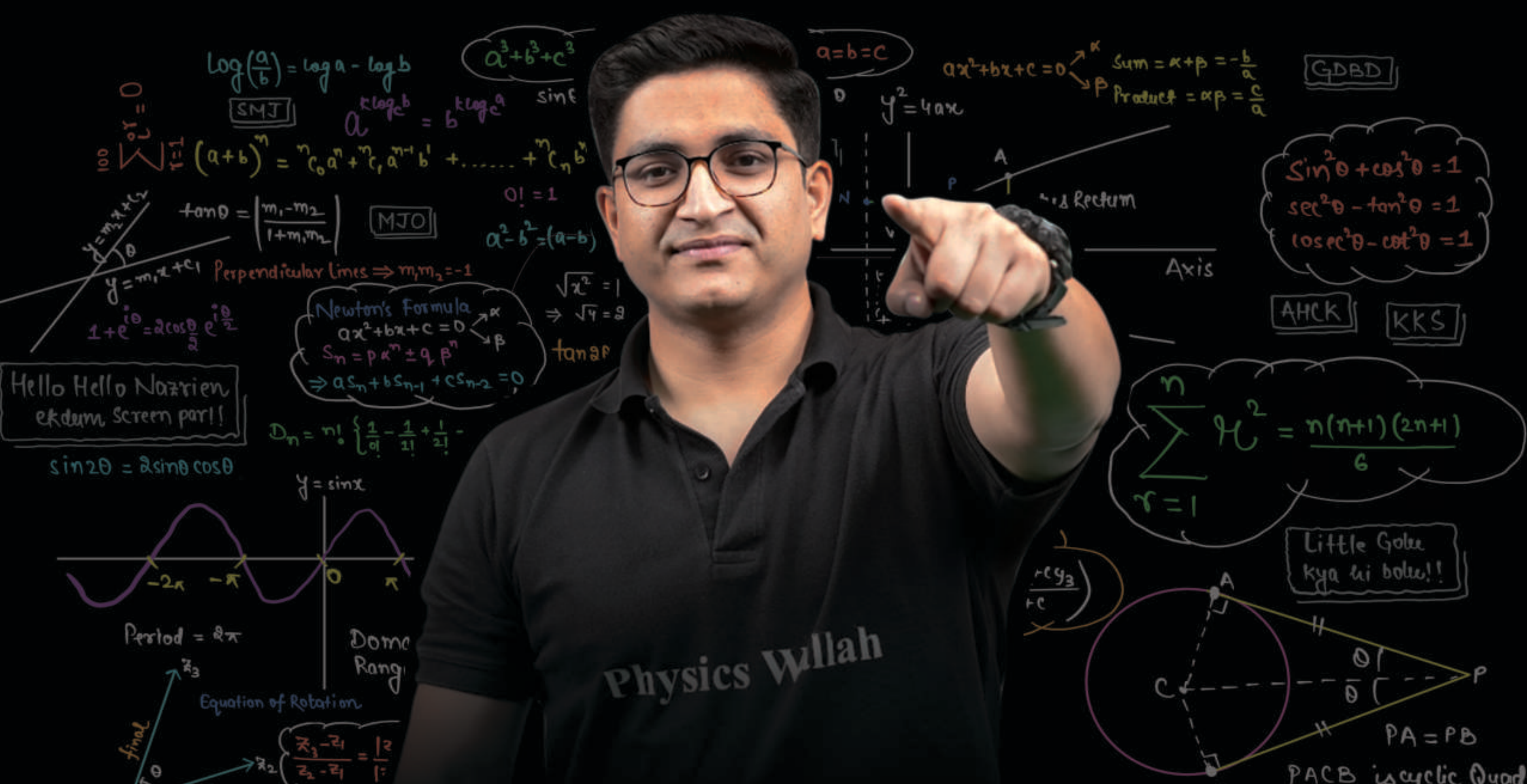
MATHS MADE eJEE

Class Notes in Handwritten Format

Class 11

JEE Main & Advanced



Sachin Jakhar
(Sacchhu Sir)

Ab BACKLOG ko Kaho BBye!!

Complete Theory | Basic to Advanced Problems | Special Tips & Tricks
PYQ's Main & Advanced | Detailed Solutions | Pure Classroom Feel

Contents

1 Basic Mathematics

BASIC POINTS TO NOTE

Note-01

Square root of positive real number is always positive.

$\sqrt{x^2} = |x|$

Note-02

Division by zero is not-defined.

Note-03

Even root of only non-negative real numbers is defined. Odd root of every real number is defined.

Note-04

Multiplication and division (cancellation) is only valid for non-zero numbers/expressions.
(काटा-पीटी–Non zero न. ki होती है)

Note-05

Square (or even powers) of any number or expression is always +ve or can be zero.

Question

Q. If a, b, c are real numbers satisfying $2a^2 + 5b^2 + 9c^2 - 4ab + 2b - 6ac + 1 = 0$, then $a + b + c =$ _____.

Sol. $2a^2 + 5b^2 + 9c^2 - 4ab + 2b - 6ac + 1 = 0$

$\Rightarrow a^2 + 4b^2 - 2(a)(2b) + 9c^2 + a^2 - 2(a)(3c) + b^2 + 2b(1) + 1^2 = 0$

$\Rightarrow (a - 2b)^2 + (3c - a)^2 + (b + 1)^2 = 0$

$\Rightarrow a = 2b,\ 3c = a,\ b = -1$

$\Rightarrow a = -2,\ b = -1,\ c = \frac{-2}{3}$

$\Rightarrow a + b + c = \frac{-11}{3}$ fnf.

IDENTITIES

एक ही नारा हमारा,
रट डाल ओ यारा

Part-01

हम जानते हैं! बचपन का ♥

(1) $(a + b)^2 = a^2 + 2ab + b^2 = (a - b)^2 + 4ab$

(2) $(a - b)^2 = a^2 - 2ab + b^2 = (a + b)^2 - 4ab$

(3) $a^2 - b^2 = (a - b)(a + b)$

$a^4 - b^4 = (a^2 - b^2)(a^2 + b^2)$

(4) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
$= a^3 + b^3 + 3a^2b + 3ab^2$

(5) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
$= a^3 - b^3 - 3a^2b + 3ab^2$

(6) $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

(7) $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

Part-02

Thodi stylish wali

(1) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

(2) $(a + b + c + d)^2$ = (Sabhi ke squares ka sum) + 2(sum of multiplication of every two)
$= (a^2 + b^2 + c^2 + d^2) + 2(ab + ac + ad + bc + bd + cd)$

Part-03

(Impressive)

(1) $a^2 + b^2 + c^2 - ab - bc - ca = \frac{(a - b)^2 + (b - c)^2 + (c - a)^2}{2}$

(2) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Note

If $\boxed{a^3 + b^3 + c^3 = 3abc}$

or $\boxed{\dfrac{a^3 + b^3 + c^3}{abc} = 3}$

$\Rightarrow (a + b + c = 0)$ or $(a = b = c)$

Question

Q. The value of

$$\frac{(1.5)^3 + (4.7)^3 + (3.8)^3 - 3 \times 1.5 \times 4.7 \times 3.8}{(1.5)^2 + (4.7)^2 + (3.8)^2 - 1.5 \times 4.7 - 1.5 \times 3.8 - 4.7 \times 3.8}$$

is

(A) 8 (B) 9

(C) 10 (D) 11

Sol.

$$\frac{(1.5)^3 + (4.7)^3 + (3.8)^3 - 3 \times 1.5 \times 4.7 \times 3.8}{(1.5)^2 + (4.7)^2 + (3.8)^2 - 1.5 \times 4.7 - 1.5 \times 3.8 - 4.7 \times 3.8}$$

say: $1.5 = a$, $4.7 = b$, $3.8 = c$

$$\frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ca}$$

$$= \frac{(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)}{(a^2 + b^2 + c^2 - ab - bc - ca)}$$

$= a + b + c = 1.5 + 4.7 + 3.8 = 10$ fnf.

