CUET (UG)

MATHEMATICS

As Per Latest NTA Pattern

CHAPTERWISE & TOPICWISE
QUESTION BANK
REVISION SHEETS WITH DETAILED REVISION NOTES

WITH CHAPTERWISE PAST YEAR QUESTIONS

HIGHLIGHTS

- Chapterwise Revision Sheet(Mind Maps)
- 1100+ NCERT Based Topicwise MCQs with Explanations
- 200+ Matching type, Statement Based and A & R type MCQs
- 125+ Case Based / Comprehension Based MCQs
- Chapterwise CUET Past Year Questions 2023 & 2022
- NTA CUET 2022 Questions with Explanations
- 4 Sample Papers as per Latest NTA Mock Test (Solved with QR code)
- 2024 & 2023 Papers Analysis
- Elaborated Solutions

2025
EXAMINATION

3rd Edition
CUET 2024 Solved Paper

Contents

Mathematics

Paper Analysis

Chapter-wise Analysis of Question Papers 2024 & 2023

S. No.	Name of the Chapter	NTA CUET Question Paper (16th May 2024)		NTA CUET Question Paper (30th May 2023)		NTA CUET Question Paper (21st May 2023)	
		No. of Ques	Percentage (%)	No. of Ques	Percentage (%)	No. of Ques	Percentage (%)
1.	Relations and Functions	2	4	3	6	2	4
2.	Inverse trigonometric Functions	1	2	2	4	3	6
3.	Matrices	6	12	5	10	4	8
4.	Determinants	3	6	5	10	6	12
5.	Continuity and Differentiability	5	10	4	8	4	8
6.	Application of Derivatives	5	10	7	14	6	12
7.	Integrals	5	10	3	6	3	6
8.	Application of integrals	3	6	3	6	4	8
9.	Differential Equations	3	6	4	8	4	8
10.	Vector Algebra	3	6	2	4	3	6
11.	Three Dimensional Geometry	4	8	3	6	3	6
12.	Linear Programming	4	8	5	10	4	8
13.	Probability	6	12	4	8	4	8

CHAPTER-11

REVISION SHEET

THREE DIMENSIONAL GEOMETRY

Shortest Distance Between Two Lines

Distance between two skew lines:
Let l_1 and l_2 be two skew lines with equations

$$\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$$

and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$

$\therefore$ Required shortest distance is

$$d = \left|\frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

Cartesian form:
The shortest distance between the lines

$$l_1 : \frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$$

and $l_2 : \dfrac{x - x_2}{a_2} = \dfrac{y - y_2}{b_2} = \dfrac{z - z_2}{c_2}$

$$d = \left|\frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}}\right|$$

Distance between parallel lines:
If two lines l_1 and l_2 are parallel, then they are coplanar. Let the lines be given by

$$\vec{r} = \vec{a}_1 + \lambda\vec{b}$$

and $\vec{r} = \vec{a}_2 + \mu\vec{b}$

The distance between the given parallel lines is

$$d = \left|\frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|}\right|$$

Angle Between Two Lines

Let θ be the acute angle between two vectors then

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|.$$

or $\cos\theta = \dfrac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$

Two lines will be perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$, or $\vec{b}_1 \cdot \vec{b}_2 = 0$

parallel if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$ or $\vec{b}_1 = \lambda\vec{b}_2$

Direction Cosines and Direction Ratios

Direction ratios: Let a, b, c be proportional to the direction cosines l, m, n then a, b, c are called the direction ratios.
If the coordinates P and Q are (x_1, y_1, z_1) and (x_2, y_2, z_2) then the direction ratios of line PQ are, $a = x_2 - x_1$, $b = y_2 - y_1$ & $c = z_2 - z_1$

Direction cosines : Let α, β, γ be angles which a directed line makes with the positive directions of the axes of x, y and z respectively, then cos α, cos β, cos γ are called the direction cosines of the line. The direction cosines are usually denoted by (l, m, n).
Thus $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$.
If l, m, n be the direction cosines and a, b, c be the direction ratios of a vector, then

$$\ell = \pm\frac{a}{\sqrt{a^2 + b^2 + c^2}}, m = \pm\frac{b}{\sqrt{a^2 + b^2 + c^2}},$$

$$n = \pm\frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

If l, m, n be the direction cosines of a line, then $l^2 + m^2 + n^2 = 1$.

Equation of a Line

Vector Equation of a line:
Let l be the line which passes through the point A and is parallel to a given vector $\vec{b}$. Let $\vec{r}$ be the position vector of an arbitrary point P on the line, then Vector equation is $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian Equation of a line:
Let the coordinates of the given point A be (x_1, y_1, z_1) and the direction ratios of the line be a, b, c. Consider the coordinates of any point P be (x, y, z). Then

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}; \quad \vec{A} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

and $\vec{B} = a\hat{i} + b\hat{j} + c\hat{k}$

Then $\dfrac{x - x_1}{a} = \dfrac{y - y_1}{b} = \dfrac{z - z_1}{c}$

This is the Cartesian equation of the line.

NTA CUET PAPER
(16TH MAY 2024)

Instructions

1. ***Section-A will have 15 Questions covering both i.e., Mathematics/Applied Mathematics which will be compulsory for all candidates.***
2. ***Section-B1 will have 35 Questions from Mathematics out of which 25 Questions need to be attempted.****
3. ***Marking Scheme of the test.***
  a. ***Correct answer or the most appropriate answer: Five marks (+5)***
  b. ***Any incorrect option marked will be given minus one mark (–1)***
  c. ***Unanswered/marked for review will be given no mark (0)***

Full Marks: 200 ***(Time: 60 Minutes)***
(*Section-B2 Applied Mathematics is not covered)

SECTION-A (MATHEMATICS/APPLIED MATHEMATICS)

1. An objective function $Z = ax + by$ is maximum at points (8, 2) and (4, 6). If $a \geq 0$ and $b \geq 0$ and $ab = 25$, then the maximum value of the function is equal to:
(*a*) 60 (*b*) 50 (*c*) 40 (*d*) 80

2. The area of the region bounded by the lines $x + 2y = 12$, $x = 2$, $x = 6$ and x-axis is:
(*a*) 34 sq units (*b*) 20 sq units (*c*) 24 sq units (*d*) 16 sq units

3. A die is rolled thrice. What is the probability of getting a number greater than 4 in the first and the second throw of dice and a number less than 4 in the third throw?
(*a*) $\frac{1}{3}$ (*b*) $\frac{1}{6}$ (*c*) $\frac{1}{9}$ (*d*) $\frac{1}{18}$

4. The corner points of the feasible region determined by $x + y \leq 8$, $2x + y \geq 8$, $x \geq 0$, $y \geq 0$ are $A(0, 8)$, $B(4, 0)$ and $C(8, 0)$. If the objective function $Z = ax + by$ has its maximum value on the line segment AB, then the relation between a and b is:
(*a*) $8a + 4 = b$ (*b*) $a = 2b$
(*c*) $b = 2a$ (*d*) $8b + 4 = a$

5. If $t = e^{2x}$ and $y = \log_e t^2$ then $\frac{d^2y}{dx^2}$ is:
(*a*) 0 (*b*) $4t$ (*c*) $\frac{4e^{2t}}{t}$ (*d*) $\frac{e^{2t}(4t-1)}{t^2}$

6. If A and B are symmetric matrices of the same order, then $AB - BA$ is a:
(*a*) symmetric matrix (*b*) zero matrix
(*c*) skew symmetric matrix (*d*) identity matrix

7. If A is a square matrix of order 4 and $|A| = 4$, then $|2A|$ will be :
(*a*) 8 (*b*) 64 (*c*) 16 (*d*) 4

8. If $[A]_{3\times 2}[B]_{x\times y} = [C]_{3\times 1}$, then :
(*a*) $x = 1, y = 3$ (*b*) $x = 2, y = 1$
(*c*) $x = 3, y = 3$ (*d*) $x = 3, y = 1$

9. If a function $f(x) = x^2 + bx + 1$ is increasing in the interval [1, 2], then the least value of b is
(*a*) 5 (*b*) 0 (*c*) –2 (*d*) –4

10. Two dice are thrown simultaneously. If X denotes the number of fours, then the expectation of X will be:
(*a*) $\frac{5}{9}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{4}{7}$ (*d*) $\frac{3}{8}$

