Part 2

OLYMPIAD WALLAH

JEEnius Edge 12

MATHEMATICS

For JEE, ISI-CMI & Olympiad Maths

Ultimate Resource for JEE, ISI-CMI & Olympiad Maths Preparation

CONTENTS

EXERCISE-1

1. $\int \frac{1-x^7}{x\left(1+x^7\right)}dx$ equals -

(A) $\ln x+\frac{2}{7}\ln\left(1+x^7\right)+C$

(B) $\ln x-\frac{2}{7}\ln\left(1-x^7\right)+C$

(C) $\ln x-\frac{2}{7}\ln\left(1+x^7\right)+C$

(D) $\ln x+\frac{2}{7}\ln\left(1-x^7\right)+C$

2. Primitive of $\frac{3x^4-1}{\left(x^4+x+1\right)^2}$ w.r.t.x is –

(A) $\frac{x}{x^4+x+1}+C$ (B) $-\frac{x}{x^4+x+1}+C$

(C) $\frac{x+1}{x^4+x+1}+C$ (D) $-\frac{x+1}{x^4+x+1}+C$

3. If $\int\frac{\cos x-\sin x+1-x}{e^x+\sin x+x}dx = \ell n\left(f(x)\right)+g(x)+C$ where C is the constant of integration and f(x) is positive, then $f(x) + g(x)$ has the value equal to

(A) $e^x + \sin x + 2x$ (B) $e^x + \sin x$

(C) $e^x - \sin x$ (D) $e^x + \sin x + x$

4. Integral of $\sqrt{1+2\cot x\left(\cot x+\operatorname{cosec} x\right)}$ w.r.t.x is

(A) $2\ell n\cos\frac{x}{2}+C$

(B) $2\ell n\sin\frac{x}{2}+C$

(C) $\frac{1}{2}\ell n\cos\frac{x}{2}+C$

(D) $\ell n \sin x - \ell n(\operatorname{cosec} x - \cot x) + C$

5. $\int x.\frac{\ln\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}dx$ equals –

(A) $\sqrt{1+x^2}\ln\left(x+\sqrt{1+x^2}\right)-x+C$

(B) $\frac{x}{2}.\ln^2\left(x+\sqrt{1+x^2}\right)-\frac{x}{\sqrt{1+x^2}}+C$

(C) $\frac{x}{2}.\ln^2\left(x+\sqrt{1+x^2}\right)+\frac{x}{\sqrt{1+x^2}}+C$

(D) $\sqrt{1+x^2}\ln\left(x+\sqrt{1+x^2}\right)+x+C$

6. Let g(x) be an antiderivative for f(x). Then $\ell n\left(1+\left(g(x)\right)^2\right)$ is an antiderivative for

(A) $\frac{2f(x)g(x)}{1+\left(f(x)\right)^2}$ (B) $\frac{2f(x)g(x)}{1+\left(g(x)\right)^2}$

(C) $\frac{2f(x)}{1+\left(f(x)\right)^2}$ (D) none

7. A function y = f(x) satisfies $f''(x)=-\frac{1}{x^2}-\pi^2\sin(\pi x); f'(2)=\pi+\frac{1}{2}$ and $f(1)=0$. The value of $f\left(\frac{1}{2}\right)$ is

(A) $\ell n2$ (B) 1

(C) $\frac{\pi}{2}-\ell n2$ (D) $1-\ell n2$

8. Consider $f(x)=\frac{x^2}{1+x^3}$; $g(t)=\int f(t)dt$.

If g (1) = 0 then g(x) equals –

(A) $\frac{1}{3}\ell n\left(1+x^3\right)$ (B) $\frac{1}{3}\ell n\left(\frac{1+x^3}{2}\right)$

(C) $\frac{1}{2}\ell n\left(\frac{1+x^3}{3}\right)$ (D) $\frac{1}{3}\ell n\left(\frac{1+x^3}{3}\right)$

9. $\int\frac{e^{\sqrt{x}}}{\sqrt{x}}(x+\sqrt{x})dx$

(A) $2e^{\sqrt{x}}\left[x-\sqrt{x}+1\right]+C$

(B) $e^{\sqrt{x}}\left[x-2\sqrt{x}+1\right]+C$

(C) $e^{\sqrt{x}}(x+\sqrt{x})+C$

(D) $e^{\sqrt{x}}(x+\sqrt{x}+1)+C$

10. $\int \frac{dx}{\sqrt[3]{x^{5/2}(x+1)^{7/2}}}$

(A) $-\left(\frac{x+1}{x}\right)^{1/6}+C$ (B) $6\left(\frac{x+1}{x}\right)^{-1/6}+C$

(C) $\left(\frac{x}{x+1}\right)^{5/6}+C$ (D) $-\left(\frac{x}{x+1}\right)^{5/6}+C$

