theta = \dfrac{3\pi}{4}$?

(*a*) 0 (*b*) 1

(*c*) –1 (*d*) None of the above

7. If α + β = 90°, then what is the value of $\sqrt{\cos\alpha\operatorname{cosec}\beta-\cos\alpha\sin\beta}$

(*a*) cos α (*b*) cos β

(*c*) sin β (*d*) sin α

8. If $\tan^2\theta = \dfrac{1-a^2}{1+a^2}$ then the value of $\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}+\dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

(*a*) $\dfrac{1}{1+a^2}$ (*b*) $\dfrac{1}{a^2}$

(*c*) $\dfrac{1}{1-a^2}$ (*d*) $\dfrac{2}{a^2}$

9. Find the value of $\operatorname{cosec}\left(-\dfrac{19\pi}{3}\right)$.

(*a*) $-\dfrac{2}{\sqrt{5}}$ (*b*) $\dfrac{2}{\sqrt{3}}$

(*c*) $-\dfrac{2}{\sqrt{3}}$ (*d*) None of these

10. Find the value of $2\sin^2\dfrac{\pi}{6}+\operatorname{cosec}^2\dfrac{7\pi}{6}\cos^2\dfrac{\pi}{3}$

(*a*) $\dfrac{1}{2}$ (*b*) 2

(*c*) $\dfrac{3}{4}$ (*d*) $\dfrac{3}{2}$

11. Find the value of $\cos^2\dfrac{\pi}{8}+\cos^2\dfrac{3\pi}{8}+\cos^2\dfrac{5\pi}{8}+\cos^2\dfrac{7\pi}{8}$

(*a*) –2 (*b*) 3

(*c*) 4 (*d*) 4

12. If $f(\alpha)=\sqrt{\sec^2\alpha-1}$, then what is $\dfrac{f(\alpha)+f(\beta)}{1-f(\alpha)f(\beta)}$ equal to?

(*a*) $f(\alpha-\beta)$ (*b*) $f(\alpha+\beta)$

(*c*) $f(\alpha)+f(\beta)$ (*d*) $f(\alpha\beta)$

13. $\dfrac{\tan\left(\dfrac{\pi}{4}+x\right)}{\tan\left(\dfrac{\pi}{4}-x\right)}$ is equal to

(*a*) $\left(\dfrac{1+\tan x}{1-\tan x}\right)^2$ (*b*) $\left(\dfrac{1-\tan x}{1+\tan x}\right)^2$

(*c*) $\left(\dfrac{1+\sin x}{1+\sin x}\right)^2$ (*d*) $\left(\dfrac{1-\sin x}{1+\sin x}\right)^2$

14. If $\tan\theta = \dfrac{p}{q}$, then the value of $\dfrac{p\sin\theta-q\cos\theta}{p\sin\theta+q\cos\theta}$ is

(*a*) $\dfrac{p^2-q^2}{p^2+q^2}$ (*b*) $\dfrac{p^2+q^2}{p^2-q^2}$

(*c*) 0 (*d*) 1

15. The value of $\tan\left(22\dfrac{1}{2}\right)^\circ$

(*a*) $\sqrt{2}+1$ (*b*) $\sqrt{2}-1$

(*c*) $\pm(\sqrt{2}-1)$ (*d*) $-\sqrt{2}-1$

16. What is the value of tan 15° + cot 15°?

(*a*) 1 (*b*) 2

(*c*) 4 (*d*) –2

17. Find the values of cot 15° is:

(*a*) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (*b*) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

(*c*) $\frac{1}{1-2\sqrt{3}}$ (*d*) $\frac{1}{1+2\sqrt{3}}$

18. If $\sin 2\alpha = \frac{3}{4}$ then $\sin^3\alpha + \cos^3\alpha$ equal to?

(*a*) $\frac{\sqrt{5}}{8}$ (*b*) $\frac{\sqrt{7}}{8}$

(*c*) $\frac{\sqrt{11}}{8}$ (*d*) None of these

19. If $\tan\alpha = \frac{1}{7}, \sin\beta = \frac{1}{\sqrt{10}}; 0 < \alpha, \beta < \frac{\pi}{2}$, then what is the value of $\cos(\alpha + 2\beta)$?

(*a*) $-\frac{1}{2}$ (*b*) $-\frac{1}{\sqrt{2}}$

(*c*) $\frac{1}{\sqrt{2}}$ d) $\frac{1}{2}$

20. If $A + B = \frac{\pi}{3}$ and $\cos A + \cos B = 1$ then $\cos(A - B) =$

(*a*) $\frac{1}{3}$ (*b*) $-\frac{1}{3}$

(*c*) $-\frac{2}{3}$ (*d*) None of these

21. Value of $\frac{\sin 5A - \sin 3A}{\cos 5A + \cos 3A}$ is:

(*a*) tan *A* (*b*) cot *A*

(*c*) sec *A* (*d*) cosec *A*

22. The value of $\cot\left(\frac{\pi}{4}+\theta\right)\cot\left(\frac{\pi}{4}-\theta\right)$ is

(*a*) –1 (*b*) 0

(*c*) 1 (*d*) Not defined

23. If $4\tan\theta\tan\phi = 3$, then $\frac{\cos(\theta-\phi)}{\cos(\theta+\phi)}$ is equal to

(*a*) 1 (*b*) 3

(*c*) 4 (*d*) 7

24. The value of $\sin\theta\sin\left(\frac{\pi}{3}-\theta\right)\sin\left(\frac{\pi}{3}+\theta\right)$ is

(*a*) $\frac{1}{2}\sin\theta$ (*b*) $\frac{1}{2}\sin 3\theta$

(*c*) $\frac{1}{4}\sin 3\theta$ (*d*) $\frac{1}{4}\sin\theta$

25. Let $\sin 2\theta = \cos 3\theta$, where θ is acute angle. What is the value of $1 + 4\sin\theta$?

(*a*) $\sqrt{3}$ (*b*) 2

(*c*) $\sqrt{5}$ (*d*) 3

26. If $\tan(45° + \theta) = 1 + \sin 2\theta$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$, then what is the value of $\cos 2\theta$?