11. For the function $f(x) = 2x^3 - 9x^2 + 12x - 5$, $x \in [0, 3]$, match List-I with List-II:

List-I		List-II	
(A)	Absolute maximum value	(I)	3
(B)	Absolute minimum value	(II)	0
(C)	Point of maxima	(III)	–5
(D)	Point of minima	(IV)	4

Choose the correct answer from the options given below:
(*a*) (A) - (IV), (B) - (II), (C) - (I), (D) - (III)
(*b*) (A) - (II), (B) - (III), (C) - (I), (D) - (IV)
(*c*) (A) - (IV), (B) - (III), (C) - (II), (D) - (I)
(*d*) (A) - (IV), (B) - (III), (C) - (I), (D) - (II)

12. The second order derivative of which of the following functions is 5^x ?

(*a*) $5^x \log_e 5$ (*b*) $5^x (\log_c 5)^2$ (*c*) $\dfrac{5^x}{\log_c 5}$ (*d*) $\dfrac{5^x}{(\log_e 5)^2}$

13. The degree of the differential equation $\left(1-\left(\dfrac{dy}{dx}\right)^2\right)^{3/2} = k\dfrac{d^2y}{dx^2}$ is:

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) $\dfrac{3}{2}$

14. $\int \dfrac{\pi}{x^{n+1}-x}dx =$

(*a*) $\dfrac{\pi}{n}\log_e\left|\dfrac{x^n-1}{x^n}\right|+C$ (*b*) $\log_e\left|\dfrac{x^n+1}{x^n-1}\right|+C$

(*c*) $\dfrac{\pi}{n}\log_e\left|\dfrac{x^n+1}{x^n}\right|+C$ (*d*) $\pi\log_e\left|\dfrac{x^n}{x^n-1}\right|+C$

15. The value of $\int_0^1 \dfrac{a-bx^2}{(a+bx^2)^2}dx$ is:

(*a*) $\dfrac{a-b}{a+b}$ (*b*) $\dfrac{1}{a-b}$ (*c*) $\dfrac{a+b}{2}$ (*d*) $\dfrac{1}{a+b}$

SECTION-B1 (MATHEMATICS)

16. The unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$, is :

(*a*) $\dfrac{1}{\sqrt{6}}\hat{i}+\dfrac{2}{\sqrt{6}}\hat{j}+\dfrac{1}{\sqrt{6}}\hat{k}$ (*b*) $-\dfrac{1}{\sqrt{6}}\hat{i}+\dfrac{1}{\sqrt{6}}\hat{j}-\dfrac{1}{\sqrt{6}}\hat{k}$

(*c*) $-\dfrac{1}{\sqrt{6}}\hat{i}+\dfrac{2}{\sqrt{6}}\hat{j}+\dfrac{2}{\sqrt{6}}\hat{k}$ (*d*) $-\dfrac{1}{\sqrt{6}}\hat{i}+\dfrac{2}{\sqrt{6}}\hat{j}-\dfrac{1}{\sqrt{6}}\hat{k}$

17. Let *X* denote the number of hours you play during a randomly selected day. The probability that *X* can take values *x*, has the following form, where *c* is some constant.

$$P(X=x)=\begin{cases}0.1, & \text{if } x=0\\ cx, & \text{if } x=1 \text{ or } x=2\\ c(5-x), & \text{if } x=3 \text{ or } x=4\\ 0, & \text{otherwise}\end{cases}$$

Match **List-I** with **List-II**:

	List-I		List-II
(A)	*c*	(I)	0.75
(B)	$P(X \le 2)$	(II)	0.3
(C)	$P(X = 2)$	(III)	0.55
(D)	$P(X \ge 2)$	(IV)	0.15

Choose the correct answer from the options given below:

(*a*) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
(*b*) (A) - (IV), (B) - (III), (C) - (II), (D) - (I)
(*c*) (A) - (I), (B) - (II), (C) - (IV), (D) - (III)
(*d*) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

18. If $\sin y = x\sin(a+y)$, then $\dfrac{dy}{dx}$ is:

(*a*) $\dfrac{\sin^2 a}{\sin(a+y)}$ (*b*) $\dfrac{\sin(a+y)}{\sin^2 a}$

(*c*) $\dfrac{\sin(a+y)}{\sin a}$ (*d*) $\dfrac{\sin^2(a+y)}{\sin a}$

19. The distance between the lines $\vec{r}=\hat{i}-2\hat{j}+3\hat{k}+\lambda\cdot(2\hat{i}+3\hat{j}+6\hat{k})$ and $\vec{r}=3\hat{i}-2\hat{j}+1\hat{k}+\mu(4\hat{i}+6\hat{j}+12\hat{k})$ is :

(*a*) $\dfrac{\sqrt{28}}{7}$ (*b*) $\dfrac{\sqrt{199}}{7}$ (*c*) $\dfrac{\sqrt{328}}{7}$ (*d*) $\dfrac{\sqrt{421}}{7}$

20. If $f(x)=2\left(\tan^{-1}(e^x)-\dfrac{\pi}{4}\right)$, then *f*(*x*) is :

(*a*) even and is strictly increasing in $(0, \infty)$
(*b*) even and is strictly decreasing in $(0, \infty)$
(*c*) odd and is strictly increasing in $(-\infty, \infty)$
(*d*) odd and is strictly decreasing in $(-\infty, \infty)$

21. For the differential equation $(x\log_e x)dy = (\log_e x - y)dx$

(A) Degree of the given differential equation is 1.
(B) It is a homogeneous differential equation.
(C) Solution is $2y\log_e x + A = (\log_e x)^2$. where *A* is an arbitrary constant
(D) Solution is $2y\log_c x + A = \log_e(\log_e x)$, where *A* is an arbitrary constant

Choose the correct answer from the options given below :

(*a*) (A) and (C) only (*b*) (A), (B) and (C) only
(*c*) (A), (B) and (D) only (*d*) (A) and (D) only

22. There are two bags. Bag-1 contains 4 white and 6 black balls and Bag-2 contains 5 white and 5 black balls. A die is rolled, if it shows a number divisible by 3 , a ball is drawn from Bag -1, else a ball is drawn from Bag - 2. If the ball drawn is not black in colour, the probability that it was not drawn from Bag-2 is :

(*a*) $\dfrac{4}{9}$ (*b*) $\dfrac{3}{8}$ (*c*) $\dfrac{2}{7}$ (*d*) $\dfrac{4}{19}$

23. Which of the following cannot be the direction ratios of the straight line $\dfrac{x-3}{2}=\dfrac{2-y}{3}=\dfrac{z+4}{-1}$

(*a*) 2, −3, −1 (*b*) −2, 3, 1 (*c*) 2, 3, −1 (*d*) 6, −9, −3

46. If $f(x)$, defined by $f(x) = \begin{cases} kx+1, & \text{if } x \le \pi \\ \cos x, & \text{if } x > \pi \end{cases}$ is continuous at $x = \pi$, then the value of k is :

(*a*) 0 (*b*) π (*c*) $\frac{2}{\pi}$ (*d*) $-\frac{2}{\pi}$

47. If $P = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ and $Q[2 \quad -4 \quad 1]$ are two matrices, then $(PQ)'$ will be :

(*a*) $\begin{bmatrix} 4 & 5 & 7 \\ -3 & -3 & 0 \\ 0 & -3 & -2 \end{bmatrix}$ (*b*) $\begin{bmatrix} -2 & 4 & 2 \\ 4 & -8 & -4 \\ -1 & 2 & 1 \end{bmatrix}$

(*c*) $\begin{bmatrix} 5 & 5 & 2 \\ 7 & 6 & 7 \\ -9 & -7 & 0 \end{bmatrix}$ (*d*) $\begin{bmatrix} -2 & 4 & 8 \\ 7 & 5 & 7 \\ -8 & -2 & 6 \end{bmatrix}$

48. $\Delta = \begin{vmatrix} 1 & \cos x & 1 \\ -\cos x & 1 & \cos x \\ -1 & -\cos x & 1 \end{vmatrix}$

(A) $\Delta = 2(1 - \cos^2 x)$
(B) $\Delta = 2(2 - \sin^2 x)$
(C) Minimum value of Δ is 2
(D) Maximum value of Δ is 4

Choose the correct answer from the options given below :

(*a*) (A), (C) and (D) only (*b*) (A), (B) and (C) only
(*c*) (A), (B), (C) and (D) (*d*) (B), (C) and (D) only