11. Let $f(x)=\frac{2\sin^2 x-1}{\cos x}+\frac{\cos x(2\sin x+1)}{1+\sin x}$ then $\int e^x\left(f(x)+f'(x)\right)dx$

(where C is the constant of integration)

(A) $e^x \tan x + C$ (B) $e^x \cot x + C$

(C) $e^x \operatorname{cosec}^2 x + C$ (D) $e^x \sec^2 x + C$

12. $\int \frac{x^2(1-\ell nx)}{\ell n^4 x - x^4}dx$ equals

(A) $\frac{1}{2}\ell n\left(\frac{x}{\ell nx}\right)-\frac{1}{4}\ell n\left(\ell n^2 x - x^2\right)+C$

(B) $\frac{1}{4}\ell n\left(\frac{\ell nx-x}{\ell nx+x}\right)-\frac{1}{2}\tan^{-1}\left(\frac{\ell n x}{x}\right)+C$

(C) $\frac{1}{4}\ell n\left(\frac{\ell nx+x}{\ell nx-x}\right)+\frac{1}{2}\tan^{-1}\left(\frac{\ell n x}{x}\right)+C$

(D) $\frac{1}{4}\left(\ell n\left(\frac{\ell nx-x}{\ell nx+x}\right)+\tan^{-1}\left(\frac{\ell n x}{x}\right)\right)+C$

13. $\int \frac{(2x+3)}{x(x+1)(x+2)(x+3)+1}=C-\frac{1}{f(x)}$, where $f(x)$ is of the form of $ax^2 + bx + c$ then $(a + b + c)$ equals

(A) 4 (B) 5

(C) 6 (D) none

14. $\int e^x\left(\frac{x^2-3}{(x-1)^2}\right)dx$ is equal to –

(A) $e^x\left(\frac{x+3}{x-1}\right)+C$ (B) $e^x\left(\frac{x-3}{x-1}\right)+C$

(C) $e^x\left(\frac{x+1}{x-1}\right)+C$ (D) $e^x\left(\frac{1}{x-1}\right)^2+C$

(where 'C" is integration constant)

15. $\int \frac{x^3}{\left(2x^2+1\right)^3}dx$ is equal to –

(A) $\frac{1}{4}\left(2+\frac{1}{x^2}\right)^{-2}+C$ (B) $-\frac{1}{4}\left(2+\frac{1}{x^2}\right)^{-2}+C$

(C) $\frac{1}{2}\left(2+\frac{1}{x^2}\right)^{-2}+C$ (D) $\frac{1}{4}\left(2+\frac{1}{x^2}\right)^{2}+C$

(where 'C' is integration constant)

EXERCISE-2

1. $\int \frac{\left(\sqrt{x}+1\right)\left(x^2-\sqrt{x}\right)}{x\sqrt{x}+x+\sqrt{x}}dx$

2. A function g defined for all positive real numbers, satisfies $g'(x^2) = x^3$ for all $x > 0$ and $g(1) = 1$. Compute $g(4)$.

3. $\int\left[\sin\alpha\sin(x-\alpha)+\sin^2\left(\frac{x}{2}-\alpha\right)\right]dx$

4. $\int \frac{x^2+3}{x^6\left(x^2+1\right)}dx$

5. $\int \frac{dx}{\cot\frac{x}{2}.\cot\frac{x}{3}.\cot\frac{x}{6}}$

6. $\int \sqrt{\frac{\operatorname{cosec}x-\cot x}{\operatorname{cosec}x+\cot x}}.\frac{\sec x}{\sqrt{1+2\sec x}}dx$

7. $\int \frac{\ln\left(\ln\left(\frac{1+x}{1-x}\right)\right)}{1-x^2}dx$

8. $\int\left[\left(\frac{x}{e}\right)^x+\left(\frac{e}{x}\right)^x\right]\ell n\, x\, dx$

9. $\int \frac{x^5+3x^4-x^3+8x^2-x+8}{x^2+1}dx$

10. $\int \frac{\left(\sqrt{x}+1\right)dx}{\sqrt{x}\left(\sqrt[3]{x}+1\right)}$

11. $\int \sin^{-1}\sqrt{\frac{x}{a+x}}dx$

12. $\int \frac{x\ell nx}{(x^2-1)^{3/2}}.dx$

13. $\int\left[\frac{\sqrt{x^2+1}[\ell n(x^2+1)-2\ell nx]}{x^4}\right]dx$

14. $\int \frac{\tan 2\theta}{\sqrt{\cos^6\theta+\sin^6\theta}}d\theta$

15. $\int \frac{3x^2+1}{\left(x^2-1\right)^3}dx$

16. $\int \frac{\left(ax^2-b\right)dx}{x\sqrt{c^2x^2-\left(ax^2+b\right)^2}}$

17. $\int \frac{\left(e^{\sqrt{x}}-e^{-\sqrt{x}}\right)\cos\left(e^{\sqrt{x}}+e^{-\sqrt{x}}+\frac{\pi}{4}\right)+\left(e^{\sqrt{x}}+e^{-\sqrt{x}}\right)\cos\left(e^{\sqrt{x}}-e^{-\sqrt{x}}+\frac{\pi}{4}\right)}{\sqrt{x}}dx$