(*a*) 0 (*b*) $\frac{1}{2}$

(*c*) 1 (*d*) 2

27. If θ = 18°, then what is the value of $4\sin^2\theta + 2\sin\theta$?

(*a*) –1 (*b*) 1

(*c*) 0 (*d*) 2

28. $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$ is equal to:

(*a*) 2 (*b*) 0

(*c*) 4 (*d*) 6

29. $\cos^2 48° - \sin^2 12°$ is equal to:

(*a*) $\frac{\sqrt{5}-1}{4}$ (*b*) $\frac{\sqrt{5}+1}{8}$

(*c*) $\frac{\sqrt{3}-1}{4}$ (*d*) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

30. The expression $\frac{\sin 8\theta\cos\theta - \sin 6\theta\cos 3\theta}{\cos 2\theta\cos\theta - \sin 3\theta\sin 4\theta}$

(*a*) tan θ (*b*) tan 2θ

(*c*) sin 2θ (*d*) cos 2θ

31. Write the general solution of the trigonometric equation $4\tan^2\theta = 3\sec^2\theta$

(*a*) $n\pi \pm \frac{\pi}{6}$ (*b*) $n\pi \pm \frac{\pi}{3}$

(*c*) $n\pi \pm \frac{5\pi}{6}$ (*d*) $n\pi \pm \frac{2\pi}{3}$

32. Solve $\tan 2x = -\cot\left(x + \frac{\pi}{3}\right)$

(*a*) $\frac{\pi}{6}$ (*b*) $n\pi + \frac{5\pi}{6}$

(*c*) $\frac{2\pi}{3}$ (*d*) None of these

33. If $\sin\left(\frac{\pi}{4}\cot\theta\right) = \cos\left(\frac{\pi}{4}\tan\theta\right)$, $\tan\theta =$

(*a*) $n\pi + \frac{\pi}{4}$ (*b*) $2n\pi \pm \frac{\pi}{4}$

(*c*) $n\pi - \frac{\pi}{4}$ (*d*) $2n\pi \pm \frac{\pi}{6}$

34. The general solution of $\sin^2\theta\sec\theta + \sqrt{3}\tan\theta = 0$ is

(*a*) $\theta = n\pi + (-1)^{n+1}\dfrac{\pi}{3}$

(*b*) $\theta = n\pi, n \in \mathbb{Z}$

(*c*) $\theta = n\pi + (-1)^{n+1}\dfrac{\pi}{6}, n \in \mathbb{Z}$

(*d*) $\theta = \dfrac{n\pi}{2}, n \in \mathbb{Z}$

35. If $r\sin\theta = 3$, $r = 4(1 + \sin\theta)$, $0 \le \theta \le 2\pi$, then $\theta =$

(*a*) $\dfrac{\pi}{6}, \dfrac{\pi}{3}$ (*b*) $\dfrac{\pi}{6}, \dfrac{5\pi}{6}$

(*c*) $\dfrac{\pi}{3}, \dfrac{\pi}{4}$ (*d*) $\dfrac{\pi}{2}, \pi$

36. If $\tan\theta + \tan 2\theta + \tan 3\theta = \tan\theta\tan 2\theta\tan 3\theta$ then the general value of θ is

(*a*) $n\pi$ (*b*) $\dfrac{n\pi}{6}$ (where n is even)

(*c*) $n\pi \pm \dfrac{\pi}{3}$ (*d*) $\dfrac{n\pi}{2}$

37. If $\tan 3\theta + \tan\theta = 2\tan 2\theta$, then θ is equal to $(n \in \mathbb{Z})$

(*a*) $n\pi$ (*b*) $\dfrac{n\pi}{4}$

(*c*) $\dfrac{n\pi}{2}$ (*d*) None of these

38. Find the value of x, of the given following equation $2\cos^2 x + 3\sin x = 0$

(*a*) $n\pi + (-1)^n\dfrac{\pi}{6}$ (*b*) $n\pi + (-1)^n\dfrac{5\pi}{6}$

(*c*) $n\pi + (-1)^n\dfrac{7\pi}{6}$ (*d*) $n\pi + (-1)^n\dfrac{\pi}{3}$

39. If $\sin x + \cos x = 1 + \sin x \cdot \cos x$, then x is

(*a*) $2n\pi, 2n\pi + \dfrac{\pi}{2}, n \in \mathbb{Z}$

(*b*) $2n\pi, n\pi + \dfrac{\pi}{4}, n \in \mathbb{Z}$

(*c*) $2n\pi - \dfrac{\pi}{2}, n\pi + \dfrac{\pi}{4}, n \in \mathbb{Z}$

(*d*) NOT

40. Find the value of x: $(2\sin x - \cos x)(1 + \cos x) = \sin^2 x$

(*a*) $(2n+1)\dfrac{\pi}{2}, n\pi + (-1)^n\dfrac{\pi}{6}$

(*b*) $(2n+1)\pi, n\pi + (-1)^{n+1}\dfrac{\pi}{6}$

(*c*) $(2n+1)\pi, n\pi + (-1)^n\dfrac{\pi}{6}$

(*d*) $(2n+1)\dfrac{\pi}{2}, n\pi + (-1)^{n+1}\dfrac{\pi}{6}$

41. Solution of the equation $\sin x + \cos x = \sqrt{2}$ is equal to

(*a*) $2n\pi - \dfrac{\pi}{4}$ (*b*) $n\pi + \dfrac{\pi}{4}$

(*c*) $2n\pi + \dfrac{\pi}{4}$ (*d*) $n\pi - \dfrac{\pi}{4}$

42. If a ΔABC have side a, b, c then $\dfrac{b^2 - c^2}{a^2}$ is equal to

(*a*) $\dfrac{\sin(B-C)}{\sin A}$ (*b*) $\dfrac{\sin(B-C)}{\sin(B+C)}$

(*c*) $\dfrac{\sin(B+C)}{\sin(B-C)}$ (*d*) None of these

43. The sides of a triangle are 60, 61 and 11. find the value of its circum radius.

(*a*) 30.5 (*b*) 30

(*c*) 25.5 (*d*) 22

44. If x, y, z are the length of the triangle and $x \le y \le z$, $x^2 + y^2 < z^2$ then the triangle is

(*a*) Acute angle triangle (*b*) Obtuse angle triangle

(*c*) Right angle triangle (*d*) None of these

45. The angle of elevation of a tower of height 30m from a point A due South of it is 60° and from a point B due East of A is 30°. Find the distance between A and B

(*a*) $20\sqrt{6}$ m (*b*) 10 m

(*c*) $10\sqrt{6}$ m (*d*) 20 m

46. From a point 10 m above a lake the angle of elevation of a cloud is 45°. The height of the cloud is $10\sqrt{3}$ m then the angle of depression of its reflection is β what is value of $\tan\beta$

(*a*) $(2-\sqrt{3})$ (*b*) $(1+\sqrt{3})$

(*c*) $(1-\sqrt{3})$ (*d*) $(2+\sqrt{3})$

47. A person was standing on a road near a mall. He was 1425 m away from the mall and able to see the top of the mall from the road in such a way that the top of a tree, which is in between in and the mall, we exactly in line of sight with the top of the mall. The height of the tree is 10 m and it is 30 m away from him. How tall (in m) is the mall?