49. $f(x) = \sin x + \frac{1}{2}\cos 2x$ in $\left[0, \frac{\pi}{2}\right]$

(A) $f'(x) = \cos x - \sin 2x$
(B) The critical points of the function are $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$
(C) The minimum value of the function is 2
(D) The maximum value of the function is $\frac{3}{4}$

Choose the correct answer from the options given below :

(*a*) (A), (B) and (D) only (*b*) (A), (B) and (C) only
(*c*) (A), (B), (C) and (D) (*d*) (B), (C) and (D) only

50. The direction cosines of the line which is perpendicular to the lines with direction ratios 1, –2, –2 and 0, 2, 1 are :

(*a*) $\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$

(*b*) $-\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$

(*c*) $\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}$

(*d*) $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$

Answer Key

Section-A

1. (*b*)	**2.** (*d*)	**3.** (*d*)	**4.** (*b*)	**5.** (*a*)	**6.** (*c*)	**7.** (*b*)	**8.** (*b*)	**9.** (*c*)	**10.** (*b*)
11. (*d*)	**12.** (*d*)	**13.** (*b*)	**14.** (*a*)	**15.** (*d*)					

Section-B1

16. (*d*)	**17.** (*b*)	**18.** (*d*)	**19.** (*c*)	**20.** (*c*)	**21.** (*a*)	**22.** (*c*)	**23.** (*c*)	**24.** (*c*)	**25.** (*b*)
26. (*d*)	**27.** (*a*)	**28.** (*a*)	**29.** (*b*)	**30.** (*c*)	**31.** (*b*)	**32.** (*d*)	**33.** (*c*)	**34.** (*b*)	**35.** (*a*)
36. (*d*)	**37.** (*b*)	**38.** (*d*)	**39.** (*a*)	**40.** (*b*)	**41.** (*b*)	**42.** (*a*)	**43.** (*c*)	**44.** (*d*)	**45.** (*d*)
46. (*d*)	**47.** (*b*)	**48.** (*d*)	**49.** (*a*)	**50.** (*a*)					

Explanations

1. (*b*) Given objective function, $Z = ax + by$, $a \geq 0$, $b \geq 0$ and $ab = 25$ which is maximum at points (8, 2) and (4, 6).

According to Question

$\Rightarrow (Z)_{(8,2)} = (Z)_{(4,6)}$

$\Rightarrow a(8) + b(2) = a(4) + b(6)$

$\Rightarrow 4a = 4b \quad \Rightarrow a = b \qquad ...(i)$

and $ab = 25 \qquad ...(ii)$ (given)

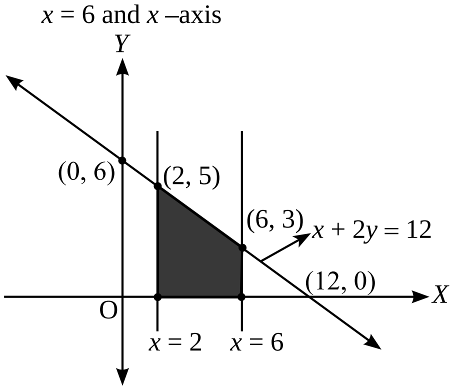
$\Rightarrow a(a) = 25 \quad \Rightarrow a^2 = 25 \quad \Rightarrow a = \pm 5$

From equation (*i*), $b = \pm 5$

$a > 0, b > 0 \quad \Rightarrow a = 5, b = 5$

$Z_{max} = 5(8) + 5(2) = 50 \quad \Rightarrow Z_{max} = 50$

2. (*d*) Given, lines $x + 2y = 12$, $x = 2$, $x = 6$ and x –axis



$\Rightarrow$ Shaded Area $= \int_{x_1}^{x_2} y\,dx$

$\Rightarrow$ Shaded Area $= \int_2^6 \left(\frac{12-x}{2}\right) dx$

$= -\frac{1}{2}\left(\frac{x^2}{2} - 12x\right)_2^6$

$= -\frac{1}{2}\left[\left(\frac{6^2}{2} - 12 \times 6\right) - \left(\frac{2^2}{2} - 12 \times 2\right)\right]$

$= -\frac{1}{2}[-54 + 22]$

$= 16$ sq. units

3. (*d*)

First throw, $x > 4 \Rightarrow 5, 6 \Rightarrow P_1 = \frac{2}{6} = \frac{1}{3}$

Second throw, $x > 4 \Rightarrow 5, 6 \Rightarrow P_2 = \frac{2}{6} = \frac{1}{3}$

Third throw, $x < 4 \Rightarrow 1, 2, 3 \Rightarrow P_3 = \frac{3}{6} = \frac{1}{2}$

These events are independent

$P = P_1 \times P_2 \times P_3 = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{2}$

$\Rightarrow P = \frac{1}{18}$

4. (*b*) Given, the objective function $Z = ax + by$ has its maximum value on the segment AB.

Point $A(0, 8)$ and point $B(4, 0)$

$Z_A = Z_B \qquad$ (Maximum value)

$\Rightarrow a(0) + b(8) = a(4) + b(0)$

$\Rightarrow 8b = 4a \Rightarrow a = 2b$

5. (*a*) Given $t = e^{2x}$, $y = \log_e t^2$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dt}(\log_e t^2) \cdot \frac{d}{dx}(e^{2x})$

$\Rightarrow \frac{dy}{dx} = \frac{1}{t^2} \cdot 2t \cdot e^{2x} \cdot 2$

$\Rightarrow \frac{dy}{dx} = \frac{4e^{2x}}{t} = \frac{4e^{2x}}{e^{2x}}$

$\Rightarrow \frac{dy}{dx} = 4 \Rightarrow \frac{d^2y}{dx^2} = 0$

6. (*c*) Given, A and B are symmetric matrices.

$A^T = A$ and $B^T = B$

Let, $M = AB - BA$

$\Rightarrow M^T = (AB - BA)^T = (AB)^T - (BA)^T$

$\Rightarrow M^T = B^T A^T - A^T B^T$

$\Rightarrow M^T = BA - AB \qquad \Rightarrow M^T = -M$

$\therefore (AB - BA)$ is a skew-symmetric matrix.

7. (*b*) Given $A = [A]_{4\times4}$, $|A| = 4$

$|2A| = 2^4|A| = 2^4 \times 4$

$\Rightarrow |2A| = 64$

8. (*b*) $[A]_{3\times2}[B]_{x\times y} = [C]_{3\times1}$

For multiplication of matrices A and B, the number of rows of B should be equal to the number of column of A.

$\Rightarrow x = 2$

$[A]_{3\times2} \cdot [B]_{x\times y} = [C]_{3\times1}$

$\Rightarrow [C]_{3\times y} = [C]_{3\times1}$

$\Rightarrow y = 1$

Hence, $x = 2$ and $y = 1$.

9. (*c*) $f(x) = x^2 + bx + 1$ is increasing in the interval [1, 2].

$f'(x) = 2x + b$

$f'(x) > 0$

$f'(1) > 0 \Rightarrow 2 + b > 0 \Rightarrow b > -2 \qquad ...(i)$

$f'(2) > 0 \Rightarrow 4 + b > 0 \Rightarrow b > -4 \qquad ...(ii)$

From equation (*i*) and (*ii*), $b > -2$

The least value of b is -2.

10. (*b*) X represents the number of fours when two dice are thrown simultaneously.