18. $\int \frac{x^2+x}{\left(e^x+x+1\right)^2}dx$

19. $\int \frac{e^{\cos x}\left(x\sin^3 x+\cos x\right)}{\sin^2 x}dx$

20. $\int \frac{5x^4+4x^5}{\left(x^5+x+1\right)^2}dx$

21. $\int (\sin x)^{-11/3}(\cos x)^{-1/3}dx$

22. $\int \frac{dx}{\sin x+\sec x}$

23. $\int \frac{4x^5-7x^4+8x^3-2x^2+4x-7}{x^2\left(x^2+1\right)^2}dx$

24. $\text{Let}\int \frac{f'(x)g(x)-g'(x)f(x)}{\left(f(x)+g(x)\right)\sqrt{f(x)g(x)-g^2(x)}}dx$

$=\sqrt{m}\tan^{-1}\left(\sqrt{\frac{f(x)-g(x)}{ng(x)}}\right)+C.$

Where m, n $\in$N and 'C' is constant of integration (g(x) > 0). Find the value of $(m^2 + n^2)$.

25. If the value

$\int \frac{1-(\cot x)^{2008}}{\tan x+(\cot x)^{2009}}dx=\frac{1}{k}\ln\left|\sin^k x+\cos^k x\right|+C,$

then find k.

26. $\int \frac{dx}{(x-\alpha)\sqrt{(x-\alpha)(x-\beta)}}$

27. Suppose $\int \frac{1-7\cos^2 x}{\sin^7 x\cos^2 x}dx=\frac{g(x)}{\sin^7 x}+C$, where C is arbitrary constant of integration. Then find the value of g' (0) + g''$\left(\frac{\pi}{4}\right)$

EXERCISE-3

1. $\int\left(\sin(101x).\sin^{99}x\right)dx$ equals

(A) $\frac{\sin(100x)(\sin x)^{100}}{100}+C$

(B) $\frac{\cos(100x)(\sin x)^{100}}{100}+C$

(C) $\frac{\cos(100x)(\cos x)^{100}}{100}+C$

(D) $\frac{\sin(100x)(\sin x)^{101}}{101}+C$

2. The evaluation of $\int \frac{px^{p+2q-1}-qx^{q-1}}{x^{2p+2q}+2x^{p+q}+1}dx$ is

(A) $-\frac{x^p}{x^{p+q}+1}+C$ (B) $\frac{x^q}{x^{p+q}+1}+C$

(C) $-\frac{x^q}{x^{p+q}+1}+C$ (D) $\frac{x^p}{x^{p+q}+1}+C$

3. The integral $\int \sqrt{\cot x}\,e^{\sqrt{\sin x}}\sqrt{\cos x}\,dx$ equals

(A) $\frac{\sqrt{\tan x}\,e^{\sqrt{\sin x}}}{\sqrt{\cos x}}+C$ (B) $2e^{\sqrt{\sin x}}+C$

(C) $-\frac{1}{2}e^{\sqrt{\sin x}}+C$ (D) $\frac{\sqrt{\cot x}\,e^{\sqrt{\sin x}}}{2\sqrt{\cos x}}+C$

MORE THAN ONE CORRECT

4. Which one of the following is FALSE ?

(A) $x.\int\frac{dx}{x}=x\ln|x|+C$

(B) $x.\int\frac{dx}{x}=x\ln|x|+Cx$

(C) $\frac{1}{\cos x}.\int\cos x\,dx=\tan x+C$

(D) $\frac{1}{\cos x}.\int\cos x\,dx=x+C$

5. If $I_n=\int(\sin x)^n dx\, n\in N$, then $5I_4 - 6I_6$ is equal to -

(A) $\sin x.(\cos x)^5+C$

(B) $\cos x.(\sin x)^5+C$

(C) $\frac{\sin 2x}{8}\left[\cos^2 2x+1-2\cos 2x\right]+C$

(D) $\frac{\sin 2x}{8}\left[\cos^2 2x+1+2\cos 2x\right]+C$

14. $\int \frac{\sin 2x + 2}{\sin 2x + \cos 2x + 1} dx = f(x) + C$, Where C is integration constant and $f(0) = 0$, then -

(A) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{8} + \frac{3}{2}\ell n2$

(B) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{8} + \frac{3}{4}\ell n2$

(C) $\lim\limits_{x \to \frac{3\pi}{4}} f(x)$ does not exist

(D) $\lim\limits_{x \to \frac{3\pi}{4}} f(x) = \frac{3\pi}{8} + \ell n3$

15. If $\int \frac{3z^3 - 8z + 5}{\sqrt{z^2 - 4z - 7}} dz = \left(z^2 + az + 36\right)\sqrt{z^2 - 4z - 7} + b\ell n\left|z - 2 + \sqrt{z^2 - 4z - 7}\right| + C$, where a, b $\in$ I and C is integration constant, then-

(A) a> b

(B) a < b

(C) a + b = 117

(D) exactly one out of a or b is a prime number.