Question [JEE Main-2021]

Q. If $a + b + c = 1$, $ab + bc + ca = 2$ and $abc = 3$ then the value of $a^4 + b^4 + c^4$ is equal to

Sol. $a + b + c = 1$

SBS: $a^2 + b^2 + c^2 + 2(ab + bc + ca) = 1$

$\Rightarrow a^2 + b^2 + c^2 = -3$

$\boxed{ab + bc + ca = 2}$

SBS: $(ab)^2 + (bc)^2 + (ca)^2 + 2(ab^2c + abc^2 + a^2bc) = 4$

$\Rightarrow a^2b^2 + b^2c^2 + c^2a^2 + 2(abc)(b + c + a) = 4$

$\Rightarrow a^2b^2 + b^2c^2 + c^2a^2 = -2$

$\boxed{a^2 + b^2 + c^2 = -3}$

SBS: $(a^2)^2 + (b^2)^2 + (c^2)^2 + 2(a^2b^2 + b^2c^2 + c^2a^2) = 9$

$\Rightarrow a^4 + b^4 + c^4 + 2(-2) = 9$

$\Rightarrow a^4 + b^4 + c^4 = 13$ fnf.

Module Question

Q. If $a, b, c \in R$ and $a, b, c \neq 0$ such that $\dfrac{a}{b} + \dfrac{b}{c} + \dfrac{c}{a} = 6$ and $\dfrac{b}{a} + \dfrac{c}{b} + \dfrac{a}{c} = 8$, then $\dfrac{a^3}{b^3} + \dfrac{b^3}{c^3} + \dfrac{c^3}{a^3} - 3$ is equal to

(A) 81 (B) 48

(C) 72 (D) 84

Sol. Put: $\boxed{\dfrac{a}{b} = x, \dfrac{b}{c} = y, \dfrac{c}{a} = z}$

$\dfrac{a}{b} + \dfrac{b}{c} + \dfrac{c}{a} = 6 \Rightarrow x + y + z = 6$ ...(1)

$\dfrac{b}{a} + \dfrac{c}{b} + \dfrac{a}{c} = 8 \Rightarrow \dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = 8$ ...(2)

and $xyz = 1$ ...(3)

SBS: eq^n (1)

$x^2 + y^2 + z^2 + 2(xy + yz + zx) = 36$

$x^2 + y^2 + z^2 = 36 - 16 = 20$

Now, $\dfrac{a^3}{b^3} + \dfrac{b^3}{c^3} + \dfrac{c^3}{a^3} - 3\ (1)$

$= \boxed{x^3 + y^3 + z^3 - 3(xyz)}$

$= (x + y + z)(x^2 + y^2 + z^2 - (xy + yz + zx))$

$= 6(20 - (8)) = 6 \times 12 = 72$ fnf

Part-04

420 wali (चार सौ बीस वाली) ★ ★ ★

$a^4 + a^2 + 1 = (a^2 - a + 1)(a^2 + a + 1)$

$(a^2)^2 + (1)^2 + 2a^2 - a^2$

$= (a^2 + 1)^2 - (a)^2$

$= (a^2 + 1 + a)(a^2 + 1 - a)$

Part-05

Term & uska Reciprocal

$$\left(x + \frac{1}{x}\right)^2 - 2 = x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$$

Question [IIT JEE-1980]

Q. If $x > y > 0$, then show that the expression $\left(\sqrt{2}\left(2x + \sqrt{x^2 - y^2}\right)\left(\sqrt{x - \sqrt{x^2 - y^2}}\right)\right)$ can be simplified to $\left(\sqrt{x + y}\right)^3 - \left(\sqrt{x - y}\right)^3$

Sol. Say: $x + y = a$ & $x - y = b$

$\Rightarrow 2x = a + b,\ 2y = a - b,\ x^2 - y^2 = ab$

$\Rightarrow 2x + \sqrt{x^2 - y^2} = a + b + \sqrt{ab}$

and $\sqrt{x - \sqrt{x^2 - y^2}} = \sqrt{\dfrac{a+b}{2} - \sqrt{ab}}$

$= \sqrt{\dfrac{\left(\sqrt{a}\right)^2 + \left(\sqrt{b}\right)^2 - 2\sqrt{ab}}{2}}$

Required expression

$= \sqrt{2}\left(a + b + \sqrt{ab}\right)\dfrac{\left(\sqrt{a} - \sqrt{b}\right)}{\sqrt{2}}$

$= \left(\sqrt{a}\right)^3 - \left(\sqrt{b}\right)^3 = \left(\sqrt{x+y}\right)^3 - \left(\sqrt{x-y}\right)^3$ HHPP

DIFFERENT TYPES OF NUMBERS

Number system (Informative)

Natural numbers: (N) = {1, 2, 3, 4, 5,...}

Whole numbers: (W) = {0, 1, 2, 3, 4, 5,....}

Integers: (I/z) = {....−3, −2, −1, 0 1, 2, 3, 4, 5,....}

Prime Numbers: Natural numbers which are divisible by 1 and itself only are called prime numbers,

Ex. {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31,....}

Composite Numbers: Let 'a' be a natural number, 'a' is said to be composite if, it has atleast three distinct factors.

Ex. {4, 6, 8, 9, 10, 12, ...}

Co-prime Numbers: Two natural numbers (not necessarily prime) are called coprime, if their H.C.F. (Highest common factor) is one.

These numbers are also called **relatively prime numbers.**

Co-prime or relative prime

नाम में Prime है, lekin numbers ka prime होना जरूरी नहीं

(a, b)

HCF (a, b) = 1

or

a & b में No common factor (except 1)

Ex. (2, 3); (3, 5); (1, 6); (8, 9)

Twin prime Numbers: If the difference between two prime numberws is two, then the numbers are called twin prime numbers.

Twin prime No ऐसे दो Prime no.'s जिनका difference '2' हो

ex: (2, 3) ×
(3, 5) ✓
(5, 7) ✓
(1, 3) ×
(11, 13) ✓

Rational Numbers:

$Q = \dfrac{p}{q}, (p \text{ \& } q \rightarrow \text{Integer})\ q \neq 0$

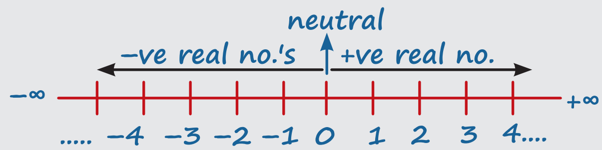
$Q = \left\{\dfrac{-1}{2}, 5, \dfrac{-3}{7}, \dfrac{1}{8}, \dfrac{11}{13}, \dfrac{6}{4}, \underbrace{1, 2, 3, 4, 5, 0, -1, -2, -3}_{\text{Integers}}\right\}$

Irrational Number:

$\bar{Q} / Q^c = \left\{\sqrt{2}, \sqrt{3}, -\sqrt{5}, \sqrt{7}, \sqrt{11}, \sqrt{8}, \sqrt{14}, ..., \pi, e...\right\}$

Real Number: $\boxed{R = \overset{\text{rational}}{Q} + \underset{\text{irrational}}{\bar{Q}}}$

Number Line:

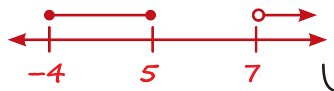
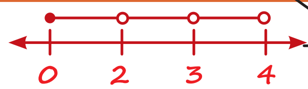
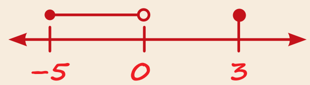


INTERVALS & SOLUTION REPRESENTATION

- ● Dark Circle → jisko include karna ho
- ○ Hollow Circle → jisko exclude karna ho
- (,) Open bracket → Jisko exclude karna ho
- [,] Closed bracket → Jisko include karna ho
- {,} Curly bracket → discrete value ke liye {kuch value ke liye}
- ∪ Union → A ∪ B ≡ combined value of A & B.
- ∩ Intersection → A ∩ B ≡ common value of A & B.
- ∈ belongs to → If 'a' set M में present है

$\Rightarrow a \in M$

Q. Match the column:

Column-I		Column-II		Column-III	
(A)	$-2 < x \le 3, x \in R$	(P)		(x)	$x \in [0, 4) - \{2, 3\}$
(B)	$0 \le x < 4$ & $x \ne 2, 3, x \in R$	(Q)		(y)	$x \in [-4, 5] \cup (7, \infty)$
(C)	$(-5 \le x < 0)$ or $(x = 3), x \in R$	(R)		(z)	$x \in [-5, 0) \cup \{3\}$
(D)	$(-4 \le x \le 5)$ or $(x > 7), x \in R$	(S)		(T)	$x \in (-2, 3]$

RULES OF INEQUALITY

Rule-01

We can add (or subtract) any number 'k' on both sides of inequality. Doing this will not change the sign of inequality.