(*a*) 564 m (*b*) 470 m

(*c*) 475 m (*d*) 467 m

48. If the angle of elevation of the sun changes from 30° to 45°, the length of the shadow of a pillar decreases by 20 meters. The height of the pillar is (in meters)

(*a*) $10(\sqrt{3}+1)$ (*b*) $10(\sqrt{3}-1)$

(*c*) $20(\sqrt{3}+1)$ (*d*) $20(\sqrt{3}-1)$

49. In $\Delta ABC, \dfrac{\sin B}{\sin(A+B)} =$

(a) $\dfrac{b}{a+b}$ (b) $\dfrac{b}{c}$

(c) $\dfrac{c}{b}$ (d) None of these

50. If the sides of a triangle are proportional to the cosines of the opposite angles then the triangle is

(a) right angled (b) equilateral

(c) obtuse angled (d) none of these

ANSWERS

1. (c)	2. (c)	3. (c)	4. (b)	5. (c)
6. (b)	7. (d)	8. (d)	9. (c)	10. (d)
11. (d)	12. (b)	13. (a)	14. (a)	15. (b)
16. (c)	17. (b)	18. (d)	19. (c)	20. (b)
21. (a)	22. (c)	23. (d)	24. (c)	25. (c)
26. (c)	27. (b)	28. (b)	29. (b)	30. (b)
31. (b)	32. (b)	33. (a)	34. (b)	35. (b)
36. (b)	37. (a)	38. (c)	39. (a)	40. (c)
41. (c)	42. (b)	43. (a)	44. (b)	45. (a)
46. (d)	47. (c)	48. (a)	49. (b)	50. (b)

EXPLANATION

1. (c) We have to find the value of

$\cos 510° \cos 330° + \sin 390° \cos 120°$

We will use formula,

$\cos(360° + x) = \cos x$

$\sin(360° - x) = \sin x$

$\cos(90° + x) = -\sin x$

So,

$\cos 510° \cos 330° + \sin 390° \cos 120°$

$= \cos(360° + 150°)\cos(360° - 30°)$

$+ \sin(360° + 30°)\cos(90° + 30°)$

$= \cos(150°)(\cos 30°) + \sin 30°(-\sin 30°)$

$= \cos(90° + 60°)(\cos 30°) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)$

$= (-\sin 60°)\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{4}$

$= -\frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{4}$

$= -\frac{3}{4} - \frac{1}{4}$

$= \frac{-3-1}{4}$

$= -1$

Thus the value of

$\cos 510° \cos 330° + \sin 390° \cos 120°$ is -1

2. (c) Given that $\sin\theta = \frac{24}{25}$ and θ lies in second quadrant so the value of $\sec\theta + \tan\theta$.

As we know that $\sec\theta$, $\tan\theta$ in second quadrant negative so,

$\sec\theta + \tan\theta = -\frac{25}{7} - \frac{24}{7}$

$= -\frac{25+24}{7}$

$= -\frac{49}{7}$

$= -7$

Hence, the value of $\sec\theta + \tan\theta$ is -7.

3. (c) Given that if $\sin A = \frac{3}{5}$, $\cos B = -\frac{12}{13}$ and both A, B lies in second quadrant then the value of $\sin(A+B)$:

As we know that sin in second quadrant is positive and cos in second quadrant is negative then,

$\cos A = -\frac{4}{5}, \sin B = \frac{5}{13}$

Now, by using formula

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

Substitute the given values and simplify,

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \frac{3}{5}\left(-\frac{12}{13}\right) + \left(-\frac{4}{5}\right)\frac{5}{13}$

$= -\frac{36}{65} - \frac{20}{65}$

$= -\frac{36+20}{65}$

$= -\frac{56}{65}$

Thus the value of $\sin(A+B)$ is $-\frac{56}{65}$.

4. (b) Given that if

$\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{m}\sin x$

then find the value of x,

By using formula

$\cos(x+y) = \cos x \cos y - \sin x \sin y$,

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{m}\sin x$

$\cos\frac{3\pi}{4}\cos x - \sin\frac{3\pi}{4}\sin x - \cos\frac{3\pi}{4}\cos x$

$-\sin\frac{3\pi}{4}\sin x = -\sqrt{m}\sin x$

$$-2\sin\frac{3\pi}{4}\sin x = -\sqrt{m}\sin x$$

$$2\sin\frac{3\pi}{4}\sin x = \sqrt{m}\sin x$$

$$2\sin\frac{3\pi}{4}\sqrt{m}$$

$$2\times\frac{1}{\sqrt{2}} = \sqrt{m}$$

$$\sqrt{2} = \sqrt{m}$$

$m = 2$

Thus the value of m is 2.

5. (*c*) Given $\operatorname{cosec}\theta - \cot\theta - \dfrac{1}{\sqrt{3}}$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow$ $(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$

$\therefore$ $\operatorname{cosec}\theta + \cot\theta = \sqrt{3}$ …(*ii*)

Adding (*i*) and (*ii*), we get:

$$2\operatorname{cosec}\theta = \sqrt{3} + \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \operatorname{cosec}\theta = \frac{2}{\sqrt{3}} \Rightarrow \quad \sin\theta = \frac{\sqrt{3}}{2}$$

Now, $\cos\theta = \sqrt{1-\sin^2\theta}$

$$\therefore \quad \cos\theta = \frac{1}{2}$$

6. (*b*) $y = \dfrac{\sin\theta + \cos\theta - \tan\theta}{\sec\theta + \operatorname{cosec}\theta - \cot\theta}$

At $\theta = \dfrac{3\pi}{4}$

$$y = \frac{\sin\frac{3\pi}{4} + \cos\frac{3\pi}{4} - \tan\frac{3\pi}{4}}{\sec\frac{3\pi}{4} + \operatorname{cosec}\frac{3\pi}{4} - \cot\frac{3\pi}{4}}$$

$$= \frac{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + 1}{-\sqrt{2} + \sqrt{2} + 1} = 1$$

7. (*d*) Given: $\alpha + \beta = 90° \quad \Rightarrow \quad \beta = 90° - \alpha$

$\Rightarrow$ $\sqrt{\cos\alpha\operatorname{cosec}(90°-\alpha) - \cos\alpha\sin(90°-\alpha)}$

$\Rightarrow$ $\sqrt{\cos\alpha\sec(\alpha) - \cos\alpha\cos(\alpha)}$

$= \sqrt{1-\cos^2\alpha} = \sqrt{\sin^2\alpha}$

$\Rightarrow$ $\sin\alpha$

8. (*d*) $\dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} + \dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$