16. If $f(x) = \frac{x}{x^2 - 1}$, then which of the following is/are correct -

(A) $\int f\left(e^x\right)dx = \frac{1}{2}\ell n\left|\frac{e^x - 1}{e^x + 1}\right| + C$

(B) $\int f(\sin x)dx = \sec x + C$

(C) $\int f(\sec\theta)d\theta = -\text{cosec}\,\theta + C$

(D) $\int f\left(x^2\right)dx = \frac{1}{4}\ell n\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\tan^{-1}x + C$

JEE MAIN PYQ

1. If $\int f(x)dx = \Psi(x)$, then $\int x^5 f\left(x^3\right)dx$ is equal to:

[JEE Main-2013]

(A) $\frac{1}{3}\left[x^3\Psi\left(x^3\right) - \int x^2\Psi\left(x^3\right)dx\right] + C$

(B) $\frac{1}{3}x^3\Psi\left(x^3\right) - 3\int x^3\Psi\left(x^3\right)dx + C$

(C) $\frac{1}{3}x^3\Psi\left(x^3\right) - \int x^2\Psi\left(x^3\right)dx + C$

(D) $\frac{1}{3}\left[x^3\Psi\left(x^3\right) - \int x^3\Psi\left(x^3\right)dx\right] + C$

2. Let $I_n = \int \tan^n x\,dx, (n > 1)$. $I_4 + I_6 = a\tan^5 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to :-

[JEE Main-2017]

(A) $\frac{1}{5}, 0$ (B) $\frac{1}{5}, 1$

(C) $\frac{1}{5}, 0$ (D) $\frac{1}{5}, -1$

3. The integral

$$\int \frac{\sin^2 x\cos^2 x}{\left(\sin^5 x + \cos^3 x\sin^2 x + \sin^3 x\cos^2 x + \cos^5 x\right)^2} dx$$

is equal to:

(A) $\frac{-1}{1 + \cot^3 x} + C$ (B) $\frac{1}{3\left(1 + \tan^3 x\right)} + C$

(C) $\frac{-1}{3\left(1 + \tan^3 x\right)} + C$ (D) $\frac{1}{1 + \cot^3 x} + C$

(Where C is an constant of integration)

[JEE Main-2018]

26. The integral $\int \frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}dx$ is equal to : (where C is a constant of integration) **[JEE Main-2021]**

(A) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{1}{4}}+C$ (B) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{5}{4}}+C$

(C) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}}+C$ (D) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{5}{4}}+C$

27. For real numbers α,β,γ and δ, if

$$\int \frac{\left(x^2-1\right)+\tan^{-1}\left(\frac{x^2+1}{x}\right)}{\left(x^4+3x^2+1\right)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$$

$$=\alpha\log_e\left(\tan^{-1}\left(\frac{x^2+1}{x}\right)\right)$$

$$+\beta\tan^{-1}\left(\frac{\gamma\left(x^2-1\right)}{x}\right)+\delta\tan^{-1}\left(\frac{x^2+1}{x}\right)+C$$

[JEE Main-2021]

where C is an arbitrary constant, then the value of $10(\alpha+\beta\gamma+\delta)$ is equal to_____.

28. If $\int \frac{\sin x}{\sin^3 x+\cos^3 x}dx=\alpha\log_e|1+\tan x|+\beta\log_e$

$$\left|1-\tan x+\tan^2 x\right|+\gamma\tan^{-1}\left(\frac{2\tan x-1}{\sqrt{3}}\right)+C$$

when C is constant of integration, then the value of $18\left(\alpha+\beta+\gamma^2\right)$ is_____. **[JEE Main-2021]**

29. If $\int \frac{dx}{\left(x^2+x+1\right)^2}=a\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+b\left(\frac{2x+1}{x^2+x+1}\right)$

$+C, x>0$

where C is the constant of integration, then the value of $9\left(\sqrt{3}a+b\right)$ is equal to.