Rule-02

We can multiply (or divide) any non-zero number 'k' on both sides of inequality and sign of inequality will change according to sign of 'k' as:

If $\boxed{k > 0,}$ then

$\boxed{\text{sign of inequality will remains same,}}$

If $\boxed{k < 0,}$ then

$\boxed{\text{sign of inequality will get reversed.}}$

WAVY CURVE METHOD

Step-01: Check structure:

$$\frac{N^r}{D^r} \begin{matrix} > \\ < \\ \ge \\ \le \end{matrix} 0 \rightarrow \text{ ek taraf zero}$$

Step-02: Find critical points (CP's)

Step-03: Plot all CP's on number line

Step-04: Check sign of given expression by putting value of x from different parts of number line.

Step-05: Jo pucha gya, use dekh key jawab do.

BAAT MAIN DUM HAI

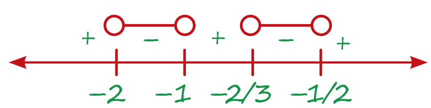
1. Roots of denominator "always" → hollow circle → always excluded.
2. In case of $\boxed{\ge, \le}$ roots of numerator → Roots of N^r पर dark circle → included
3. In case of $\boxed{>, <}$ roots → सभी पर Hollow circle

Question [IIT JEE-1987]

Q. Find the set of all x for which

$$\frac{2x}{2x^2+5x+2} > \frac{1}{x+1}.$$

Sol.
$$\frac{2x}{2x^2+5x+2} - \frac{1}{(x+1)} > 0$$
$$\frac{2x^2+2x-2x^2-5x-2}{(2x^2+5x+2)(x+1)} > 0$$
$$\frac{(3x+2)^1}{(x+2)^1(2x+1)^1(x+1)^1} < 0$$



$$x \in (-2,-1) \cup \left(-\frac{2}{3},-\frac{1}{2}\right) \text{fnf.}$$

Acidic Module Question

Q. Number of non-negative integral value of x satisfying the inequality

$$\frac{2}{x^2-x+1} - \frac{1}{x+1} - \frac{2x-1}{x^3+1} \ge 0 \text{ is}$$

(A) 0 (B) 1 (C) 2 (D) 3

Sol. $\left(\dfrac{2}{x^2-x+1}-\dfrac{1}{x+1}\right)-\dfrac{2x-1}{x^3+1}\geq 0$

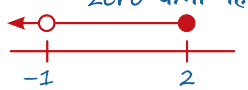
$\Rightarrow \left(\dfrac{2x+2-x^2+x-1}{(x+1)(x^2-x+1)}\right)-\dfrac{2x-1}{x^3+1}\geq 0$

$\Rightarrow \dfrac{3x-x^2+1-2x+1}{(x^3+1)}\geq 0$

$\Rightarrow \dfrac{x-x^2+2}{(x^3+1)}\geq 0 \Rightarrow \dfrac{x^2-x-2}{x^3+1}\leq 0$

$\Rightarrow \dfrac{(x+1)(x-2)}{(x+1)(x^2-x+1)}\leq 0$

zero कभी नहीं होगा



$x \in (-\infty,-1)\cup(-1,2]$ or

$x \in (-\infty,2]-\{-1\}$ fnf.

TRICK ★★★

1. Make coefficient 'x', positive in each linear factor.
2. Now start marking (+/−/+/−/+/−) from right to left.
3. Remember:

Jinn factors ki net power odd hai, uske about → sign change hota hai.
Jinn factors ki net power even hai, uske about → sign change nahi hota

Question

Q. The complete solution set of inequality $\dfrac{(x-5)^{1005}(x+8)^{1008}(x-1)}{x^{1006}(x-2)^3(x-3)^5(x-6)(x+9)^{1010}}\leq 0$ is

(A) $(-\infty,-9)\cup(-8,0)\cup(0,1)\cup(2,3)\cup[5,6)$

(B) $(-\infty,-9)\cup(-9,0)\cup(0,1)\cup(2,3)\cup(5,6)$

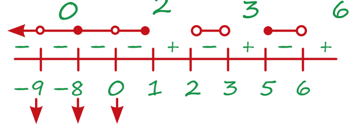
(C) $(-\infty,-9)\cup(-9,0)\cup(0,1]\cup(2,3)\cup[5,6)$

(D) $(-\infty,0)\cup(0,1]\cup(2,3)\cup[5,6)$

Sol. $\dfrac{(x-5)^{1005}(x+8)^{1008}(x-1)}{x^{1006}(x-2)^3(x-3)^5(x-6)(x+9)^{1010}}\leq 0$

5, −8, 1

0, 2, 3, 6, −9



Answer (c) fnf.

DOUBLE INEQUALITY

$h(x) < f(x) \leq g(x)$

$\boxed{h(x) < f(x)} \cap \boxed{f(x) \leq g(x)}$

Question

Q. The number of the integral solutions of $x^2+9<(x+3)^2<8x+25$ is

(A) 1 (B) 2

(C) 4 (D) 5

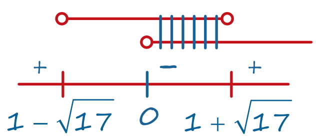
Sol.

$x^2+9<(x+3)^2<8x+25$

$x^2+9<x^2+9+6x$ | $x^2+9+6x<8x+25$

$0<6x$ | $\boxed{x^2-2x-16<0}$

$x>0$ | $(x-(1+\sqrt{17}))(x-(1-\sqrt{17}))<0$



$x\in(0,1+\sqrt{17})\equiv(0,5.\text{something})$

Integers $\Rightarrow x = 1, 2, 3, 4, 5$

ROLE OF QUADRATIC

$\boxed{D=b^2-4ac}$

$D\geq 0\rightarrow$ factorisable

$\boxed{D<0\rightarrow \text{non-factorisable}}$

Type: Non-factorisable quadratic

Quadratic $\equiv (ax^2+bx+c)$

$\Rightarrow$ If $\boxed{a>0}$ and $\boxed{D<0}$ $\Rightarrow$ हमेशा +ve होगी

$\Rightarrow$ If $\boxed{a<0}$ and $\boxed{D<0}$ $\Rightarrow$ हमेशा −ve होगी

Q. If $f(x)=\dfrac{(x^2-x+6)(x-5)}{(x+2)(3x-8-x^2)}$ and solution set of $f(x)\leq 0$ is $x\in(-\infty,-a)\cup[b,\infty)$ where $a, b\in N$, then value of GCD $(2a^4+4, b^2-1)$ is ________.