Divide the equation by $\cos\theta$

$$= \frac{\frac{\cos\theta - \sin\theta}{\cos\theta}}{\frac{\cos\theta + \sin\theta}{\cos\theta}} + \frac{\frac{\cos\theta + \sin\theta}{\cos\theta}}{\frac{\cos\theta - \sin\theta}{\cos\theta}}$$

$$= \frac{1-\tan\theta}{1+\tan\theta} + \frac{1+\tan\theta}{1-\tan\theta}$$

$$= \frac{(1-\tan\theta)^2 + (1+\tan\theta)^2}{(1+\tan\theta)(1-\tan\theta)}$$

$$= \frac{(1+\tan^2\theta - 2\tan\theta) + (1+\tan^2\theta + 2\tan\theta)}{(1-\tan^2\theta)}$$

$$= \frac{2+2\tan^2\theta}{1-\tan^2\theta} = \frac{2(1+\tan^2\theta)}{1-\tan^2\theta}$$

Putting the value of $\tan^2\theta$

$$= \frac{2\left(1+\frac{1-a^2}{1+a^2}\right)}{1-\frac{1-a^2}{1+a^2}} = \frac{\frac{2(1+a^2+1-a^2)}{1+a^2}}{\frac{1+a^2-(1-a^2)}{1+a^2}}$$

$$= \frac{\frac{2(2)}{1+a^2}}{\frac{2a^2}{1+a^2}}$$

$$= \frac{4}{2a^2} = \frac{2}{a^2}$$

9. (*c*) $\operatorname{cosec}\left(-\dfrac{19\pi}{3}\right) = -\operatorname{cosec}\left(\dfrac{19\pi}{3}\right)$

$(\because \operatorname{cosec}(-x) = -\operatorname{cosec} x)$

$$= -\operatorname{cosec}\left(6\pi + \frac{\pi}{3}\right) = -\operatorname{cosec}\left(3\times 2\pi + \frac{\pi}{3}\right)$$

$$= -\operatorname{cosec}\frac{\pi}{3}$$

$$= -\frac{2}{\sqrt{3}}$$

10. (*d*) $\operatorname{cosec}\dfrac{7\pi}{6} = \operatorname{cosec}\left(\pi + \dfrac{\pi}{6}\right) = -\operatorname{cosec}\left(\dfrac{\pi}{6}\right) =$

$$\therefore \quad 2\sin^2\frac{\pi}{6} + \operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3} =$$

$$= 2\times\left(\frac{1}{2}\right)^2 + (-2)^2\times\left(\frac{1}{2}\right)^2 = 2\times\frac{1}{4} + 4\times\frac{1}{4}$$

$$= \frac{1}{2} + 1 = \frac{3}{2}$$

11. (*d*) $\cos\frac{5\pi}{8} = \cos\frac{4\pi+\pi}{8} = \cos\left(\frac{\pi}{2}+\frac{\pi}{8}\right) = -\sin\left(\frac{\pi}{8}\right)$

$\cos\frac{7\pi}{8} = \cos\frac{4\pi+3\pi}{8} = \cos\left(\frac{\pi}{2}+\frac{3\pi}{8}\right) = -\sin\left(\frac{3\pi}{8}\right)$

Now, $\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\cos^2\frac{5\pi}{8}+\cos^2\frac{7\pi}{8}$

$= \cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\left(-\sin\frac{\pi}{8}\right)^2+\left(-\sin\frac{3\pi}{8}\right)^2$

$= \cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\left(\sin\frac{\pi}{8}\right)^2+\left(\sin\frac{3\pi}{8}\right)^2$

$= \left(\cos^2\frac{\pi}{8}+\sin^2\frac{\pi}{8}\right)+\left(\cos^2\frac{3\pi}{8}+\sin^2\frac{3\pi}{8}\right)$

$= 1 + 1 = 2$

12. (*b*) Given, $f(\alpha) = \sqrt{\sec^2\alpha - 1}$

$\Rightarrow\ f(\alpha) = \sqrt{\tan^2\alpha}$

$\Rightarrow\ f(\alpha) = \tan\alpha$ …(*i*)

$\Rightarrow\ f(\beta) = \tan\beta$

Put values in $\frac{f(\alpha)+f(\beta)}{1-f(\alpha)f(\beta)}$,

$\Rightarrow\ \frac{f(\alpha)+f(\beta)}{1-f(\alpha)f(\beta)} = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$\Rightarrow\ \frac{f(\alpha)+f(\beta)}{1-f(\alpha)f(\beta)} = \tan(\alpha+\beta)$

From (*i*),

$\Rightarrow\ \frac{f(\alpha)+f(\beta)}{1-f(\alpha)f(\beta)} = f(\alpha+\beta)$

13. (*a*) $\therefore\ \dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$

$$= \frac{\dfrac{\tan\left(\frac{\pi}{4}\right)+\tan x}{1+\tan\left(\frac{\pi}{4}\right)\tan x}}{\dfrac{\tan\left(\frac{\pi}{4}\right)-\tan x}{1+\tan\left(\frac{\pi}{4}\right)\tan x}}$$

$$= \frac{\dfrac{1+\tan x}{1-\tan x}}{\dfrac{1-\tan x}{1+\tan x}}$$

$$= \frac{(1+\tan x)^2}{(1-\tan x)^2}$$

$$= \left(\frac{1+\tan x}{1-\tan x}\right)^2$$

Hence option (*a*) is correct.

14. (*a*) Given that, $\tan\theta = \frac{p}{q}$

Now, $\frac{p\sin\theta - q\cos\theta}{p\sin\theta - q\cos\theta}$

$= \frac{p\tan\theta - q}{p\tan\theta + \theta}$

[dividing denominator and numerator by $\cos\theta$]

$$= \frac{p\times\frac{p}{q}-q}{p\times\frac{p}{q}+q}$$

$= \frac{p^2-q^2}{p^2+q^2}$

Hence option (*a*) is correct.

15. (*b*) $\tan(45°)$

$= \tan\left(\frac{45°}{2}+\frac{45°}{2}\right)$

$$= \frac{2\tan\left(\frac{45°}{2}\right)}{\left(1-\tan^2\left(\frac{45°}{2}\right)\right)}$$

(Using expansion for $\tan(2x)$

This implies,

$$1 = \frac{2\tan\left(\frac{45°}{2}\right)}{\left(1-\tan^2\left(\frac{45°}{2}\right)\right)}$$

Rearranging terms,

$\tan^2\left(\frac{45°}{2}\right)+2\tan\left(\frac{45°}{2}\right)-1=0$

Solving the quadratic equation

$x^2+2x-1=0$ gives

$x = \left(\sqrt{2}-1\right)$ or $\left(-\sqrt{2}-1\right)$

But $\tan\left(\frac{45°}{2}\right)$ lies in the first quadrant therefore it should be positive.