[JEE Main-2021]

30. If $\int_0^{\pi}\left(\sin^3 x\right)e^{-\sin^2 x}dx=\alpha-\frac{\beta}{e}\int_0^1\sqrt{t}e^t dt$ then $\alpha+\beta$ is equal to **[JEE Main-2021]**

31. Let $I_n(x)=\int_0^x \frac{1}{\left(t^2+5\right)^n}dt$, n = 1, 2, 3,....Then **[JEE Main-2022]**

(A) $50I_6-9I_5=xI_5'$ (B) $50I_6-11I_5=xI_5'$

(C) $50I_6-9I_5=I_5$ (D) $50I_6-11I_5=I_5'$

32. If $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx=g(x)+c, g(1)=0$, then $g\left(\frac{1}{2}\right)$ is equal to : **[JEE Main-2022]**

(A) $\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)+\frac{\pi}{3}$ (B) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)+\frac{\pi}{3}$

(C) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)-\frac{\pi}{3}$ (D) $\frac{1}{2}\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)-\frac{\pi}{6}$

33. The integral $\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$ is equal to **[JEE Main-2022]**

(A) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|+C$

(B) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\left(\frac{x}{2}+\frac{\pi}{3}\right)}\right|+C$

(C) $\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}\right|+C$

(D) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}-\frac{\pi}{6}\right)}\right|+C$

34. For $I(x)=\int\frac{\sec^2 x-2022}{\sin^{2022} x}dx$, if $I\left(\frac{\pi}{4}\right)=2^{1011}$, then **[JEE Main-2022]**

(A) $3^{1010}I\left(\frac{\pi}{3}\right)-I\left(\frac{\pi}{6}\right)=0$

(B) $3^{1010}I\left(\frac{\pi}{6}\right)-I\left(\frac{\pi}{3}\right)=0$

(C) $3^{1011}I\left(\frac{\pi}{3}\right)-I\left(\frac{\pi}{6}\right)=0$

(D) $3^{1011}I\left(\frac{\pi}{6}\right)-I\left(\frac{\pi}{3}\right)=0$

35. Let $f(x)=\int\frac{2x}{\left(x^2+1\right)\left(x^2+3\right)}dx$.

If $f(3)=\frac{1}{2}\left(\log_e 5-\log_e 6\right)$, then $f(4)$ is equal to **[JEE Main-2023]**

(A) $\frac{1}{2}\left(\log_e 19-\log_e 17\right)$

(B) $\log_e 17-\log_e 18$

(C) $\log_e 19-\log_e 20$

(D) $\frac{1}{2}\left(\log_e 17-\log_e 19\right)$

36. For $\alpha, \beta, \gamma, \delta \in N$, if $\int\left(\left(\frac{x}{e}\right)^{2x}+\left(\frac{e}{x}\right)^{2x}\right)\log_e x dx$ $=\frac{1}{\alpha}\left(\frac{x}{e}\right)^{\beta x}-\frac{1}{\gamma}\left(\frac{e}{x}\right)^{\delta x}+C$. Where $e=\sum_{n=0}^{\infty}\frac{1}{n!}$ and C is constant of integration, then $\alpha+2\beta+3\gamma-4\delta$ is equal to: **[JEE Main-2023]**

(A) 1 (B) -4

(C) -8 (D) 4

37. Let $I(x)=\int\frac{(x+1)}{x\left(1+xe^x\right)^2}dx, x>0$. If $\lim_{x\to\infty} I(x)=0$ then I(A) is equal to **[JEE Main-2023]**

(A) $\frac{e+2}{e+1}-\log_e(e+1)$ (B) $\frac{e+1}{e+2}+\log_e(e+1)$

(C) $\frac{e+1}{e+2}-\log_e(e+1)$ (D) $\frac{e+2}{e+1}+\log_e(e+1)$

38. If $I(x)=\int e^{\sin^2 x}\left(\cos x\sin 2x-\sin x\right)dx$ and I(0) = 1, then $I\left(\frac{\pi}{3}\right)$ is equal to **[JEE Main-2023]**

(A) $-\frac{1}{2}e^{\frac{3}{4}}$ (B) $e^{\frac{3}{4}}$

(C) $\frac{1}{2}e^{\frac{3}{4}}$ (D) $-e^{\frac{3}{4}}$

39. $\int_0^{\infty}\frac{6}{e^{3x}+6e^{2x}+11e^x+6}dx$ **[JEE Main-2023]**

(A) $\log_e\left(\frac{512}{81}\right)$ (B) $\log_e\left(\frac{32}{27}\right)$

(C) $\log_e\left(\frac{256}{81}\right)$ (D) $\log_e\left(\frac{64}{27}\right)$

40. $\int_0^1\frac{1}{\left(5+2x-2x^2\right)\left(1+e^{(2-4x)}\right)}dx=\frac{1}{\alpha}\log_e\left(\frac{\alpha+1}{\beta}\right)$, $\alpha, \beta > 0$, then $\alpha^4-\beta^4$ is equal to: **[JEE Main-2023]**