Sol. $\boxed{a > 0 \quad D < 0} \Rightarrow$ +ve always

$$\Rightarrow f(x) = \frac{(x^2 - x + 6)(x - 5)}{(x + 2)(3x - 8 - x^2)} \le 0$$

$\boxed{a < 0 \quad D < 0} \Rightarrow$ −ve always

$$\Rightarrow \boxed{\frac{x - 5}{x + 2} \ge 0}$$

$+ \quad -2 \quad - \quad 5 \quad +$ $\Rightarrow x \in (-\infty, -2) \cup [5, \infty)$

$a = 2,\ b = 5$

$GCD(2a^4 + 4,\ b^2 - 1)$

HCF $(36, 24) = 12$ fnf

Question [IIT JEE 1982] ★ ★ ★ New to me

Q. The largest interval for which $x^{12} - x^9 + x^4 - x + 1 > 0$ is

(A) $-4 < x \le 0$ (B) $0 < x < 1$

(C) $-100 < x < 100$ (D) $-\infty < x < \infty$

Sol. $x \ge 1 \Rightarrow \underbrace{x^{12} - x^9}_{+ve} + \underbrace{x^4 - x}_{+ve} + 1 > 0 \rightarrow$ +ve

$x \le 0 \Rightarrow \underbrace{(x^{12} + x^4 + 1)}_{+ve} \underbrace{- (x^9 + x)}_{(-)\,(-ve,\ -ve)} \rightarrow$ +ve

$0 < x < 1 \Rightarrow \underbrace{1 - x}_{+ve} + \underbrace{x^4 - x^9}_{+ve} + \underbrace{x^{12}}_{+ve} \rightarrow$ +ve

$\Rightarrow$ Hence $x \in R$ or $(-\infty < x < \infty)$ fnf.

EQUATIONS CONTAINING SQUARE ROOT

$\boxed{\text{Irrational equations}}$ → Power is in fraction

Squaring करने से Extra roots आ सकते हैं।

#Verify → all values obtained after squaring

Q. Solve: $\sqrt{6 - 4x - x^2} = x + 4$

Sol. $\sqrt{6 - 4x - x^2} = x + 4$

SBS $\Rightarrow 6 - 4x - x^2 = (x + 4)^2$

$\Rightarrow 6 - 4x - x^2 = x^2 + 16 + 8x$

$\Rightarrow 0 = 2(x^2 + 6x + 5)$

$\Rightarrow 0 = 2(x + 5)(x + 1)$

$x = -5, -1$, now check these values

$\sqrt{6 - 4x - x^2} = x + 4$

$\sqrt{6 - 4(-5) - (-5)^2} = -5 + 1 \Rightarrow 1 = -1$ ✗

$\boxed{\#\ x = -5}$ ← Reject (extra root)

$\sqrt{6 - 4x - x^2} = x + 4$

$\sqrt{6 - 4(-1) - (-1)^2} = -1 + 4 \Rightarrow 3 = 3$ ✓

$\boxed{\#\ x = -1}$ ← Accepted & fnf.

Question [IIT-1997]

Q. The equation $\sqrt{x + 1} - \sqrt{x - 1} = \sqrt{4x - 1}$ has

(A) no solution (B) one solution

(C) two solution (D) more than 2 sol.

Sol. $\sqrt{x + 1} - \sqrt{x - 1} = \sqrt{4x - 1}$

SBS: $(x + 1) + (x - 1) - 2\sqrt{x + 1}\sqrt{x - 1} = 4x - 1$

$\Rightarrow -2\sqrt{x + 1}\sqrt{x - 1} = 2x - 1$

SBS: $4(x + 1)(x - 1) = 4x^2 + 1 - 4x$

$\Rightarrow (4x^2 - 4) = 4x^2 - 4x + 1$

$\Rightarrow 4x = 5 \Rightarrow x = \frac{5}{4}$ (Rejected) $\Rightarrow$ No sol.

Question [JEE Main-2023]

Q. The number of real roots of the equation $\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$, is

(A) 0 (B) 1 (C) 3 (D) 2

Sol. $\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$

$$\boxed{\sqrt{(x - 3)(x - 1)} + \sqrt{(x - 3)(x + 3)} - \sqrt{(x - 3)(4x - 2)} = 0}$$

$\sqrt{x - 3}\left(\sqrt{x - 1} + \sqrt{x + 3} - \sqrt{4x - 2}\right) = 0$

$\sqrt{x - 3} = 0 \rightarrow x = 3$

$\sqrt{x - 1} + \sqrt{x + 3} = \sqrt{4x - 2}$

SBS: $x - 1 + x + 3 + 2\sqrt{x - 1}\sqrt{x + 3} = 4x - 2$

$\Rightarrow 2\sqrt{x - 1}\sqrt{x + 3} = 2(x - 2)$

SBS: $(x - 1)(x + 3) = x^2 + 4 - 4x$

$\Rightarrow x^2 + 2x - 3 = x^2 + 4 - 4x$

$\Rightarrow 6x = 7 \Rightarrow x = \frac{7}{6}$ (rejected.)

Hence only one root $x = 3$ fnf.

IRRATIONAL INEQUALITIES

Another rules of inequality

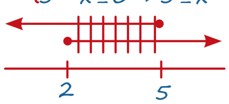
Squaring both side in an inequality is only allowed when both sides are non-negative (+ve or zero)

Q. $\sqrt{x-2} \geq \sqrt{5-x}$

Sol. $\boxed{\sqrt{x-2} \geq \sqrt{5-x}}$

Defining

$\cap \begin{cases} x-2 \geq 0 \rightarrow x \geq 2 \\ 5-x \geq 0 \rightarrow 5 \geq x \end{cases}$



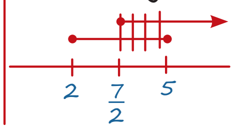
$\boxed{x \in [2, 5]}$

Solution SBS:

$x - 2 \geq 5 - x$

$2x \geq 7 \Rightarrow \boxed{x \geq \frac{7}{2}}$

fnf = defining ∩ solution



$x \in \left[\frac{7}{2}, 5\right]$

Q. Solve: $2 - \frac{2x}{3} \geq \sqrt{x+4}$

Sol. Sumit: **Defining:** $x + 4 \geq 0 \Rightarrow \boxed{x \geq -4}$

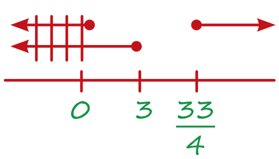
Case-I: Where LHS is +ve/0

$\Rightarrow 2 - \frac{2x}{3} \geq 0 \Rightarrow \boxed{3 \geq x}$ (initial condition)

$\Rightarrow 2 - \frac{2x}{3} \geq \sqrt{x+4}$

$\Rightarrow SBS : \frac{4}{9}(9 + x^2 - 6x) \geq x + 4$

$\Rightarrow x(4x - 33) \geq 0$

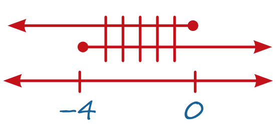


C-I : $\boxed{x \in (-\infty, 0]}$

Case-II: When LHS is –ve

$\underbrace{2 - \frac{2x}{3}}_{-ve} \geq \underbrace{\sqrt{x+4}}_{+ve} \Rightarrow \phi$

fnf = Defining ∩ solution

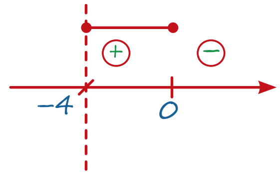


$x \in [-4, 0]$fnf

Santosh: Defining $x + 4 \geq 0 \Rightarrow \boxed{x \geq -4}$

$\Rightarrow$ WCM $\Rightarrow 2 - \frac{2x}{3} - \sqrt{x+4} \geq 0$

As, Roots of LHS is $\boxed{x = 0}$



$x \in [-4, 0]$ fnf

MODULUS FUNCTION

Also called as Absolute value function

$$\boxed{\begin{aligned} |x| &= x, \ x \geq 0 \\ &= -x, \ x < 0 \end{aligned}}$$

Ex: $|x - 5| = x - 5, \quad x - 5 \geq 0$

$\quad\quad = -(x - 5), \ x - 5 < 0$

Ex: $|(x^2 - 5x + 6)| = x^2 - 5x + 6, \ x \leq 2$

$\quad\quad = -(x^2 - 5x + 6), \ 2 < x < 3$

$\quad\quad = x^2 - 5x + 6, \quad x \geq 3$

PROPERTIES OF MOD

1. जब भी modulus aaye to sabse pehle use हटाओ
2. $\sqrt{x^2} = |x|$
3. $|x^2| = |x|^2 = x^2$
4. $|ab| = |a||b|$
5. $|a^n| = |a|^n$
6. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
7. $|a \pm b| \neq |a| \pm |b|$ (in general)