$P(X = 0) = P(\bar{4})P(\bar{4})$

$= \left(1 - \frac{1}{6}\right) \times \left(1 - \frac{1}{6}\right) = \frac{25}{36}$

$P(X = 1) = P(\bar{4})P(4) + P(4)P(\bar{4})$

$= \frac{5}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{5}{6} = \frac{10}{36}$

$P(X = 2) = P(4)P(4) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

X	0	1	2
$P(X)$	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

$\therefore E(X) = \sum X_i P(X_i)$

$= 0 \times \frac{25}{36} + \frac{1 \times 10}{36} + 2 \times \frac{1}{36} = \frac{12}{36} \Rightarrow E(X) = \frac{1}{3}$

11. (*d*) Function $f(x) = 2x^3 - 9x^2 + 12x - 5$

$x \in [0, 3]$

$f'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2)$

$\Rightarrow f'(x) = 6(x - 1)(x - 2)$

For maxima and minima

$\Rightarrow f'(x) = 0 \Rightarrow 6(x - 1)(x - 2) = 0$

$\Rightarrow x = 1, x = 2$

$f(0) = 2(0)^3 - 9(0)^2 + 12 \times 0 - 5 = -5$ (Absolute Minimum)

$f(1) = 2(1)^3 - 9(1)^2 + 12 \times 1 - 5 = 0$

$f(2) = 2(2)^3 - 9(2)^2 + 12 \times 2 - 5 = -1$

$f(3) = 2(3)^3 - 9(3)^2 + 12 \times 3 - 5 = 4$ (Absolute Maximum)

Point of Maxima, $x = 3$

Point of Minima, $x = 0$

(A) - (IV), (B) - (III), (C) - (I), (D) - (II)

12. (*d*) $\frac{d^2y}{dx^2} = 5^x \quad \Rightarrow d\left(\frac{dy}{dx}\right) = 5^x dx$

Take integration both sides, we get

$\Rightarrow \int d\left(\frac{dy}{dx}\right) = \int 5^x dx \quad \Rightarrow \frac{dy}{dx} = \frac{5^x}{\log_e 5} + C$

$\Rightarrow dy = \frac{5^x}{\log_e 5} dx + C$

Again integrating, we get

$\Rightarrow \int dy = \int \frac{5^x}{\log_e 5} dx + C \quad \Rightarrow y = \frac{5^x}{(\log_e 5)^2}$

1

Chapter

RELATIONS AND FUNCTIONS

TOPIC-WISE QUESTIONS

Types of Relation

1. Let $R = \{(x, y) : x^2 + y^2 = 1, x, y \in R\}$ be a relation in R. The relation R is

(*a*) Reflexive (*b*) Symmetric
(*c*) Transitive (*d*) Anti-symmetric

2. The relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)} on set A = {1, 2, 3} is

(*a*) Reflexive but not symmetric
(*b*) Reflexive but not transitive
(*c*) Symmetric and transitive
(*d*) Neither symmetric nor transitive

3. Let R = {(a, a)} be a relation on a set A. Then R is

(*a*) Symmetric
(*b*) Antisymmetric
(*c*) Symmetric and antisymmetric
(*d*) Neither symmetric nor anti-symmetric

4. In the set A = {1, 2, 3, 4, 5}, a relation R is defined by $R = \{(x, y) | x, y \in A \text{ and } x < y\}$. Then R is

(*a*) Reflexive (*b*) Symmetric
(*c*) Transitive (*d*) None of these

5. The void relation on a set A is

(*a*) Reflexive
(*b*) Symmetric and transitive
(*c*) Reflexive and symmetric
(*d*) Reflexive and transitive

6. Let R = {(3,3), (6,6), (9,9), (12,12), (6,12), (3,9), (3,12), (3,6)} be a relation on the set A = {3, 6, 9, 12}. The relation is

(*a*) An equivalence relation
(*b*) Reflexive and symmetric only
(*c*) Reflexive and transitive only
(*d*) Reflexive only

7. The number of reflexive relations of a set with four elements is equal to

(*a*) 2^{16} (*b*) 2^{12}
(*c*) 2^{8} (*d*) 2^{4}

8. If A is the set of even natural numbers less than 8 and B is the set of prime numbers less than 7, then the number of relations from A to B is

(*a*) 2^9 (*b*) 9^2
(*c*) 3^2 (*d*) $2^9 – 1$

9. If A = {a, b, c, d}, then a relation R = {(a, b), (b, a), (a, a)} on A is

(*a*) Transitive only
(*b*) Symmetric only
(*c*) Reflexive and transitive
(*d*) Symmetric and transitive only

10. Let A = {1, 2, 3}. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is

(*a*) 1 (*b*) 4
(*c*) 3 (*d*) 2

11. Let A and B be the finite sets containing m and n elements respectively. The number of relations that can be defined from A to B is

(*a*) 2^{mn} (*b*) $2^{m + n}$
(*c*) mn (*d*) 0

12. Let R be the relation in the set N given by R = {(a, b) : a = b –2, b > 6} choose the correct answer.

(*a*) $(3, 8) \in R$ (*b*) $(2, 4) \in R$
(*c*) $(6, 8) \in R$ (*d*) $(8, 7) \in R$

13. Let R is reflexive relation on finite set A having n element, and let there be m ordered pairs R. Then

(*a*) $m \geq n$ (*b*) $m = n$
(*c*) $m \leq n$ (*d*) none of these

14. Let A = {7, 8, 9, 10} and R = {(8, 8) (9, 9), (10, 10), (7, 8)} be a relation on A, then R is

(*a*) Transitive (*b*) Symmetric
(*c*) Reflexive (*d*) None of these

15. If A = {7, 8, 9}, then the relation R = {(8, 9)} in A is

(*a*) Symmetric only
(*b*) Symmetric and transitive only
(*c*) Transitive only
(*d*) Equivalence

16. Let A be the finite set containing n distinct elements. The numbers of relations that can be defined on A is

(*a*) n^2 (*b*) 2^n
(*c*) 2^{n^2} (*d*) 2^{n-1}

17. Let R be the relations defined on the set N of natural numbers by the rule x R y iff x + 2 y = 8, then domain R is

(*a*) {2, 4, 8} (*b*) {2, 4, 6}
(*c*) {2, 4, 6, 8} (*d*) {1, 2, 3, 4}

18. If a relation R on the set {1, 2, 3} be defined by R = {(1, 2)}, then R is

(*a*) Symmetric (*b*) Transitive
(*c*) Reflexive (*d*) None of these

19. Let us define a relation R in R as a R b if a ≥ b. Then, R is

R = {(a, b) : a ≥ b}

(*a*) An equivalence relation
(*b*) Reflexive, transitive but not symmetric
(*c*) Symmetric, transitive but to reflexive
(*d*) Neither transitive nor reflexive but symmetric

20. The relation R defined on the set A = {1, 2, 3, 4, 5} by R = {(a, b) : $|a^2 - b^2| < 7$} is given by

(*a*) {(2, 2), (3, 2), (4, 2), (2, 4)}
(*b*) {(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)}
(*c*) {(3, 3), (4, 3), (5, 4), (3, 4)}
(*d*) {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 2), (2, 3)}

21. Let R be a relation on the set N of natural numbers defined by nRm if n divides m. Then R is

(*a*) Reflexive and symmetric
(*b*) Transitive and symmetric
(*c*) Equivalence
(*d*) Reflexive, transitive but not symmetric

22. Let L denote the set of all straight lines in a plane, Let relation R be defined by *l* Rm, iff *l* perpendicular to m for all *l* ∈ L Then, R is

(*a*) Reflexive (*b*) Symmetric
(*c*) Transitive (*d*) None of these

23. For real numbers x and y, we write xRy ⇒ $x - y + \sqrt{2}$ is an irrational number. Then the relation R is

(*a*) Reflexive (*b*) Symmetric
(*c*) Transitive (*d*) None of these

24. Consider a non-empty set consisting of children in family and a relation R defined as aRb, if a is brother of b. Then R is

(*a*) Symmetric but not transitive
(*b*) Transitive but not symmetric
(*c*) Neither symmetric not transitive
(*d*) Both symmetric and transitive

25. The maximum number of equivalence relations on the set A = {1, 2, 3} is

(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 5

26. If the set A contains 5 elements and the set B contains 6 elements, then the number of relations from A to B is 2P. Find p

(*a*) 720 (*b*) 120
(*c*) 30 (*d*) None of these

Types of Function

27. The function f : N → N (N is the set of natural numbers) defined by f(n) = 2n + 3 is

(*a*) Surjective only (*b*) Injective only
(*c*) Bijective (*d*) Neither one-one nor onto

28. Set A has 3 elements and set B has 4 elements. The number of injections that can be defined from A into B is

(*a*) 144 (*b*) 12
(*c*) 24 (*d*) 64

29. The function f : R → R, $f(x) = x^2$ is

(*a*) One-one but not onto
(*b*) Onto but not one-one
(*c*) One-one onto
(*d*) None of these

30. The function f: R → R defined by $f(x) = 2^x + 2^{|x|}$ is

(*a*) One-one and into (*b*) Many-one and into
(*c*) One-one and into (*d*) Many-one and into

31. If the set A contains 5 elements and the set B contains 6 elements, then the number of onto mapping from A to B is

(*a*) 720 (*b*) 120
(*c*) 0 (*d*) None of these

32. Let f : R → R be defined as f(x) = 3x. Choose the correct answer.

(*a*) f is one-one onto
(*b*) f is many one onto
(*c*) f is one-one but not onto
(*d*) f is neither one-one nor onto