(A) 21 (B) 0

(C) 19 (D) –21

41. Let $I(x)=\int\sqrt{\frac{x+7}{x}}dx$ and $I(9) = 12 + 7\log_e 7$. If $I(1)=\alpha+7\log_e\left(1+2\sqrt{2}\right)$, then α^4 is equal to **[JEE Main-2023]**

42. Let $f(x)=\int\frac{dx}{\left(3+4x^2\right)\sqrt{4-3x^2}}, |x|<\frac{2}{\sqrt{3}}$.

If $f(0) = 0$ and $f(1)=\frac{1}{\alpha\beta}\tan^{-1}\left(\frac{\alpha}{\beta}\right)$, $\alpha, \beta>0$, then $\alpha^2+\beta^2$ is equal to **[JEE Main-2023]**

43. If $\int\sqrt{\sec 2x-1}dx=\alpha\log_e$ $\left|\cos 2x+\beta+\sqrt{\cos 2x\left(1+\cos\frac{1}{\beta}x\right)}\right|+$ constant, then $\beta-\alpha$ is equal to _____. **[JEE Main-2023]**

ISI PYQ

1. Let $f(x)=(\tan x)^{\frac{3}{2}}-3\tan x+\sqrt{\tan x}$. Consider the three integrals $I_1=\int_0^1 f(x)dx$, $I_2=\int_{0.3}^{1.3} f(x)dx$, $I_3=\int_{0.5}^{1.5} f(x)dx$. Then **[ISI – 2011]**

(A) $I_1 > I_2 > I_3$ (B) $I_2 > I_1 > I_3$
(C) $I_3 > I_1 > I_2$ (D) $I_1 > I_3 > I_2$

2. Suppose f is continuously differentiable up to 3rd order and satisfies

$$\int_0^1 \{6f(x)+x^3 f'''(x)\}dx = f''(1)$$

Which of the following must be true? **[ISI – 2011]**

(A) f(1) = 0 (B) f '(1) = 2f(1)
(C) f '(1) = f(1) (D) f '(1) = 0

3. Let [x] denote the greatest integer less than or equal to x. The value of the integral $\int_1^n [x]^{x-[x]}dx$ is equal to **[ISI – 2016]**

(A) $1+\frac{2^3}{\log_e 2}-\frac{2^2}{\log_e 2}+\frac{3^4}{\log_e 3}-\frac{3^3}{\log_e 3}+\ldots+\frac{(n-1)^n}{\log_e(n-1)}-\frac{(n-1)^{n-1}}{\log_e(n-1)}$

(B) $1+\frac{1}{\log_e 2}+\frac{2}{\log_e 3}+\ldots+\frac{n-2}{\log_e(n-1)}$

(C) $\frac{1}{2}+\frac{2^2}{3}+\ldots+\frac{n^{n+1}}{n+1}$

(D) $\frac{2^3-1}{3}+\frac{3^4-2^3}{4}+\ldots+\frac{n^{n+1}-(n-1)^n}{n+1}$

4. Let f be a continuous strictly increasing function from $[0,\infty)$ onto $[0,\infty)$ and $g=f^{-1}$ (that is, f(x) = y if and only if g(y) = x). Let a, b > 0 and $a \neq b$. Then $\int_0^a f(x)dx+\int_0^b g(y)dy$ is **[ISI – 2016]**

(A) greater than or equal to ab
(B) less than ab
(C) always equal to ab
(D) always equal to $\frac{af(a)+bg(b)}{2}$

JEE ADVANCED PYQ

1. $\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}dx$ is equals to – **[JEE-2006, (3M, -1M)]**

(A) $\frac{\sqrt{2x^4-2x^2+1}}{x^2}+c$
(B) $\frac{\sqrt{2x^4-2x^2+1}}{x^3}+c$
(C) $\frac{\sqrt{2x^4-2x^2+1}}{x}+c$
(D) $\frac{\sqrt{2x^4-2x^2+1}}{2x^2}+c$

2. Let $f(x)=\frac{x}{(1+x^n)^{1/n}}$ for n ≥ 2 and g(x) = $\underbrace{f\ of\ of\ \ldots.of}_{f\ occurs\ n\ times}(x)$. Then $\int x^{n-2}\, g(x)\, dx$ equals. **[JEE-2007, 3]**

(A) $\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}}+K$
(B) $\frac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}}+K$
(C) $\frac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}}+K$
(D) $\frac{1}{n+1}(1+nx^n)^{1+\frac{1}{n}}+K$

3. Let F(x) be an indefinite integral of $\sin^2 x$. **[JEE-2007, 3]**

Statement-1: The function F(x) satisfies $F(x+\pi)=F(x)$ for all real x.
because
Statement-2: $\sin^2(x+\pi)=\sin^2 x$ for all real x.