MODULUS EQUATION

100% Question aayega!! KKPNCC

1. If $\alpha > 0$ and $|x| = \alpha$, then $\boxed{x = \pm \alpha}$
2. If $\alpha \in R$ and $|x| = |\alpha|$, then $\boxed{x = \pm \alpha}$

Question [JEE Main-2023]

Q. The sum of all the roots of the equation $|x^2 - 8x + 15| - 2x + 7 = 0$ is

(A) $9 + \sqrt{3}$ (B) $11 + \sqrt{3}$

(C) $9 - \sqrt{3}$ (D) $11 - \sqrt{3}$

Sol. **Sunita:**

$\boxed{|x^2 - 8x + 15| = 2x - 7}$

⊕	⊖
$\Rightarrow x^2 - 8x + 15 = 2x - 7$	$x^2 - 8x + 15 = -(2x - 7)$
$\Rightarrow x^2 - 10x + 22 = 0$	$\Rightarrow x^2 - 6x + 8 = 0$
$x = 5 \pm \sqrt{3}$	$x = 2, 4$

For $|x| = \alpha \Rightarrow \alpha \geq 0$

$2x - 7 \geq 0 \rightarrow x \geq \frac{7}{2}$

$x = \underset{\times}{2}, \underset{\checkmark}{4}, \underset{\checkmark}{5 + \sqrt{3}}, \underset{\times}{5 - \sqrt{3}} \Rightarrow \text{sum} = 9 + \sqrt{3}$ fnf

Sushmita: $|(x - 5)(x - 3)| - 2x + 7 = 0$

Mod हटाओ

C-I: $\boxed{x \in (-\infty, 3] \cup [5, \infty)}$ + 3 − 5 +

$\Rightarrow x^2 - 8x + 15 - 2x + 7 = 0$

$x^2 - 10x + 22 = 0$

$x = 5 \pm \sqrt{3}$

$\Rightarrow$ only $\boxed{x = 5 + \sqrt{3}}$ is accepted

$x \in (-\infty, 3] \cup [5, \infty)$

C-II: Now, for $\boxed{x \in (3, 5)}$

$-(x^2 - 8x + 15) - 2x + 7 = 0$

$\Rightarrow -x^2 + 6x - 8 = 0$

$\Rightarrow x = 2, 4$

$\Rightarrow$ only $\boxed{x = 4}$ is accepted.

Question [JEE Adv.-2021]

Q. For $x \in R$, the number of real roots of the equation: $3x^2 - 4|x^2 - 1| + x - 1 = 0$ is

Sol. + −1 − 1 +

$3x^2 - 4|(x - 1)(x + 1)| + x - 1 = 0$

Case-I: $\boxed{x \in (-\infty, -1] \cup [1, \infty)}$ (initial cond^n.)

$\Rightarrow 3x^2 - 4(x^2 - 1) + x - 1 = 0$

$\Rightarrow x^2 - x - 3 = 0$

$x = \frac{1 \pm \sqrt{1 - 4(-3)}}{2} = \frac{1 \pm \sqrt{13}}{2}$ (both accepted)

Case-II: $\boxed{x \in (-1, 1)}$ (initial condition)

$3x^2 + 4(x^2 - 1) + x - 1 = 0$

$\Rightarrow 7x^2 + x - 5 = 0$

$\Rightarrow x = \frac{-1 \pm \sqrt{1 - 4(7)(-5)}}{2(7)} = \frac{1 \pm \sqrt{141}}{14}$

(both are accepted)

$\Rightarrow$ Number of real roots = 4 fnf.

Question [JEE Main-2024]

Q. The number of real solutions of the equation $x|x + 5| + 2|x + 7| - 2 = 0$ is

Sol. $x < -7$ | $-7 \leq x < -5$ | $x \geq -5$ (points −7, −5 on the number line)

Case-I: $\boxed{x < -7}$ (initial condition)

$x(-x - 5) + 2(-x - 7) - 2 = 0$

$\Rightarrow -x^2 - 5x - 2x - 14 - 2 = 0$

$\Rightarrow x^2 + 7x + 16 = 0 \Rightarrow x \in \phi$ (No real roots)

Case-II: $\boxed{-7 \leq x < -5}$ (initial condition)

$-x(x + 5) + 2(x + 7) - 2 = 0$

$\Rightarrow -x^2 - 5x + 2x + 14 - 2 = 0$

$\Rightarrow x^2 - 3x - 12 = 0$

$\Rightarrow x = \frac{-3 \pm \sqrt{57}}{2}$ ⊖ → −5. something (✓); ⊕ → 2. something (×)

Case-III: $\boxed{x \geq -5}$ (initial condition)

$x(x + 5) + 2(x + 7) - 2 = 0$

$\Rightarrow x^2 + 5x + 2x + 14 - 2 = 0$

$\Rightarrow x^2 + 7x + 12 = 0$

$\Rightarrow (x + 3)(x + 4) = 0$

$\Rightarrow x = -3, -4$ (both accepted)

$\Rightarrow$ Number of real roots = 3 fnf.

Q. Number of solution equation $|5x + 2| + |3 - x| = 7x + 5$

(A) 0 (B) 1

(C) 2 (D) 3

Sol. Case-I: $x < -2/5$

$-(5x + 2) + 3 - x = 7x + 5$

$\Rightarrow -5x - 2 + 3 - x = 7x + 5$

$\Rightarrow 1 - 5 = 13x \Rightarrow \boxed{\frac{-4}{13} = x}$ Rejected

Case-II: $-2/5 \le x < 3$

$(5x+2) + (3-x) = 7x + 5$

$\Rightarrow 4x + 5 = 7x + 5$

$\boxed{x = 0}$ Accepted

Case-III: $x \ge 3$

$5x + 2 + (-(3-x)) = 7x + 5$

$\Rightarrow 5x + 2 - 3 + x = 7x + 5$

$\boxed{x = -6}$ Rejected

$\Rightarrow$ No. of sol = 1 fnf.

MODULUS INEQUALITY

1. If $|x| < \alpha \Rightarrow -\alpha < x < \alpha$
2. If $|x| \le \alpha \Rightarrow -\alpha \le x \le \alpha$
3. If $|x| > \alpha \Rightarrow x < -\alpha$ union $x > \alpha$
4. If $|x| \ge \alpha \Rightarrow x \le -\alpha$ union $x \ge \alpha$

Question

Q. If solution set of inequality $|x-2| - 3 \le 2$ is $x \in [-a, b] \cup [a, c]$ where $b < a < c$, then $a + b + c$ is ________.

Sol. Say: $|x-2| - 3 = \alpha$

$\Rightarrow |\alpha| \le 2 \Rightarrow -2 \le \alpha \le 2$

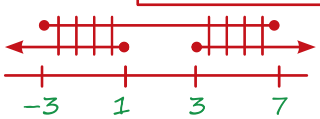
$-2 \le |x-2| - 3 \le 2$

$\boxed{1 \le |x-2| \le 5}$

$1 \le |x-2| \quad \cap \quad |x-2| \le 5$

$(x - 2 \le -1) \cup (1 \le x - 2) \quad | \quad -5 \le x - 2 \le 5$

$\boxed{x \le 1 \cup 3 \le x} \quad | \quad \boxed{-3 \le x \le 7}$



$x \in [-3, 1] \cup [3, 7]$ fnf

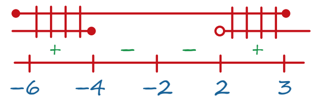
$\Rightarrow a + b + c = 3 + 1 + 7 = 11$ fnf.