$\tan\left(\frac{45°}{2}\right) = \left(\sqrt{2}-1\right)$

16. (*c*) $\tan 15° + \cot 15°$

$\Rightarrow\ \frac{\sin 15°}{\cos 15°}+\frac{\cos 15°}{\sin 15°}$

$$\Rightarrow \quad \frac{\sin^2 15° + \cos^2 15°}{\cos 15° \cos 15°}$$

$$\Rightarrow \quad \frac{1}{\sin 15° \cos 15°}$$

Now multiply and divide by 2

$$\Rightarrow \quad \frac{2}{2 \sin 15° \cos 15°}$$

$$\Rightarrow \quad \frac{2}{\sin (15 \times 2)°}$$

$$\Rightarrow \quad \frac{2}{\sin (30)°}$$

$$\Rightarrow \quad \frac{2}{\frac{1}{2}}$$

$$\Rightarrow \quad 2 \times 2 = 4$$

17. (*b*) $\therefore \quad \cot (15°)$

$$= \frac{1}{\tan (15°)}$$

$$= \frac{1}{\tan (45° - 30°)}$$

$$= \frac{1}{\dfrac{\tan (45°) - \tan (30°)}{1 + \tan (45°) \tan (30°)}}$$

$$= \frac{1}{\dfrac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}}$$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

Hence option (*b*) is correct.

18. (*d*) $\sin^3 \alpha + \cos^3 \alpha$

$= (\sin \alpha + \cos \alpha)(\sin^2 \alpha + \cos^2 \alpha - \sin \alpha \cos \alpha)$

$$= (\sin \alpha + \cos \alpha)\left(1 - \frac{\sin 2\alpha}{2}\right)$$

$$= \sqrt{(1 + \sin 2\alpha)}\left(1 - \frac{\sin 2\alpha}{2}\right)$$

$$= \sqrt{\left(1 + \frac{3}{4}\right)}\left(1 - \frac{3}{8}\right)$$

$$= \frac{\sqrt{7}}{2} \times \frac{5}{8}$$

$$= \frac{5\sqrt{7}}{16}$$

19. (*c*) $\tan \alpha = \dfrac{1}{7}, \sin \beta = \dfrac{1}{\sqrt{10}}; 0 < \alpha, \beta < \dfrac{\pi}{2}$

$$\sin \alpha = \frac{1}{5\sqrt{2}}, \cos \alpha = \frac{7}{5\sqrt{2}}$$

$\cos (\alpha + 2\beta) = \cos \alpha \times \cos 2\beta - \sin \alpha \times \sin 2\beta$

$$= \frac{7}{5\sqrt{2}} \times \frac{4}{5} - \frac{1}{5\sqrt{2}} \times \frac{3}{5}$$

$$= \frac{28 - 3}{25\sqrt{2}} = \frac{1}{\sqrt{2}}$$

20. (*b*) We have, $\cos A + \cos B = 1$

$$\Rightarrow \quad 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} = 1$$

$$\Rightarrow \quad 2 \cos \frac{\pi}{6} \cos \frac{A-B}{2} = 1$$

$$\Rightarrow \quad \cos \frac{A-B}{2} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad 2 \cos^2 \frac{A-B}{2} = \frac{2}{3}$$

$$\Rightarrow \quad 2 \cos^2 \frac{A-B}{2} - 1 = \frac{2}{3} - 1$$

$$\Rightarrow \quad \cos (A - B) = -\frac{1}{3}$$

21. (*a*) We have to find the value of $\dfrac{\sin 5A - \sin 3A}{\cos 5A + \cos 3A}$:

By using formula $\dfrac{\sin C - \sin D}{\cos C + \cos D} = \tan \dfrac{C - D}{2}$

$$\frac{\sin 5A - \sin 3A}{\cos 5A + \cos 3A} = \tan \frac{5A - 3A}{2}$$

$$= \tan \frac{2A}{2}$$

$= \tan A$

Thus the value of $\dfrac{\sin 5A - \sin 3A}{\cos 5A + \cos 3A}$ is $\tan A$.

22. (*c*) Given the trigonometric identity is,

$$\cot\left(\frac{\pi}{4} + \theta\right)\cot\left(\frac{\pi}{4} - \theta\right)$$

$$= \frac{\cot\left(\frac{\pi}{4} + \theta\right)\cot\left(\frac{\pi}{4} - \theta\right)}{\sin\left(\frac{\pi}{4} + \theta\right)\sin\left(\frac{\pi}{4} - \theta\right)}$$

$$= \frac{2 \cos\left(\frac{\pi}{4} + \theta\right)\cot\left(\frac{\pi}{4} - \theta\right)}{2 \sin\left(\frac{\pi}{4} + \theta\right)\sin\left(\frac{\pi}{4} - \theta\right)}$$

$$= \frac{\cos\left(\left(\frac{\pi}{4}+\theta\right)-\left(\frac{\pi}{4}-\theta\right)\right)+\cos\left(\left(\frac{\pi}{4}+\theta\right)+\left(\frac{\pi}{4}-\theta\right)\right)}{\cos\left(\left(\frac{\pi}{4}+\theta\right)-\left(\frac{\pi}{4}-\theta\right)\right)-\cos\left(\left(\frac{\pi}{4}+\theta\right)+\left(\frac{\pi}{4}-\theta\right)\right)}$$

$$= \frac{\cos\left(\frac{\pi}{4}+\theta-\frac{\pi}{4}+\theta\right)+\cos\left(\frac{\pi}{4}+\theta+\frac{\pi}{4}-\theta\right)}{\cos\left(\frac{\pi}{4}+\theta-\frac{\pi}{4}+\theta\right)-\cos\left(\frac{\pi}{4}+\theta+\frac{\pi}{4}-\theta\right)}$$

$$= \frac{\cos 2\theta+\cos\frac{\pi}{2}}{\cos 2\theta-\cos\frac{\pi}{2}}$$

$$= \frac{\cos 2\theta+0}{\cos 2\theta-0} \qquad \left[\because\ \cos\frac{\pi}{2}=0\right]$$

$$= \frac{\cos 2\theta}{\cos 2\theta} = 1$$

23. (*d*) Given that the trigonometric relation is, $4\tan\theta\tan\phi = 3$

$$\Rightarrow \tan\theta\tan\phi = \frac{3}{4}$$

$$\Rightarrow \frac{\sin\theta\sin\phi}{\cos\theta\cos\phi} = \frac{3}{4}$$

$$\Rightarrow \frac{\cos\theta\cos\phi}{\sin\theta\sin\phi} = \frac{3}{4}$$

$$\Rightarrow \frac{\cos\theta\cos\phi+\sin\theta\sin\phi}{\cos\theta\cos\phi+\sin\theta\sin\phi} = \frac{4+3}{4-3}$$

$$\Rightarrow \frac{\cos(\theta-\phi)}{\cos(\theta+\phi)} = \frac{7}{1}$$

$$\Rightarrow \frac{\cos(\theta-\phi)}{\cos(\theta+\phi)} = 7$$

24. (*c*) $\sin\theta\sin\left(\frac{\pi}{3}-\theta\right)\sin\left(\frac{\pi}{3}+\theta\right)$