(A) Statement – 1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.
(B) Statement – 1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement – 1 is True, Statement-2 is False.
(D) Statement – 1 is False, Statement-2 is True.

4. Let $I=\int\frac{e^x}{e^{4x}+e^{2x}+1}dx,\ J=\int\frac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx.$

Then, for an arbitrary constant c, the value of J – I equals. **[JEE-2008, 3 (-1)]**

(A) $\frac{1}{2}\ell n\left(\frac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right)+c$

(B) $\frac{1}{2}\ell n\left(\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right)+c$

(C) $\frac{1}{2}\ell n\left(\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right)+c$

(D) $\frac{1}{2}\ell n\left(\frac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right)+c$

5. The integral $\int\frac{\sec^2 x}{(\sec x+\tan x)^{9/2}}dx$ equals (for some arbitrary constant K)

[JEE-2012, 3M,-1M]

(A) $-\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

(B) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

(C) $-\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

(D) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

ADVANCED & OLYMPIAD CHALLENGER QUESTIONS

1. $\int\frac{\tan(\ln x)\tan\left(\ln\frac{x}{2}\right)\tan(\ln 2)}{x}dx$

2. $\int\frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}dx$

3. $\int\frac{dx}{\left(x+\sqrt{x(1+x)}\right)^2}$

4. $\int\frac{dx}{\sqrt{\sin^3 x\sin(x+\alpha)}}$

5. $\int\frac{x}{(7x-10-x^2)^{3/2}}dx$

6. $\int\frac{(1+x^2)dx}{1-2x^2\cos\alpha+x^4}\ \alpha\in(0,\pi)$

7. $\int\sqrt{\frac{\sin(x-a)}{\sin(x+a)}}dx$

8. $\int\frac{\cos^2 x}{1+\tan x}dx$

9. Let f(x) is a quadratic function such that f (0) = 1 and $\int\frac{f(x)dx}{x^2(x+1)^3}$ is a rational function, find the value of f'(0)

10. $\int\frac{\cos x-\sin x}{7-9\sin 2x}dx$

11. $\int\cos 2\theta.\ \ell n\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}d\theta$

MATCH THE COLUMN

12. $I_1 = \int \tan x \tan(ax+b)dx$ and

$I_2 = \int \cot x \cot(ax+b)dx$

Column-I		Column-II	
(A)	Value of I_1 for a =1 is	(P)	$x - \cot b\, \ell n \dfrac{\cos(x-b)}{\cos x} + C$
(B)	Value of I_2 for a =1 is	(Q)	$\cot b\, \ell n \dfrac{\sin x}{\sin(x+b)} - x + C$
(C)	Value of I_1 for a =– 1 is	(R)	$\cot b\, \ell n \left(\dfrac{\cos x}{\cos(x+b)}\right) - x + C$
(D)	Value of I_2 for a =– 1 is	(S)	$x + \cot b\, \ell n \left(\dfrac{\sin x}{\sin(b-x)}\right) + C$

ADDITIONAL EXERCISE

1. $\int \dfrac{\log_6 \sqrt[6]{((\sin x)6^{\cos x})}}{\tan x} dx$

2. $\int \dfrac{dx}{\sec x + \operatorname{cosec} x} dx$

3. $\int \dfrac{\cot x dx}{(1-\sin x)(\sec x + 1)}$

4. $\int \dfrac{\sin(x+\alpha)}{\cos^3 x} \sqrt{\dfrac{\operatorname{cosec} x + \sec x}{\operatorname{cosec} x - \sec x}} dx$

5. $\int \dfrac{a^2 \sin^2 x + b^2 \cos^2 x}{a^4 \sin^2 x + b^4 \cos^2 x} dx$

6. $\int \dfrac{x^3 + x + 1}{x^4 + x^2 + 1} dx$

7. $\int \dfrac{\sqrt{\cot x} - \sqrt{\tan x}}{1 + 3\sin 2x} dx$

8. $\int \dfrac{\sqrt{\sin^4 x + \cos^4 x}}{\sin^3 x \cos x} dx, x \in \left(0, \dfrac{\pi}{2}\right)$