Question [JEE Main-2022]

Q. Let $S = \left\{x \in [-6, 3] - \{-2, 2\} : \frac{|x+3| - 1}{|x| - 2} \ge 0\right\}$ and $T = \{x \in Z : x^2 - 7|x| + 9 \le 0\}$. Then the number of elements of $S \cap T$ is

(A) 7 (B) 5 (C) 4 (D) 3

Sol. $\boxed{\frac{|x+3| - 1}{|x| - 2} \ge 0}$ $x = -2, -4$; $x = 2, -2$



$S \Rightarrow \boxed{x \in [-6, -4] \cup (2, 3]}$

intergers are $x = -6, -5, -4, 3$

Check these 4 integers in T

$\Rightarrow x^2 - 7|x| + 9 \le 0$

$\boxed{x = -6, -5, -4, 3}$

$\Rightarrow x = -6$

$36 - 7 \times 6 + 9 = 3 \le 0$ ×

$\Rightarrow x = -5$

$25 - 35 + 9 = -1 \le 0$ √

$\Rightarrow x = -4$

$16 - 28 + 9 = -3 \le 0$ √

$\Rightarrow x = 3$

$9 - 21 + 9 = -3 \le 0$ √

$\Rightarrow$ No. of element in $(S \cap T) = 3$ fnf.

SPECIAL RESULT

IMPORTANT FORMAT

1. $|\alpha| + |\beta| = |\alpha + \beta| \Rightarrow$ solution: $\boxed{\alpha\beta \ge 0}$
2. $|\alpha| + |\beta| > |\alpha + \beta| \Rightarrow$ solution: $\boxed{\alpha\beta < 0}$
3. $|\alpha| + |\beta| = |\alpha - \beta| \Rightarrow$ solution: $\boxed{\alpha\beta \le 0}$
4. $|\alpha| + |\beta| > |\alpha - \beta| \Rightarrow$ solution: $\boxed{\alpha\beta > 0}$

Blue बादल (KKPNCC)

अगर same sign वाली satisfy करे तो $\Rightarrow \alpha\beta > 0$

अगर opposite sign वाली satisfy करे तो $\Rightarrow \alpha\beta < 0$

equality ($\ge$, $\le$) के लिए किसी एक α or β को zero put करके देखो

Blue बादल (KKPNCC)

Hint: ek traf दो अलग-अलग mod & ek side ek hi mod.

Q. Solve for values of x:

(A) $|3x - 5| + |8 - x| = |3 + 2x|$

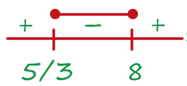
(B) $|x^2 - 5x + 6| + |x^2 - 4| = 5|x - 2|$

Sol. (A) $\underbrace{|3x - 5|}_{\alpha} + \underbrace{|8 - x|}_{\beta} = \underbrace{|3 + 2x|}_{\alpha + \beta}$

$\boxed{|\alpha| + |\beta| = |\alpha + \beta|} \Rightarrow \alpha\beta \ge 0$

$(3x - 5)(8 - x) \geq 0$

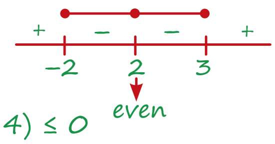
$(3x - 5)(x - 8) \leq 0$



$\Rightarrow x \in \left[\frac{5}{3}, 8\right]$ fnf

(B) $|\overbrace{x^2 - 5x + 6}^{\alpha}| + |\overbrace{x^2 - 4}^{\beta}| = 5\overbrace{|x - 2|}^{|\beta - \alpha|}$

$\boxed{|\alpha| + |\beta| = |\beta - \alpha|}$



$\Rightarrow \alpha\beta \leq 0$

$(x^2 - 5x + 6)(x^2 - 4) \leq 0$

$\Rightarrow (x - 2)^2(x - 3)(x + 2) \leq 0$

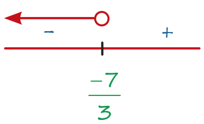
$x \,\varepsilon\, [-2, 3]$ fnf.

Q. Solve: $|(x^2 + 5x + 9)| < |x^2 + 2x + 2| + |3x + 7|$.

Sol. $|\overbrace{x^2 + 2x + 2}^{\alpha} + \overbrace{3x + 7}^{\beta}| < |\overbrace{x^2 + 2x + 2}^{\alpha}| + |\overbrace{3x + 7}^{\beta}|$

$|\alpha + \beta| < |\alpha| + |\beta| \Rightarrow \alpha\beta < 0$

$\underbrace{(x^2 + 2x + 2)}_{\text{+ve always}}(3x + 7) < 0$



$\boxed{x \in \left(-\infty, \frac{-7}{3}\right)}$ fnf.

Question [JEE Main-2021]

Q. If $A = \{x \in R;\ |x - 2| > 1\}$, $B = \{x \in R: \sqrt{x^2 - 3} > 1\}$, $C = \{x \in R;\ |x - 4| \geq 2\}$ and Z is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^C \cap Z$ is

Sol. Set A: $|x - 2| > 1$

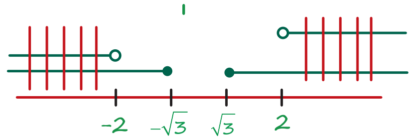
$x - 2 < -1 \ \cup \ x - 2 > 1$

$x < 1 \cup x > 3$

A : $\boxed{x\ (-\infty, 1) \cup (3, \infty)}$

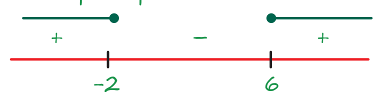
Set B: $\sqrt{x^2 - 3} > 1$

Solⁿ: SBS	defining
$x^2 - 3 > 1$	$x^2 - 3 \geq 0$
$x^2 - 4 > 0$	$(x - \sqrt{3})(x + \sqrt{3}) \geq 0$
$(x - 2)(x + 2) > 0$	

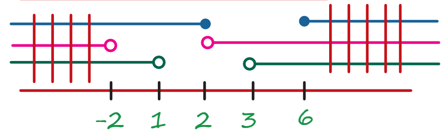


B: $\boxed{x \in (-\infty, -2) \cup (2, \infty)}$

set C: $|x - 4| - 2 \geq 0$



C : $\boxed{x \in (-\infty, -2) \cup [6, \infty)}$



$A \cap B \cap C \Rightarrow \boxed{x \in (-\infty, -2) \cup [6, \infty)}$

Complement $\Rightarrow (A \cap B \cap C)^C \,\varepsilon\, [-2, 6)$

$[-2, 6) \cap Z \rightarrow$ no. of elem $= 8$

$\Rightarrow$ number of subsets $= 2^8 = 256$ fnf

Question [JEE Main-2023]

Q. Let $\lambda \in R$ and let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then the largest element in the set $S = (x + \lambda : x$ is an integer solution of E) is

Sol. E: $\underbrace{|x|^2 - 2|x|}_{\text{Integer}} + \underbrace{|\lambda - 3|}_{\text{Integer}} = 0,\ x \in Z$

$\Rightarrow |x|^2 - 2|x| + 1 + |\lambda - 3| - 1 = 0$

$(|x| - 1)^2 + |\lambda - 3| = 1$

Case-I: $|x| - 1 = 0$ & $|\lambda - 3| = 1$

$x = \pm 1$ & $\lambda = 4$ or 2

Case-II: $|x| - 1 = 1$ & $|\lambda - 3| = 0$

$x = \pm 2$ & $\lambda = 3$

$\Rightarrow$ Largest value of $x + \lambda = 5$ fnf.