33. Let f : R → R defined by $f(x) = \begin{cases} 3x, & \text{if } x > 3 \\ x^2, & \text{if } 1 < x \le 3 \\ x, & \text{if } x \le 1 \end{cases}$

Then f(–2) + f(0) + f(2) + f(5) is equal to

(*a*) –4 (*b*) 17
(*c*) 0 (*d*) one of these

34. Let $f : R \to [0, \frac{\pi}{2})$ defined by $f(x) = \tan^{-1}(x^2 + x + a)$, then the set of values of a for which f is onto is

(*a*) [0, ∞) (*b*) $[\frac{1}{4}, \infty)$
(*c*) [2, 1] (*d*) None of these

35. Let A = {x, y, z} and B = {a, b} then the number of onto function from A to B is

(*a*) 3 (*b*) 0
(*c*) 6 (*d*) 8

36. If A and B have 4 elements respectively then the number of one-one function A to B is

(*a*) 6^4 (*b*) 4^6
(*c*) 360 (*d*) 240

37. If A and B have 4 elements each then the number of one-one onto (bijective) function from A to B is

(*a*) 4^2 (*b*) 24
(*c*) 0 (*d*) None of these

38. The function $y = \frac{x}{1+|x|}, x \in R, y \in R$ is

(*a*) Onto but one-one
(*b*) One-one onto
(*c*) One-one but not onto
(*d*) None of these

39. Number of onto (subjective) functions from A to B if n(A) = 6 and n(B) = 3 are

(*a*) $3^6 - 3$ (*b*) $2^6 - 2$
(*c*) 540 (*d*) None of these

40. The function f: $[0, \infty) \rightarrow R$ given by $f(x) = \frac{x}{x+1}$, is

(*a*) One-one and onto
(*b*) One-one but not onto
(*c*) Onto but not one-one
(*d*) Neither one-one nor onto

41. The number of bijective functions from set A to itself when A contain 106 elements is

(*a*) $(106)^2$ (*b*) 106
(*c*) 106! (*d*) 2^{106}

42. The function f : R → R given by $f(x) = x^2 + x$, is

(*a*) One-one and onto
(*b*) One-one and into
(*c*) Many-one and onto
(*d*) Many one and into

Composition of Functions [RC]

43. If f : R → R, f(x) = 2x – 1 and g : R → R, $g(x) = x^2 + 2$, then (gof) (x) equals-

(*a*) $2x^2 - 1$ (*b*) $(2x - 1)^2$
(*c*) $2x^2 + 3$ (*d*) $4x^2 - 4x + 3$

44. $f(x) = \sqrt{|x-1|}$ and g (x) = sin x then (fog) (x) equals

(*a*) $\sin\left\{\sqrt{|x-1|}\right\}$ (*b*) $\left|\sin\frac{x}{2} - \cos\frac{x}{2}\right|$
(*c*) | sin x – cos x | (*d*) None of these

45. If $g(x) = x^2 + x - 2$ and $\frac{1}{2}$ (gof) (x) $= 2x^2 - 5x + 2$, then f(x) is equal to

(*a*) 2x – 3 (*b*) 2x + 3
(*c*) $2x^2 + 3x + 1$ (*d*) $2x^2 - 3x - 1$

46. If f (x) = |x| and g(x) = [x], then value of fog $\left(-\frac{1}{4}\right)$ + gof $\left(-\frac{1}{4}\right)$ is

(*a*) 0 (*b*) 1
(*c*) –1 (*d*) 1/4

47. If $f(x) = \sin^2 x$, $g(x) = \sqrt{x}$ and $h(x) = \cos^{-1} x$, $0 \le x \le 1$, then

(*a*) hogof(x) = gofoh(x)
(*b*) gofoh(x) = fohog(x)
(*c*) fohog(x) = hogof(x)
(*d*) None of these

48. Which of the following is correct for three function f, g, h defined from R to R?

(*a*) (f – g) oh = foh + goh
(*b*) $\left(\frac{f}{g}\right)$oh $= \frac{foh}{goh}$
(*c*) (fg) oh = (foh) (goh)
(*d*) (fog) oh = (foh) o (goh)

49. If f(x) = 2 sin x, g(x) = cos2x, then (f + g) $\left(\frac{\pi}{3}\right)$ =

(*a*) 1 (*b*) $\frac{2\sqrt{3}+1}{4}$
(*c*) $\sqrt{3} + \frac{1}{4}$ (*d*) None of these

50. Let: f: R → R be defined by f(x) = sin x and g : R → R be defined by $g(x) = x^2$ then fog is

(*a*) $x^2 \sin x$ (*b*) $(\sin x)^2$
(*c*) $\sin x^2$ (*d*) $\frac{\sin x}{x^2}$

51. Let f : R → R be defined by $f(x) = 3x^2 - 5$ and g : R → R by $g(x) = \frac{x}{x^2+1}$. Then g of is

(*a*) $\frac{3x^2-5}{9x^4-30x^2+26}$ (*b*) $\frac{3x^2-5}{9x^4-6x^2+26}$
(*c*) $\frac{3x^2}{x^4+2x^2-4}$ (*d*) $\frac{3x^2}{9x^4+30x^2-4}$

52. If $f(x) = \sin^2 x$ and the composite function g(f(x)) = |sin x|, then g(x) is equal to

(*a*) $\sqrt{x-1}$ (*b*) $\sqrt{x}$
(*c*) $\sqrt{x+1}$ (*d*) $-\sqrt{x}$

53. If $f(x) = (25 - x^4)^{1/4}$ for $0 < x < \sqrt{5}$, then $f\left(f\left(\frac{1}{2}\right)\right) =$

(*a*) 2^{-4} (*b*) 2^{-5}

(*c*) 2^{-2} (*d*) 2^{-1}

54. If $g(x) = 1 + \sqrt{x}$ and $f(g(x)) = 3 + 2\sqrt{x} + x$ then f(x) is equal to

(*a*) $1 + 2x^2$ (*b*) $2 + x^2$

(*c*) $1 + x$ (*d*) $2 + x$

55. If $f : R \to R$ and $g : R \to R$ are defined by $f(x) = 2x + 3$ and $g(x) = x^2 + 7$, then the values of x such that $g(f(x)) = 8$ are

(*a*) –1, –2 (*b*) –1, 2

(*c*) 1, 2 (*d*) 1, –2

Invertible Function [RC]

56. The inverse of a function exists if it is

(*a*) An injection (*b*) A surjection

(*c*) A bijection (*d*) Identity function

57. If $f : R \to R$, $f(x) = 2x + 1$ and $g : R \to R$, $g(x) = x^3$,then $(gof)^{-1}(27)$ equals -

(*a*) –1 (*b*) 0

(*c*) 1 (*d*) 2

58. Let $f : R \to R$ be defined by $f(x) = 3x - 4$. Then $f^{-1}(x)$ is given by

(*a*) $\frac{x+4}{3}$ (*b*) $\frac{x}{3} - 4$

(*c*) $3x + 4$ (*d*) None of these

59. Let $f : R \to R$ be the functions defined by $f(x) = x^3 + 5$. Then f^{-1} is

(*a*) $(x+5)^{\frac{1}{3}}$ (*b*) $(x-5)^{\frac{1}{3}}$

(*c*) $(5-x)^{\frac{1}{3}}$ (*d*) $5 - x$

60. Let $f : R - \left\{\frac{3}{5}\right\} \to R$ be defined by $f(x) = \frac{3x+2}{5x-3}$.

(*a*) $f^{-1}(x) = f(x)$ (*b*) $f^{-1}(x) = -f(x)$

(*c*) $fof(x) = x$ (*d*) Both (*a*) and (*c*)

61. If $f(x) = \frac{1-x}{1+x}, x \neq -1$, then $f^{-1}(x)$ equal to

(*a*) $f(x)$ (*b*) $\frac{1}{f(x)}$

(*c*) $-f(x)$ (*d*) $-\frac{1}{f(x)}$

62. Let $f : A \to B$ and $g : B \to C$ be bijections, then $(fog)^{-1} =$

(*a*) $f^{-1}og^{-1}$ (*b*) fog

(*c*) $g^{-1}of^{-1}$ (*d*) gof

63. Let $f : R \to R$, $g : R \to R$ be two functions, given by $f(x) = 2x - 3$, $g(x) = x^3 + 5$. Then $(fog)^{-1}(x)$ is equal to