$$\sin\theta\left(\sin^2\frac{\pi}{3}-\sin^2\theta\right)$$

$$\sin\theta\left(\frac{3}{4}-\sin^2\theta\right)$$

$$\frac{1}{4}\sin 3\theta$$

25. (*c*) $\Rightarrow\ \sin 2\theta = \cos 3\theta$

$\Rightarrow\ \sin 2\theta = \sin(90° - 3\theta)$

$\Rightarrow\ 2\theta = 90° - 3\theta$

$\Rightarrow\ 5\theta = 90°$

$\therefore\ \theta = 18°$

$$\Rightarrow\ \sin\theta = \frac{\sqrt{5}-1}{4}$$

Now, $1 + 4\sin\theta = 1 + 4\left\{\frac{\sqrt{5}-1}{4}\right\} = \sqrt{5}$

26. (*c*) $\tan(45° + \theta) = 1 + \sin 2\theta$

$$\Rightarrow \frac{1+\tan\theta}{1-\tan\theta} = 1+\frac{2\tan\theta}{1+\tan^2\theta}$$

$$\Rightarrow \frac{1+\tan\theta}{1-\tan\theta} = \frac{1+\tan^2\theta+2\tan\theta}{1+\tan^2\theta}$$

$$\Rightarrow \frac{1+\tan\theta}{1-\tan\theta} = \frac{(1+\tan\theta)^2}{1+\tan^2\theta}$$

$$\Rightarrow \frac{1}{1-\tan\theta} = \frac{1+\tan\theta}{1+\tan^2\theta}\ (\text{As } 1+\tan\theta \neq 0)$$

$\Rightarrow\ 1 + \tan^2\theta = 1 - \tan^2\theta$

$\Rightarrow\ \tan^2\theta = 0$

Now, $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$$\Rightarrow\ \tan 2\theta = \frac{1-0}{1+0}$$

$\Rightarrow\ \cos 2\theta = 1$

27. (*b*) $4\sin^2\theta + 2\sin\theta$

$= 4\sin^2 18° + 2\sin 18°$

$$= 4\left(\frac{\sqrt{5}-1}{4}\right)^2 + 2\left(\frac{\sqrt{5}-1}{4}\right)$$

$$= 2\left(\frac{\sqrt{5}-1}{4}\right)\left\{\left(2\times\frac{\sqrt{5}-1}{4}\right)+1\right\}$$

$$= 2\left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{2}\right)$$

$$= 2\times\frac{4}{8} = 1$$

28. (*b*) $\sin^4\theta + \cos^4\theta = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$

And, $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

Here $[a^3 + b^3 = (a + b)(a^2 + b^2 - ab)]$

After put the value to the given Eqn we get

$2(1 - 3\sin^2\theta\cos^2\theta) - 3(1 - 2\sin^2\theta\cos^2\theta)$

$2 - 6\sin^2\theta\cos^2\theta - 3 + 6\sin^2\theta\cos^2\theta + 1$

$3 - 3 = 0$

29. (*b*) We know that

$\cos^2 A - \sin^2 B = \cos(A + B)\cos(A - B)$

After put the formula we get $\cos 60°\cos 36°$

$$\left(\frac{1}{2}\right)\left(\frac{\sqrt{5}+1}{4}\right) = \frac{\sqrt{5}+1}{8}$$

And the final answer is $\frac{\sqrt{5}+1}{8}$

30. (*b*) We know that,

$2 \sin A \cos B = \sin (A + B) + \sin (A – B)$

$2 \cos A \cos B = \cos (A + B) + \cos (A – B)$

$2 \sin A \sin B = \cos (A – B) – \cos (A + B)$

the given equation is divided and multiply by 2

$$= \frac{2 \sin 8\theta \cos \theta - 2 \sin 6\theta \cos 3\theta}{2 \cos 2\theta \cos \theta - 2 \sin 3\theta \sin 4\theta}$$

$$= \frac{\sin 9\theta + \sin 7\theta - (\sin 9\theta + \sin 3\theta)}{\cos 3\theta + \cos \theta - (\cos \theta - \cos 7\theta)}$$

$$= \frac{\sin 7\theta - \sin 3\theta}{\cos 3\theta + \cos 7\theta}$$

$$= \frac{2 \cos 5\theta \sin 2\theta}{2 \cos 5\theta \cos 2\theta}$$

$= \tan 2\theta$

31. (*b*) $4 \tan^2 \theta = 3\sec^2 \theta$

$4 \tan^2 \theta = 3(1 + \tan^2 \theta)$

$4 \tan^2 \theta = 3 + 3 \tan^2 \theta$

$\tan^2 \theta = 3 = \tan^2 \frac{\pi}{3}$

$\theta = n\pi \pm \frac{\pi}{3}$

And the final answer is $n\pi \pm \frac{\pi}{3}$

32. (*b*) $\tan 2x = -\cot\left(x + \frac{\pi}{3}\right)$

$\tan 2x = \tan\left(\frac{\pi}{2} + x + \frac{\pi}{3}\right)$

$2x = n\pi + \frac{\pi}{2} + x + \frac{\pi}{3}$

$x = n\pi + \frac{5\pi}{6}$

And the final answer is $n\pi + \frac{5\pi}{6}$

33. (*a*) $\sin\left(\frac{\pi}{4}\cot\theta\right) = \cos\left(\frac{\pi}{4}\tan\theta\right)$

$\sin\left(\frac{\pi}{4}\cot\theta\right) = \sin\left[\frac{\pi}{4} - \left(\frac{\pi}{4}\tan\theta\right)\right]$

$\frac{\pi}{4}\cot\theta = \frac{\pi}{4} - \frac{\pi}{4}\tan\theta$

$\frac{\pi}{4}\tan\theta + \frac{\pi}{4}\cot\theta = \frac{\pi}{2}$

$\frac{1}{4}\left(\tan\theta + \frac{1}{\tan\theta}\right) = \frac{1}{2}$

$\tan\theta + \frac{1}{\tan\theta} = 2$

$\frac{1 + \tan^2\theta}{\tan\theta} = 2$

$1 + \tan^2 \theta = 2 \tan \theta$

$(1 - \tan\theta)^2 = 0$

$\tan\theta = 1 = \tan \frac{\pi}{4}$

$\theta = n\pi + \frac{\pi}{4}$

And the final answer is $n\pi + \frac{\pi}{4}$

34. (*b*) $\sin^2\theta \sec\theta + \sqrt{3}\tan\theta = 0$

$\sin\theta \frac{\sin\theta}{\cos\theta} + \sqrt{3}\tan\theta = 0$

$\tan\theta \sin\theta + \sqrt{3}\tan\theta = 0$

$\tan\theta\left(\sin\theta + \sqrt{3}\right) = 0$

either , $\tan \theta = 0 = \tan 0$

$\theta = n\pi$, $n \in Z$

Or, $\sin\theta + \sqrt{3} = 0$

$\sin\theta = -\sqrt{3}$ (not possible)