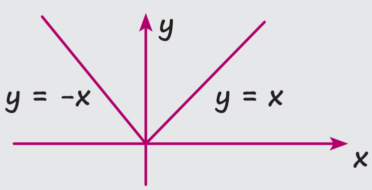
DOMAIN, RANGE & GRAPH OF MOD

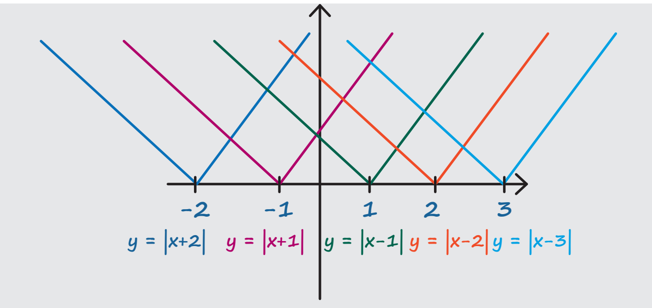
$f(x) = |x|$

▲ $x \in R$

▲ Range (value of mod) $\in [0, \infty)$

Graph:

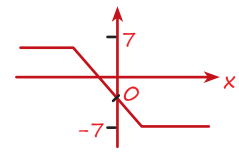
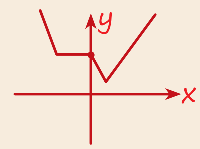
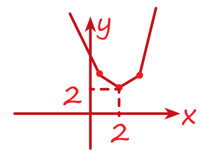
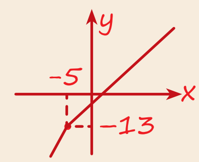




Blue बादल (KKPNCC)

Use madam method (x ki value put kro, y nikalo, fir point बनाकर plot कर दो, Now join all points) to draw graph Mod of Linear function

Q. Match The Graph:

Column-I:		Column-II:	
(P)	$f(x) = \|x-1\| + \|x-2\| + \|x-3\|$	(A)	
(Q)	$f(x) = 3\|x-2\| + \|x+5\| - 2\|x\|$	(B)	
(R)	$f(x) = 2x - 3 - \|x+5\|$	(C)	
(S)	$f(x) = \|x-3\| - \|x+4\|$	(D)	

Sol. P-(C), Q-(B), R-(D), S-(A)

LOGARITHM

100% Question aayega!!

KKPNCC

Direct Question from log in Mains + Adv.

1. $\log_B A$ = log 'A' with base 'B'
2. Value of $\log_B A$ = base(B) ki power में क्या रखें ki vo (A) बन जाए।
3. $\log_B A$ is only defined when

$\Rightarrow \{A > 0\} \cap \{B > 0\} \cap \{B \neq 1\}$ ★★★

Q. Values of x for which $f(x) = \log_{|x-3|}\left(\frac{|x|-5}{2-|x|}\right)$ is defined.

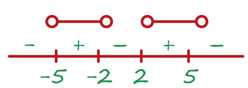
Sol. $f(x) = \log_{|x-3|}\left(\frac{|x|-5}{2-|x|}\right)$

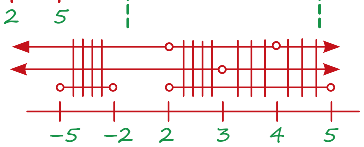
$$\left\{\frac{|x|-5}{2-|x|} > 0\right\} \cap \{|x-3| > 0\} \cap \{|x-3| \neq 1\}$$

$x \in R - \{3\}$

$x - 3 \neq \pm 1$

$\Rightarrow x \neq 4, 2$





fnf $\Rightarrow \{x \in (-5, -2) \cup (2, 5) - \{3, 4\}\}$

PROPERTIES ★★★

Sara khel properties (jayedaad) ka hai !!

Point-01: If $\boxed{\log_b a = C} \Leftrightarrow \boxed{a = b^c}$

Point-02: $\log_a m + \log_a n = \log_a (mn)$

$\log_a m + \log_a n + \log_a p = \log_a (mnp)$

Point-03: $\log_a m - \log_a n = \log_a \left(\frac{m}{n}\right)$

Point-04: (i) $\log_{(a)^q} (m)^p = \frac{p}{q} \log_a m$

(ii) $\log_a (m)^{2n} = 2n \log_a |m|$

Point-05: Base change property:

(i) Apni marzi ka base: $\log_b a = \frac{\log_c a}{\log_c b}$

(ii) Jab log ki terms multiplication mein ho

$\log_b a \cdot \log_c b = \log_c a$

(iii) Log wali term ka reciprocal: $\log_b a = \frac{1}{\log_a b}$

Point-06: $(a)^{\log_c b} = (b)^{\log_c a}$

Also: (i) $(a)^{(k \log_c b)} = b^{k \log_c a}$

(ii) $(v_1)^{v_2} = (e)^{v_2 \ln(v_1)}$

Memorize Following: $\log_{10} x = \frac{\ln x}{2.303}$

$\ln 2 = 0.693$, $\ln 3 = 1.098$, $\ln 5 = 1.609$, $\ln 7 = 1.945$,

Note

Line of thinking

(i) Try to make "base" same every where.

(ii) Terms ko halka krne ki koshish kro.

(iii) Property khud bta degi ki kaunsi lagegi.

Q. Simplify following:

(i) $\log_{1/3} \sqrt[4]{729 \cdot \sqrt[3]{9^{-1}} \cdot 27^{-4/3}}$

(ii) $\frac{\log_5 250}{\log_{50} 5} - \frac{\log_5 10}{\log_{1250} 5}$ ★★★

Sol. (i) $\log_{1/3} \sqrt[4]{729 \cdot \sqrt[3]{9^{-1}} \cdot 27^{-4/3}}$

$= \log_{\frac{1}{3}} \left(\sqrt[4]{729 \times \frac{1}{9}}\right) = \log_{\frac{1}{3}} (3^4)^{\frac{1}{4}} = \log_{\frac{1}{3}} 3$

$= -1$ fnf

(ii) $\log_5 250 = \log_5 125 + \log_5 2 = 3 + \log_5 2$

$\log_5 10 = \log_5 5 + \log_5 2 = 1 + \log_5 2$

$\log_{1250} 5 = \frac{1}{\log_5 1250} = \frac{1}{\log_5 625 + \log_5 2}$

$= \frac{1}{4 + \log_5^2}$

$\log_{50} 5 = \frac{1}{\log_5 50} = \frac{1}{\log_5 25 + \log_5 2}$

$= \frac{1}{2 + \log_5 2}$

मान लो: $\log_5 2 = \alpha$

$\Rightarrow \frac{\log_5 250}{\log_{50} 5} - \frac{\log_5 10}{\log_{1250} 5} = \frac{(3+\alpha)}{\left(\frac{1}{2+\alpha}\right)} - \frac{(1+\alpha)}{\left(\frac{1}{4+\alpha}\right)}$

$= (3 + \alpha)(2 + \alpha) - (1 + \alpha)(4 + \alpha)$

$= 6 + 2\alpha + 3\alpha + \alpha^2 - 4 - 4\alpha - \alpha - \alpha^2$

$= 2$ fnf

Acidic Question

Q. Value of: $a^{\sqrt{\log_a b}} - b^{\sqrt{\log_b a}} =$ ________ .

(A) $\sqrt{a} - \sqrt{b}$ (B) $b - a$

(C) $\sqrt{b} - \sqrt{a}$ (D) Zero

Sol. $(a)^{\sqrt{\log_a b}} - (a)^{\frac{\log_a b}{\sqrt{\log_a b}}} = (a^{\log_a b})^{\frac{1}{\sqrt{\log_a b}}}$

$= (b^{\log_b a})^{\frac{1}{\sqrt{\log_a b}}} = (b)^{\sqrt{\log_b a}}$

$\Rightarrow (a)^{\sqrt{\log_a b}} - (b)^{\sqrt{\log_b a}} = 0$ fnf

CHALLENGER QUESTION

Q. Simplify following:

$\sqrt{\log_2 3 . \log_2 12 \cdot \log_2 48 \cdot \log_2 192 + 16} - \log_2 12 \cdot \log_2 48 =$ ________ .