(*a*) $\left(x - \frac{7}{2}\right)^{1/3}$ (*b*) $\left(\frac{x+7}{2}\right)^{1/3}$

(*c*) $\left(\frac{x-2}{7}\right)^{1/3}$ (*d*) $\left(\frac{x-7}{2}\right)^{1/3}$

64. If $f(x) = \frac{3x+2}{5x-3}$, then

(*a*) $f^{-1}(x) = f(x)$ (*b*) $(fof)(x) = -x$

(*c*) $f^{-1}(x) = -f(x)$ (*d*) $f^{-1}(x) = -\frac{1}{19}f(x)$

65. $f : R \to R$ is a function defined by $f(x) = 10x - 7$. If $g = f^{-1}$, then $g(x) =$

(*a*) $\frac{1}{10x+7}$ (*b*) $\frac{1}{10x-7}$

(*c*) $\frac{x+7}{10}$ (*d*) $\frac{x-7}{10}$

RANK BOOSTER

Match the Column MCQs

1. Match Column-I with Column-II.

	Column-I		Column-II
A.	f is an invertible function defined as $f(x) = \frac{3x-5}{5}$, then write $f^{-1}(x)$	(i)	$\frac{2x-1}{x-1}$
B.	If $A = R - \{2\}$, $B = R - \{1\}$ & $f : A \to B$ is a function defined by $f(x) = \frac{x-1}{x-2}$ is invertible. Hence, find f^{-1}.	(ii)	$\frac{-1+\sqrt{4x-3}}{2}$
C.	Function $f : N \to N$ is defined by $f(x) = x^2 + x + 1$ is one-one but not onto. Find inverse of $f : N \to S$, where S is a range of f.	(iii)	$\frac{x+4}{3}$

D.	Function $f : R \to R$ is defined by $f(x) = 3x - 4$, find f^{-1}, where f is invertible	(iv)	$\frac{5x+4}{3}$

Choose the correct answer from the options given below:

(*a*) A-(iii), B-(i), C-(iv), D-(ii)
(*b*) A-(iv), B-(i), C-(ii), D-(iii)
(*c*) A-(iii), B-(i), C-(ii), D-(iii)
(*d*) A-(iv), B-(i), C-(iii), D-(ii)

2. Match Column-I with Column-II.

	Column-I		Column-II
A.	$f : R \to R$ defined by $f(x) = ax + b$ is	(i)	injection but not surjection
B.	$f : R \to [0, \infty)$ defined by $f(x) = \|x\|$ is	(ii)	surjection but not injection
C.	$f : R \to R$ defined by $f(x) = [x]$ is	(iii)	bijection
D.	$f : N \to N$ defined by $f(x) = x^3$ is	(iv)	neither injection nor surjection

Choose the correct answer from the options given below:

(*a*) A-(ii), B-(iv), C-(i), D-(iii)
(*b*) A-(ii), B-(iv), C-(iii), D-(i)
(*c*) A-(iii), B-(ii), C-(iv), D-(i)
(*d*) A-(iii), B-(ii), C-(i), D-(iv)

3. Match Column-I with Column-II.

	Column-I		Column-II
A.	If $f : R \to R$ is defined by $f(x) = 3x + 2$, define $f[f(x)]$	(i)	x
B.	If $f : R \to R$ and $g : R \to R$ are given by $f(x) = \|x\|$ & $g(x) = \|5x - 2\|$, write fog	(ii)	$9x + 8$
C.	Write fog, if $f : R \to R$ & $g : R \to R$ are given by $f(x) = 8x^3$ and $g(x) = (x)^{1/3}$	(iii)	8x
D.	If $f : R \to R$ is defined by $f(x) = (3 - x^3)^{1/3}$, then find $fof(x)$	(iv)	$\|5x - 2\|$

Choose the correct answer from the options given below:

(*a*) A-(ii), B-(iv), C-(iii), D-(i)
(*b*) A-(ii), B-(iii), C-(i), D-(iv)
(*c*) A-(ii), B-(iv), C-(i), D-(iii)
(*d*) A-(iv), B-(iii), C-(i), D-(ii)

4. Match Column-I with Column-II.

	Column-I		Column-II
A.	Relation R on R defined as $R = \{(a, b) : (a \le b)\}$ is	(i)	an equivalence relation
B.	$f : X \to Y$ is a function. Define relation R on X given by $R = \{(a, b) : f(a) = f(b)\}$ is	(ii)	symmetric but not reflexive or transitive
C.	Relation R on N defined as $R = \{(x, y) : x \in N, y \in N \text{ \& } 2x + y = 24\}$ is	(iii)	not symmetric but reflexive & transitive
D.	Let $A = \{5, 6, 7\}$. Define a relation R on A as $R = \{(5, 6) (6, 5)\}$ is	(iv)	neither symmetric nor transitive nor reflexive

Choose the correct answer from the options given below:

(*a*) A-(i), B-(ii), C-(iii), D-(iv)
(*b*) A-(ii), B-(iv), C-(iii), D-(i)
(*c*) A-(iii), B-(i), C-(iv), D-(ii)
(*d*) A-(iv), B-(i), C-(ii), D-(iii)

5. Match Column-I with Column-II.

	Column-I		Column-II
A.	R on set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b)$: 2 divides $(a - b)\}$ write equivalence class [0]	(i)	$\{(1, 3), (2, 4)\}$
B.	If $A = \{1, 2, 3, ..., 9\}$ and R is the relations $A \times A$ defined by (a, b) R (c, d), if $a + d = b + c$ for (a, b), (c, d) in $A \times A$. write equivalence class [(2, 5)]	(ii)	$\{(1, 4) (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$
C.	Relations S defined on set $A = \{1, 2, 3, 4\}$ by (a, b) S (c, d) $\Rightarrow a + d = b + c$. Write equivalence class [(1, 3)]	(iii)	$\{1, 3, 5\}$ $\{2, 4\}$
D.	Relation R in set $A = \{1, 2, 3, 4, 5\}$ given $R = \{(a, b) : \|a - b\|$ is divisible by 2$\}$. Write all the equivalence classes of R	(iv)	$\{0, 2, 4\}$

Choose the correct answer from the options given below:

(*a*) A-(ii), B-(i), C-(iii), D-(iv)
(*b*) A-(iv), B-(i), C-(ii), D-(iii)
(*c*) A-(iv), B-(ii), C-(iii), D-(i)
(*d*) A-(iv), B-(ii), C-(i), D-(iii)

Assertion & Reason MCQs

Directions: These questions consist of two statements each, printed as Assertion and Reason. While answering these questions, you are required to choose any one of the following four responses.

(*a*) Both Assertion and Reason are True and the Reason is a correct explanation of the Assertion.

(*b*) Both Assertion and Reason are True but Reason is not a correct explanation of the Assertion.

(*c*) Assertion is True but the Reason is False.

(*d*) Assertion is False but Reason is True.

6. **Assertion (A):** Every even function is many one.

Reason (R): Any function which is symmetrical about a line is many one.

7. **Assertion (A):** $y = f(x) = \frac{x^2 - 2x + 4}{x^2 - 2x + 5}$, $x \in R$, range of f(x) is [3/4, 1) ***[RC]***

Reason (R): $(x-1)^2 = \frac{4y-3}{1-y}$

8. **Assertion (A):** Even functions are not one-one.

Reason (R): Even functions are symmetrical about y-axis.

9. **Assertion (A):** Let $f(x) = x^2$, $g(x) = \cos x$. Then $fog \neq gof$ ***[RC]***

Reason (R): $(fog)(x) = f(x)g(x)$

10. **Assertion (A):** Let f: [0, 3] $\to$ [1, 13] is defined by $f(x) = x^2 + x + 1$ then inverse is ***[RC]***

$f^{-1}(x) = \frac{-1+\sqrt{4x-3}}{2}$

Reason (R): Many-one function is not invertible

Statement Based MCQs

Directions: These questions consist of two statements each, printed as Statement-I and Statement-II. While answering these questions, you are required to choose any one of the following four responses.

(*a*) Both Statement-I and Statement-II are correct.

(*b*) Both Statement-I and Statement-II are incorrect.

(*c*) Statement-I is correct & Statement-II is incorrect.

(*d*) Statement-I is incorrect & Statement-II is correct.