And the final answer is $\theta = n\pi$, $n \in Z$

35. (*b*) $r \sin \theta = 3$, $r = 4(1 + \sin \theta)$

than,

$4(1 + \sin \theta) \sin \theta = 3$

$4 \sin \theta + 4 \sin^2 \theta = 3$

$4 \sin^2 \theta + 4 \sin \theta - 3 = 0$

$4 \sin^2 \theta + 6 \sin \theta - 2 \sin \theta - 3 = 0$

$(2 \sin \theta - 1)(2 \sin \theta + 3) = 0$

either, $2 \sin \theta + 3 = 0$

$\sin \theta = \frac{-3}{2}$ (Not possible)

Or, $2 \sin \theta - 1 = 0$

$\sin \theta = \frac{1}{2} = \sin \frac{\pi}{6}$

And the final answer is $\frac{\pi}{6}$, $\frac{5\pi}{6}$

36. (*b*) $\tan \theta + \tan 2\theta + \tan 3\theta = \tan \theta \tan 2\theta \tan 3\theta$

$\tan \theta + \tan 2\theta = \tan \theta \tan 2\theta \tan 3\theta - \tan 3\theta$

$\tan \theta + \tan 2\theta = \tan 3\theta (\tan \theta \tan 2\theta - 1)$

$\frac{\tan\theta + \tan 2\theta}{\tan\theta \tan 2\theta - 1} = \tan 3\theta$

$\frac{\tan\theta + \tan 2\theta}{1 - \tan\theta \tan 2\theta} = \tan(3\theta)$

$\tan 3\theta = \tan (-3\theta)$

$n\pi + 3\theta = -3\theta$

$6\theta = -n\pi$

$\theta = \frac{n}{6}\pi$, here we know

$\{n \in Z$ and $-n \in Z\}$

And the final answer is $\frac{n\pi}{6}$

37. (*a*) $\tan 3\theta + \tan\theta = 2\tan 2\theta$

$\tan 3\theta - \tan 2\theta = \tan 2\theta - \tan\theta$

$$\frac{\sin 3\theta}{\cos 3\theta} - \frac{\sin 2\theta}{\cos 2\theta} = \frac{\sin 2\theta}{\cos 2\theta} - \frac{\sin\theta}{\cos\theta}$$

$$\frac{\sin 3\theta\cos\theta - \sin 2\theta\cos 3\theta}{\cos 3\theta\cos 2\theta} = \frac{\sin 2\theta\cos\theta - \cos 2\theta\sin\theta}{\cos 3\theta\cos\theta}$$

$$\frac{\sin(3\theta - 2\theta)}{\cos 3\theta\cos 2\theta} = \frac{\sin(2\theta - \theta)}{\cos 2\theta\cos\theta}$$

$\sin\theta\cos\theta = \sin\theta\cos 3\theta$

$\sin\theta(\cos\theta - \cos 3\theta) = 0$

$$\sin\theta\left[2\sin\left(\frac{\theta + 3\theta}{2}\right)\sin\left(\frac{3\theta - \theta}{2}\right)\right] = 0$$

$2\sin\theta\sin 2\theta\sin\theta = 0$

either, $\sin\theta = 0$, than, $\theta = n\pi$

Or, $\sin 2\theta = 0$, than $2\theta = n\pi$

$\theta = \dfrac{n\pi}{2}$

And the final answer is $n\pi$

38. (*c*) $2\cos^2 x + 3\sin x = 0$

$2[1 - \sin^2 x] + 3\sin x = 0$

$2 - 2\sin^2 x + 3\sin x = 0$

$2\sin^2 x - 4\sin x + \sin x - 2 = 0$

$(2\sin x + 1)(\sin x - 2) = 0$

either, $2\sin x + 1 = 0$

than, $\sin x = \dfrac{-1}{2} = -\sin\left(\dfrac{\pi}{6}\right) = \sin\left(\pi + \dfrac{\pi}{6}\right)$

$\sin x = \sin\dfrac{7\pi}{6}$

Or, $\sin x - 2 = 0$

$\sin x = 2$ (Not possible)

And the final answer is $x = n\pi + (-1)^n \dfrac{7\pi}{6}$

39. (*a*) $\sin x + \cos x = 1 + \sin x\cos x$

$\sin x - 1 + \cos x - \sin x\cos x = 0$

$(1 - \cos x)(\sin x - 1) = 0$

either, $1 - \cos x = 0$

$\cos x = 1 = \cos 0$

$x = 2n\pi$

Or, $\sin x - 1 = 0$

$\sin x = 1 = \sin\dfrac{\pi}{2}$

$x = 2n\pi + \dfrac{\pi}{2}$

And the final answer is $2n\pi$, $2n\pi + \dfrac{\pi}{2}$ $n \in Z$

40. (*c*) $(2\sin x - \cos x)(1 + \cos x) = \sin^2 x$

$(2\sin x - \cos x)(1 + \cos x) - (1 - \cos^2 x) = 0$

$(1 + \cos x)(2\sin x - \cos x - 1 + \cos x) = 0$

$(1 + \cos x)(2\sin x - 1) = 0$

either, $\cos x = -1$

$x = (2n + 1)\pi$

Or, $2\sin x - 1 = 0$

$\sin x = \dfrac{1}{2} = \sin\dfrac{\pi}{6}$

$x = n\pi + (-1)^n\dfrac{\pi}{6}$

And the final answer is

$(2n + 1)\pi$, $n\pi + (-1)^n\dfrac{\pi}{6}$

41. (*c*) $\sin x + \cos x = \sqrt{2}$

$$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{\sqrt{2}}{\sqrt{2}}$$

$\sin x\sin\dfrac{\pi}{4} + \cos x\cos\dfrac{\pi}{4} = 1$

$\cos\left(x - \dfrac{\pi}{4}\right) = 1$

$\cos\left(x - \dfrac{\pi}{4}\right) = \cos 2n\pi$

$x - \dfrac{\pi}{4} = 2n\pi$

$x = 2n\pi + \dfrac{\pi}{4}$

And the final answer is $2n\pi + \dfrac{\pi}{4}$

42. (*b*) By Sine formula

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = K$$

$\therefore$ $a = K \text{ Sin } A$, $b = K \text{ Sin } B$, $c = K \text{ Sin } C$

$$\frac{b^2 - c^2}{a^2}$$

$$= \frac{K^2\sin^2 B - K^2\sin^2 C}{K^2\sin^2 A} = \frac{\sin^2 B - \sin^2 C}{\sin^2 A}$$

$$= \frac{\sin(B + C)\sin(B - C)}{\sin^2(180 - (B + C))} = \frac{\sin(B + C)\sin(B - C)}{\sin^2(B + C)}$$

$$= \frac{\sin(B - C)}{\sin(B + C)}$$

43. (*a*) Since $61^2 = 60^2 + 11^2$, hence its a right angled triangle, and in a right triangle circumradius is half of the hypotenuse

hence $R = \dfrac{61}{2}$

44. (*b*) We have,

$x^2 + y^2 < z^2$

$\Rightarrow x^2 + y^2 < x^2 + y^2 - 2xy\cos Z$

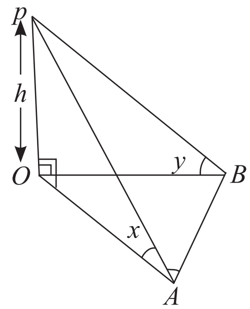
By cosine rule $\left[\because \cos Z = \dfrac{x^2 + y^2 - z^2}{2xy}\right]$