9. $\int \dfrac{x^2}{(x\cos x - \sin x)(x \sin x + \cos x)} dx$

10. $\int \dfrac{dx}{\cos^3 x - \sin^3 x}$

MATCH THE COLUMN

11. Match the following:

Column-I			Column-II
(A)	$\int \dfrac{x^4 - 1}{x^2\sqrt{x^4 + x^2 + 1}} dx$	(P)	$\ln\left(\dfrac{(x^2+1) + \sqrt{x^4+1}}{x}\right) + c$
(B)	$\int \dfrac{x^2 - 1}{x\sqrt{1 + x^4}} dx$	(Q)	$C - \dfrac{1}{\sqrt{2}} \ln\left(\dfrac{\sqrt{x^4+1} - \sqrt{2}x}{(x^2-1)}\right)$
(C)	$\int \dfrac{1 + x^2}{(1-x^2)\sqrt{1+x^4}} dx$	(R)	$C - \tan^{-1}\left(\sqrt{\sqrt{1 + \dfrac{1}{x^4}} - 1}\right)$
(D)	$\int \dfrac{1}{(1+x^4)\sqrt{\sqrt{1+x^4} - x^2}} dx$	(S)	$\dfrac{\sqrt{x^4 + x^2 + 1}}{x} + C$

Answer Key

EXERCISE - 1

1.	C	**2.**	B	**3.**	B	**4.**	B	**5.**	A	**6.**	B	**7.**	D
8.	B	**9.**	A	**10.**	B	**11.**	A	**12.**	B	**13.**	B	**14.**	C
15.	A												

EXERCISE - 2

1. $\frac{x^2}{2} - x + C$

2. $\frac{67}{5}$

3. $\frac{1}{2}(x - \sin x) + C$

4. $C - \frac{2}{x} + \frac{2}{3x^3} - \frac{3}{5x^5} - 2\tan^{-1} x$

5. $2ln\left(\sec\frac{x}{2}\right) - 3ln\left(\sec\frac{x}{3}\right) - 6ln\left(\sec\frac{x}{6}\right) + C$

6. $\sin^{-1}\left(\frac{1}{2}\sec^2\frac{x}{2}\right) + C$

7. $\frac{1}{2}\left[\ln\left(\frac{1+x}{1-x}\right).\ln\left(\ln\frac{1+x}{1-x}\right) - \ln\left(\frac{1+x}{1-x}\right)\right] + C$

8. $\left(\frac{x}{e}\right)^x - \left(\frac{e}{x}\right)^x + C$

9. $\frac{x^4}{4} + x^3 - x^2 + 5x + \frac{1}{2} ln\,(x^2 + 1) + 3\tan^{-1} x + C$

10. $6\left[\frac{t^4}{4} - \frac{t^2}{2} + t + \frac{1}{2}\ln(1+t^2) - \tan^{-1} t\right] + C$ where $t = x^{1/6}$

11. $(a + x)\ \text{arc}\tan\sqrt{\frac{x}{a}} - \sqrt{ax} + C$

12. $\text{arc}\sec x - \frac{\ln x}{\sqrt{(x^2 - 1)}} + C$

13. $\frac{(x^2+1)\sqrt{x^2+1}}{9x^3}\left[2 - 3\ln\left(1 + \frac{1}{x^2}\right)\right]$

14. $ln\left(\frac{1+\sqrt{1+3\cos^2 2\theta}}{\cos 2\theta}\right) + C$

15. $C - \frac{x}{(x^2 - 1)^2}$

16. $\sin^{-1}\left(\frac{ax^2 + b}{cx}\right) + k$

17. $2\sqrt{2}\left(\cos(e^{-\sqrt{x}})\right)\left(\sin(e^{\sqrt{x}}) + \cos(e^{\sqrt{x}})\right) + C$

18. $C - ln(1 + (x + 1)e^{-x}) - \frac{1}{1 + (x+1)e^{-x}}$

19. $C - e^{\cos x}(x + \text{cosec}x)$

20. $C - \frac{x+1}{x^5 + x + 1}$ or $C + \frac{x^5}{x^5 + x + 1}$

21. $-\frac{3(1 + 4\tan^2 x)}{8(\tan x)^{8/3}} + C$

22. $\frac{1}{2\sqrt{3}} ln\frac{\sqrt{3} + \sin x - \cos x}{\sqrt{3} - \sin x + \cos x} + \text{arc}\tan(\sin x + \cos x) + C$

23. $4\ln x + \frac{7}{x} + 6\tan^{-1}(x) + \frac{6x}{1 + x^2} + C$

24. 8 **25.** 2010 **26.** $\frac{-2}{\alpha - \beta}\sqrt{\frac{x - \beta}{x - \alpha}} + C$ **27.** 5

OLYMPIAD WALLAH

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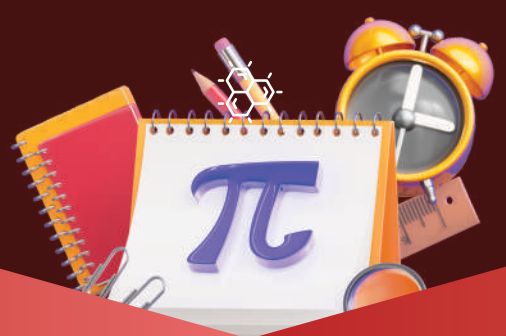

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