11. **Statement-I:** Let L be the collection of all lines in a plane and R_1 be the relation on L as $R_1 = \{(L_1, L_2) : L_1 \perp L_2\}$ is a symmetric relation.

Statement-II: A relation R is said to be symmetric if $(a, b) \in R \Rightarrow (b, a) \in R$.

12. **Statement-I:** Let R be the relation on the set of integers Z given by R = {(a, b) : 2 divides (a – b)} is an equivalence relation.

Statement-II: A relation R in a set A is said to be an equivalence relation if R is reflective, symmetric and transitive.

13. **Statement-I:** Let f : R $\to$ R give by f(x) = x, then f is a one-one function.

Statement-II: A function g : A $\to$ B is said to be onto function if for each $b \in B$, $\exists\, a \in A$ such that g(a) = b.

Case Based Questions

Case Based-I

In two different societies, there are some school going students - including girls as well as boys. Satish forms two sets with these students, as his college project.

Let $A = \{a_1, a_2, a_3, a_4, a_5\}$ and $B = \{b_1, b_2, b_3, b_4\}$ where a_i's and b_i' s are the school going students of first and second society respectively.

Satish decides to explore these sets for various types of relations and functions.

Based on the above information, answer the following questions.

14. Satish wishes to know the number of reflexive defined on set A. How many such relations are possible?

(*a*) 0 (*b*) 2^5

(*c*) 2^{10} (*d*) 2^{20}

15. Let R :A $\to$ A, R = {(x, y) : x and y are students of same sex}. Then relation R is

(*a*) Reflexive only

(*b*) Reflexive and symmetric but not transitive

(*c*) Reflexive and transitive but not symmetric

(*d*) An equivalence relation

16. Satish and his friend Rajat are interested to know the number of symmetric relations defined on both the sets A and B, separately. Satish decides to find the symmetric relation on set A, while Rajat decides to find the symmetric relation on set B. what is difference between their results?

(*a*) 1024 (*b*) 2^{10} (15)

(*c*) 2^{10}(31) (*d*) 2^{10}(63)

17. Let R : A $\to$ B, $R = \{(a_1, b_1), (a_1, b_2), (a_2, b_1), (a_3, b_3), (a_4, b_2), (a_5, b_2)\}$, then R is

(*a*) A function

(*b*) One-one but, not onto

(*c*) Only onto, but not one-one

(*d*) Not a function

18. To help Satish in his project , Rajat decides to form onto function form set A to B. How many such functions are possible?

(a) 342 (b) 240

(c) 729 (d) 1024

Case Based-II

A relation R on a set a is said to be an equivalence relation on A iff it is

- Reflexive i.e., $(a, a) \in R\ \forall\ a \in A$.
- Symmetric i.e., $(a, b) \in R \Rightarrow (b, a) \in R\ \forall\ a, b \in A$.
- Transitive i.e., $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R\ \forall\ a, b, c \in A$.

Based on the above information, answer the following questions.

19. If the relation R = {(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)} defined on the set A = {1, 2, 3}, then R is

(a) Reflexive (b) Symmetric

(c) Transitive (d) Equivalence

20. If the relation R = {(1, 2), (2, 1), (1, 3), (3, 1)} defined on the set A = {1, 2, 3}, then R is

(a) Reflexive (b) Symmetric

(c) Transitive (d) Equivalence

21. If the relation R on the set N of all natural numbers defined as R = {(x, y) : y = x + 5 and x < 4}, then R is

(a) Reflexive (b) Symmetric

(c) Transitive (d) Equivalence

22. If the relation R on the set A = {1, 2, 3, ..., 13, 14} defined as R = {(x, y) : 3x – y = 0}, then R is

(a) Reflexive (b) Symmetric

(c) Transitive (d) Equivalence

23. If the relation R on the set A = {1, 2, 3} defined as R = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2) (2, 3), (3, 1), (3, 2), (3, 3)}, then R is

(a) Reflexive only (b) Symmetric only

(c) Transitive only (d) Equivalence

Case Based-III

Consider the mapping $f : A \to B$ is defined by $f(x) = \frac{x-1}{x-2}$ such that f is a bijection.

Based on the above information, answer the following questions.

24. Domain of f is

(a) R – {2} (b) R

(c) R – {1, 2} (d) R – {0}

25. Range of f is

(a) R (b) R –{1}

(c) R–{0} (d) R–{1, 2}

26. If $g : R - \{R\} \to R - \{1\}$ is defined by g(x) = 2f(x) – 1, then g(x) in terms of x is

(a) $\frac{x+2}{x}$ (b) $\frac{x+1}{x-2}$

(c) $\frac{x-2}{x}$ (d) $\frac{x}{x-2}$

27. The function g defined above, is

(a) One-one (b) Many-one

(c) Into (d) None of these

28. A function f(x) is said to be one-one iff

(a) $f(x_1) = f(x_2) \Rightarrow -x_1 = x_2$

(b) $f(-x_1) = f(-x_2) \Rightarrow -x_1 = x_2$

(c) $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

(d) None of these

PAST YEAR QUESTIONS

1. Relation R on Real Numbers is defined as R = {(a, b) : a ≤ b}.The relation is:

(CUET 2023)

(a) Reflexive and Symmetric but not Transitive

(b) Symmetric and Transitive but not Reflexive

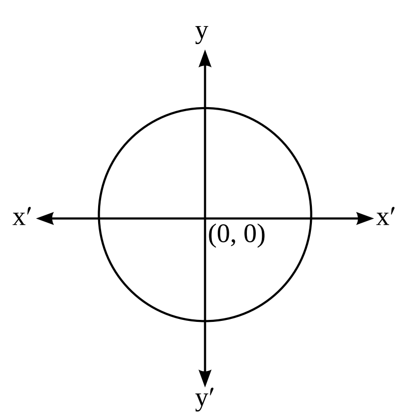
(c) Reflexive and Transitive but not Symmetric

(d) Equivalence relation

2. Which of the following graphs represent a function?

(CUET 2023)

(a)



10. A. A relation R on a set is called an equivalence relation, if it is reflexive, symmetric and transitive. **(CUET 2022)**

B. The function f : R → R defined by $f(x) = e^x$ is not one-one.

C. The one-one function is also known as injective function.

D. The onto function is also known as surjective function.

E. A function f : X → Y is said to be many-one, if two or more than two elements in set X have the different image in set Y.

Choose the correct answer from the option given below:

(*a*) A, C, D only
(*b*) B, C, D only
(*c*) C, D, E only
(*d*) B, D, E only

11. Which of the following is not an equivalence relation on Z? **(CUET 2022)**

(*a*) a R b ⇔ a + b is an even integer
(*b*) a R b ⇔ a – b is an even integer
(*c*) a R b ⇔ a ≤ b
(*d*) a R b ⇔ a = b

12. The function f : R – {–1} → R – {1} defined by $f(x) = \frac{x}{x+1}$ is **(CUET 2022)**

(*a*) Both one-one and onto
(*b*) Only one-one
(*c*) Only onto
(*d*) Neither one-one nor onto

13. If $f(x) = \sqrt{x}$ and g(x) = 2x – 3, then domain of fog(x) is **(NTA CUET 2022)**

(*a*) [3/2, ∞)
(*b*) [1/2, ∞)
(*c*) (3/2, ∞)
(*d*) [5/2, ∞)

14. If A = {1, 2, 3} and B = {1, 5, 7, 9}, then the number of one-one function from A into B is **(NTA CUET 2022)**

(*a*) 12 (*b*) 24
(*c*) 36 (*d*) 6

Answer Key

Topic-Wise Questions

1. (*b*)	**2.** (*a*)	**3.** (*c*)	**4.** (*c*)	**5.** (*b*)	**6.** (*c*)	**7.** (*d*)	**8.** (*a*)	**9.** (*d*)	**10.** (*a*)
11. (*a*)	**12.** (*c*)	**13.** (*a*)	**14.** (*a*)	**15.** (*c*)	**16.** (*c*)	**17.** (*b*)	**18.** (*b*)	**19.** (*b*)	**20.** (*d*)
21. (*d*)	**22.** (*b*)	**23.** (*a*)	**24.** (*b*)	**25.** (*d*)	**26.** (*c*)	**27.** (*b*)	**28.** (*c*)	**29.** (*d*)	**30.** (*c*)
31. (*c*)	**32.** (*a*)	**33.** (*b*)	**34.** (*b*)	**35.** (*c*)	**36.** (*c*)	**37.** (*b*)	**38.** (*c*)	**39.** (*c*)	**40.** (*b*)
41. (*c*)	**42.** (*d*)	**43.** (*d*)	**44.** (*b*)	**45.** (*a*)	**46.** (*b*)	**47.** (*d*)	**48.** (*b*)	**49.** (*c*)	**50.** (*c*)
51. (*a*)	**52.** (*b*)	**53.** (*d*)	**54.** (*b*)	**55.** (*a*)	**56.** (*c*)	**57.** (*c*)	**58.** (*a*)	**59.** (*b*)	**60.** (*d*)
61. (*a*)	**62.** (*c*)	**63.** (*d*)	**64.** (*a*)	**65.** (*c*)					