$\Rightarrow 2xy\cos Z < 0$

$\Rightarrow \cos Z < 0$

Hence, Z is an obtuse angle.

45. (*a*)



Here OP be the tower of height $h = 30m$

Angle at point A is $x = 60°$

Angle at point B is $y = 30°$

$OA = h \cot 60° = \dfrac{h}{\sqrt{3}}$

$OB = h \cot 30° = h\sqrt{3}$

In right angled ΔOAB

$OB^2 = OA^2 + AB^2$

$3h^2 = \dfrac{h^2}{3} + AB^2$

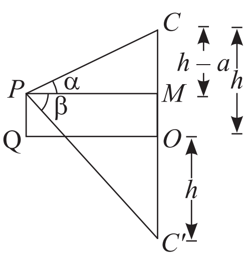
$3 \times 900 = 300 + AB^2$

$AB^2 = 3 \times 800$

$AB = 20\sqrt{6}$

The distance between A and B $AB = 20\sqrt{6}$

46. (*d*)



Distance of observation point a = 10 m

In $\Delta\, PMC$ $\tan\alpha = \tan 45° = \dfrac{h-a}{PM}$

$PM = h - a$

$\tan\beta\ PM = (h + a)$

$(h - a)\tan\beta = (h + a)$

$(10\sqrt{3} - 10)\tan\beta = (10\sqrt{3} + 10)$

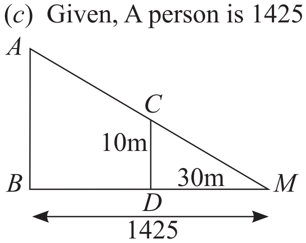
$h + 10 = (h - 10)\sqrt{3}$

$\tan\beta\,(\sqrt{3}\ 3 - 1) = (\sqrt{3} + 1)$

$\tan\beta = \dfrac{(\sqrt{3}+1)^2}{2} = (2+\sqrt{3})$

$\tan\beta = \left(2+\sqrt{3}\right)$

47. (*c*) Given, A person is 1425 m away from the mall



now, let the height of the mall (AB) be x m

$CD = 10$ m

$DM = 30$ m

$BM = 1425$ m

Let $\angle AMB = \angle CMD = \theta$

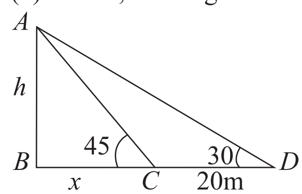
from ΔAMB and ΔCMD get,

$\dfrac{AB}{BM} = \dfrac{CD}{DM} \Rightarrow \dfrac{x}{1425} = \dfrac{10}{30}$

$x = 475$m

therefore, the height of the mall is 475 m tall.

48. (*a*) Given, The angle of elevation= 30° and 45°



now, let the height of the pillar be (AB) = h and $BC = x$

In $\Delta\, ABC$,

$\tan(45°) = \dfrac{AB}{BC} \Rightarrow 1 = \dfrac{h}{x}$

$\Rightarrow h = x$ …(*i*)

In $\Delta\, ABC$,

$\tan(30°) = \dfrac{AB}{BD} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h}{(x+20)}$

$\Rightarrow h = \dfrac{(x+20)}{\sqrt{3}}$ …(*ii*)

from equation (*i*) and (*ii*)

$x = \dfrac{(x+20)+}{\sqrt{3}} \Rightarrow \sqrt{3x} = x + 20$

$\sqrt{3x} - x = 20 \Rightarrow x\left(\sqrt{3}-1\right) = 20 \Rightarrow x = \dfrac{20}{(\sqrt{3}-1)}$

$x = \dfrac{20}{(\sqrt{3}-1)} \times \dfrac{(\sqrt{3}+1)}{(\sqrt{3}+1)} \Rightarrow x = \dfrac{20(\sqrt{3}+1)}{2}$

$\Rightarrow x = 10(\sqrt{3}+1)$

as we know from the equation 1, $h = x$

therefore, height of the pillar is $10(\sqrt{3+1})$

49. (*b*) $\dfrac{\sin B}{\sin(A+B)} = \dfrac{\sin B}{\sin C} = \dfrac{b}{c}$

50. (*b*) $\dfrac{\cos A}{\cos B} = \dfrac{\sin A}{\sin B}$

$\cos A \sin B = \sin A \cos B$

$\sin(A - B) = 0$

$A = B$

Similarly, $\dfrac{\cos B}{\cos C} = \dfrac{b}{c}$

$\dfrac{\cos B}{\cos C} = \dfrac{\sin B}{\sin C}$

$\sin(B - C) = 0$

$B = C$

$A = B = C$, Hence the triangle must be equilateral triangle

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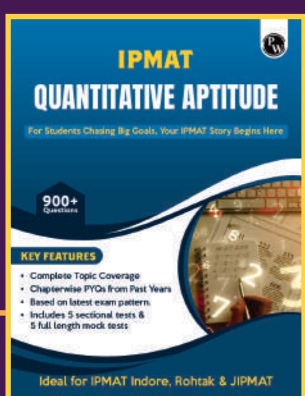

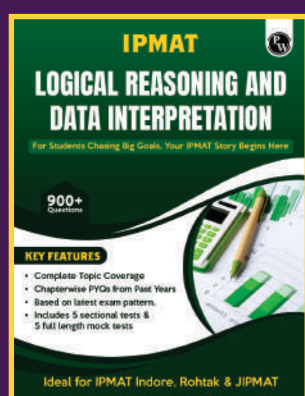

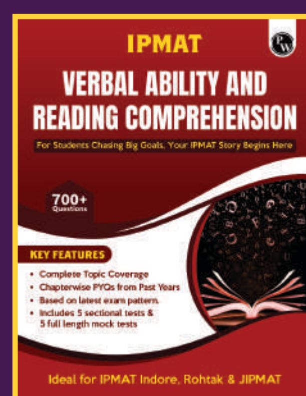

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