Rank Booster

1. (*b*)	**2.** (*c*)	**3.** (*a*)	**4.** (*c*)	**5.** (*d*)	**6.** (*c*)	**7.** (*a*)	**8.** (*a*)	**9.** (*c*)	**10.** (*b*)
11. (*a*)	**12.** (*a*)	**13.** (*a*)	**14.** (*d*)	**15.** (*d*)	**16.** (*c*)	**17.** (*d*)	**18.** (*b*)	**19.** (*a*)	**20.** (*b*)
21. (*c*)	**22.** (*d*)	**23.** (*d*)	**24.** (*a*)	**25.** (*b*)	**26.** (*d*)	**27.** (*a*)	**28.** (*c*)		

Past Year Questions

1. (*c*)	**2.** (*c*)	**3.** (*b*)	**4.** (*d*)	**5.** (*d*)	**6.** (*b*)	**7.** (*a*)	**8.** (*d*)	**9.** (*d*)	**10.** (*a*)
11. (*c*)	**12.** (*a*)	**13.** (*a*)	**14.** (*b*)						

Explanations

Topic-Wise Questions

1. (*b*) We have R = {(x, y) : $x^2 + y^2 = 1$; x, y ∈ R}.
At x = 4, y = 4,
$(4)^2 + (4)^2 = 32 \neq 1$. ∴ (4, 4) ∉R.
∴ R is not reflexive.
Let (x, y) ∈ R.
∴ $x^2 + y^2 = 1 \Rightarrow y^2 + x^2 = 1$
⇒ (y, x) ∈R
∴ R is symmetric.
(0, 1), (1, 0) ∈ R because
$(0)^2 + (1)^2 = 1$ and $(1)^2 + (0)^2 = 1$.
Also $(0)^2 + (0)^2 = 0 \neq 1$
∴ (0, 0) ∉R.
∴ R is not transitive.

2. (*a*) Since (1, 1); (2, 2); (3, 3) ∈ R therefore R is reflexive. (1, 2) ∈ R but (2, 1) ∉ R, therefore R is not symmetric. It can be easily seen that R is transitive as (1, 2) ∈ R, (2, 3) ∈ R and (1, 3) ∈ R.

3. (*c*) R = {(a, a)}
{a, a} ⇔ {a, a}
Hence, relation R is symmetric and antisymmetric.

4. (*c*) Since $x \nless x$, therefore R is not reflexive. Also x < y does not imply that y < x. So R is not symmetric. Let x Ry and y R z. Then, x < y and y < z ⇒ x < z i.e., x R z. Hence R is transitive.

5. (*b*) The void relation R on A is not reflexive as (a, a) ∉ R for any a ∈ A. The void relation is symmetric and transitive.

6. (*c*) Here (3, 3), (6, 6), (9, 9), (12, 12) ∈ R [Reflexive];
(3, 6) ∈ R, (6, 12) ∈ R then (3, 12) ∈ R [Transitive].
Hence, reflexive and transitive only.

7. (*d*) Total number of reflexive relations in a set with n elements = 2^n.
Therefore, total number of reflexive relation set with 4 elements = 2^4.

8. (*a*) A = {2, 4, 6}; B = {2, 3, 5}
∴ A × B contains 3 × 3 = 9 elements.
Hence, number of relation from A to B = 2^9.

9. (*d*) On the set A = {a, b, c, d} given relation R = {(a, b), (b, a), (a, a)}.
Since, (a, b) ∈ R ⇒ (b, a) ∈ R, therefore it is symmetric
Also, (a, b) ∈ R and (b, a) ∈ R ⇒ (a, a) ∈ R, so it is also transitive. As (b, b), (c, c) and (d, d) does not belong to R hence R is not reflexive.
Hence relation R is symmetric and transitive only.

10. (*a*) Required relation is reflexive and symmetric but not transitive is given by R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3), (3, 1)}
which is reflexive as (a, a) ∈ R ∀ a ∈ A
which is symmetric as (a, b) ∈ R ⇒ (b, a) ∈ R for a, b ∈ A
But (2, 1), (1, 3) ∈ R ⇒ (2, 3) ∉ R
Hence R is not transitive.
There is only one such relation.

11. (*a*) We have n(A) = m, n(B) = n.
∴ Number of relations defined form A to B
= number of possible subsets of A × B
= 2^{mn}

12. (*c*) ∵ b > 6 and a = b – 2
⇒ (6, 8) ∈ R as 8 > 6 and 6 = 8 – 2

13. (*a*) A set has n elements and the relation is said to be reflexive if it consists of at least n ordered pairs. There exists m ordered pairs.
Therefore, the minimum number of ordered pair in R is n.
⇒ m ≥ n.

14. (*a*) As (7, 7) ∉ R, so R can not be reflexive
Again (7, 8) ∈ R but (8, 7) ∉ R, so R is not symmetric.
As (7, 8), (8, 8) ∈ R ⇒ (7, 8) ∈ R ⇒ R is transitive.

15. (*c*) Given R = {(8, 9)} as (8, 9) belongs to R but (9, 8) does not belongs to R so R is not symmetric and R is transitive as there is only 1 element in R.

16. (*c*) Number of relations that can be defined on A = 2^{n^2}

17. (*b*) x, y, belongs to N ⇒ x, y = 1, 2, ... As x + 2y = 8 & y = 1, 2, ...
⇒ x = {5, 4, 2} for y = 1, 2, 3
∴ domain R = {2, 4, 6}.

18. (*b*) R = {(1, 2)}, A = {1, 2, 3}
Clearly R is neither reflexive nor symmetric.
As (1, 2) ∈ R but ∄ (2, b) ∈ R for b ∈ A such that (1, b) ∈ R.
Hence R is a transitive relation on A.

19. (*b*) R = {(a, b) : a ≥ b}
For reflexive
Clearly (a, a) ∈ R ∀ a ∈ R.
Hence R is reflexive.
For symmetric:
∵ (2, 1) ∈ R but (1, 2) ∉ R, 1 ≱ 2
Hence, R is not symmetric
For transitive
Let (a, b) and (b, c) ∈ R
⇒ a ≥ b and b ≥ c
⇒ a ≥ c
Hence (a, b) and (b, c) ∈ R ⇒ (a, c) ∈ R
⇒ R is a transitive relation on R.

20. (*d*) R = {(x, y) : $|x^2 - y^2| < 7$}
R = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 2), (2, 3)}

21. (*d*) Since n divides n, ∀ n ∈ N, R is reflexive. R is not symmetric since for 3, 6 ∈ N, 3 R 6 ≠ 6 R 3. R is transitive since for n, m, r whenever n/m and m/r ⇒ n/r, i.e., n divides m and m divides r, then n will divide r.

22. (*b*) We know that if line *l* is perpendicular to line m, line m must also be perpendicular to line *l*. So, *l* Rm ⇔ mR*l* and thus Relation R is symmetric.

23. (*a*) For each value of x ∈ R, x – x + $\sqrt{2}$ that is $\sqrt{2}$ is an irrational number.
It is reflexive.
Let x = $\sqrt{2}$ and y = 2 then x – y + $\sqrt{2}$ = $2\sqrt{2}$ – 2 which is irrational but when y = $\sqrt{2}$ and x = 2, x – y + $\sqrt{2}$ = 2 is not irrational.
It is not symmetric
Let x – y + $\sqrt{2}$ is irrational and y – z + $\sqrt{2}$ is irrational then in above case let x = 1; y = $\sqrt{2}$ × 2 and z = 2
Hence x – z + $\sqrt{2}$ is not irrational, so, the relation is not transitive.

24. (*b*) The relation R defined on a non-empty set consisting of children in a family is not reflexive. Since a can't

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