Sol. $\sqrt{\log_2 3 \cdot (2 + \log_2 3)(4 + \log_2 3)(6 + \log_2 3) + 16} - (2 + \log_2 3)(4 + \log_2 3)$

Put: $\log_2 3 = t$

$\Rightarrow \sqrt{t(2+t)(4+t)(6+t) + 16} - (2+t)(4+t)$

$\Rightarrow \sqrt{(t^2+6t)(t^2+6t+8)+16}-(t^2+6t+8)$

$m = t^2 + 6t$

$\Rightarrow \sqrt{m(m+8)+16}-(m+8)$

$= \sqrt{(m+4)^2}-(m+8) = |m+4|-(m+8)$

as $(m > 0)$

$(m+4)-(m+8) = -4$ fnf.

Question [JEE (Adv.)-2018]

Q. The value of $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$ is ________.

Sol. $(7^{\frac{1}{2}})^{\log_7 4} = 4^{\frac{1}{2}\log_7 7} = 4^{\frac{1}{2}} = 2$

Now, $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}}$

put: $\log_2 9 = t$

$(t^2)^{\frac{1}{\log_2 t}} = (t^2)^{\log_t 2} = 2^{2\log_t t} = 2^2 = 4$

fnf $= 4 \times 2 = 8$

LOGARITHMIC EQUATIONS

Check your solution by re-substitution in original equation

Variable agar power mein ho to $\Rightarrow$ Take "log both side"

Any expression/term is repeating $\Rightarrow$ assume it as 't'.

Note

If $a + b = cd$

Take $\log_{10}(\ )$ both sides:

$\Rightarrow \log a + \log b = \log c \log d$ → **Sushmita:** (✗)

$\Rightarrow \log(a+b) = \log(cd)$ → **Sunita:** (✓)

Question [JEE Main-2021]

Q. The number of solutions of the equation $\log_{(x+1)}(2x^2+7x+5) + \log_{(2x+5)}(x+1)^2 - 4 = 0$, $x > 0$, is ______.

Sol. $2x^2 + 7x + 5 = \underset{\alpha}{(x+1)}\,\underset{\beta}{(2x+5)}$

Given eq^n: $\log_\alpha \alpha\beta + \log_\beta(\alpha)^2 - 4 = 0$

$\Rightarrow \log_\alpha \alpha + \log_\alpha \beta + 2\log_\beta \alpha - 4 = 0$

$\Rightarrow \log_\alpha \beta + 2\log_\beta \alpha - 3 = 0$

put: $\boxed{\log_\beta \alpha = t} \Rightarrow \log_\alpha \beta = \frac{1}{t}$

$\frac{1}{t} + 2t - 3 = 0 \Rightarrow 2t^2 - 3t + 1 = 0$

$\Rightarrow (2t-1)(t-1) = 0 \Rightarrow t = 1/2$, or 1

$\Rightarrow \log_\beta \alpha = \frac{1}{2}$ or $1 \Rightarrow \boxed{\alpha = \sqrt{\beta}}$ or $\boxed{\alpha = \beta}$

for $\alpha = \sqrt{\beta} \Rightarrow (x+1) = \sqrt{2x+5} \Rightarrow$ SBS : $x = \pm 2$

for $\alpha = \beta \Rightarrow x + 1 = 2x + 5 \Rightarrow x = -4$

$\Rightarrow$ Only 2 is satisfying, hence fnf = 1

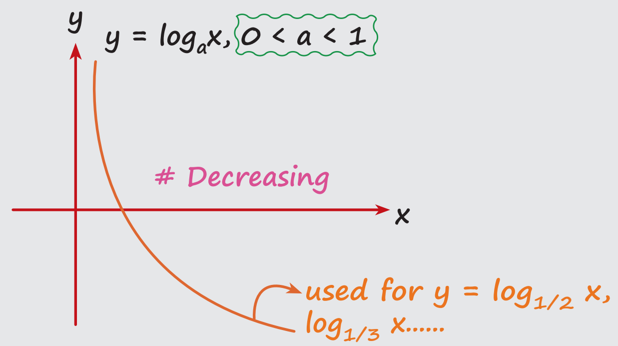
DOMAIN, RANGE & GRAPH OF LOG

$y = f(x) = \log_a x$, $a > 0$, $a \neq 1$

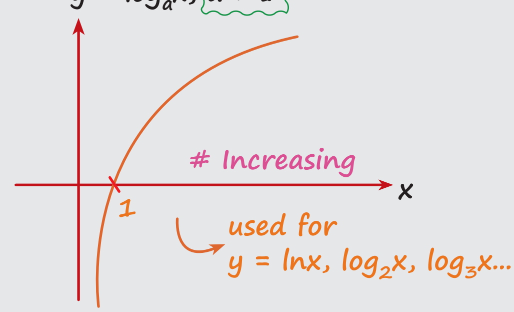
$x \in (0, \infty) \rightarrow$ Domain

$y \in (-\infty, \infty) \rightarrow$ Range

$y = \log_a x$, $0 < a < 1$



$y = \log_a x$, $a > 1$



MODULE CHALLENGER QUESTION

Q. Find the number of integral solution of the equation

$$\log_{\sqrt{x}}(x+|x-2|) = \log_x(5x-6+5|x-2|)$$

Sol. $\log_x(x+|x-2|)^2 = \log_x(5x-6+5|x-2|)$

$(x+|x-2|)^2 = 5(x+|x-2|) - 6$

put: $\boxed{x+|x-2| = m}$

$m^2 = 5m - 6$

$\Rightarrow m^2 - 5m + 6 = 0 \Rightarrow (m-2)(m-3) = 0$

$\Rightarrow m = 2, 3$

Case–I: $\boxed{x+|x-2| = 2}$

$x \geq 2$: $|x-2| = 2-x$

$x - 2 = 2 - x$

$2x = 4$

$x = 2$

$\cup$

$\boxed{x < 2}$ all are sol^n.

$-(x-2) = 2 - x$

Case–II:

$x \geq 2 \Rightarrow x - 2 = 3 - x$

$\Rightarrow x = \frac{5}{2}$ ✓

For $\boxed{x < 2} \Rightarrow \phi$

from C-I U C-II

$\Rightarrow \boxed{x = 2}$ is only integer satisfying.

LOG INEQUALITY

Point-01: If $\boxed{\text{base} > 1}$ $\Rightarrow$ log hatega या lagega to sign of inequality will remain same.

Point-02: If $\boxed{0 < \text{base} < 1}$ $\Rightarrow$ log hatega या to sign of inequality will reversed.

Point-03: $\boxed{\text{Base is variable}}$ $\Rightarrow$ Make cases according to point – 01 & 02.

Point-04: Before final answer take intersection with Domain of Log.

Blue बादल (KKPNCC)

Log – wali Inequality

defining part ∩ Solution Part

Question

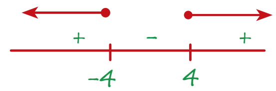
Q. Solve: $\log_{1/3}(x^2 - 7) \leq -2$

Sol. $\log_{1/3}(x^2 - 7) \leq -2$

$\Rightarrow x^2 - 7 \geq 9$

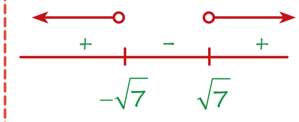
$\Rightarrow x^2 - 16 \geq 0$

$\Rightarrow (x-4)(x+4) \geq 0$

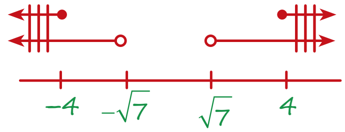


Def: $x^2 - 7 > 0$

$(x-\sqrt{7})(x+\sqrt{7}) > 0$



fnf: sol^n. ∩ Def:



$\Rightarrow \boxed{x \in (-\infty, -4] \cup [4, \infty)}$ fnf.

When base is variable

Consider two cases:

Case–I: Base > 1

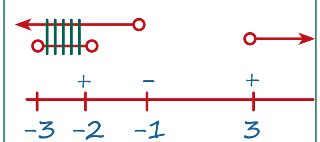
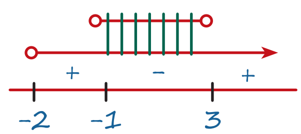
Case–II: 0 < Base < 1

$\Big\}$ Solution: (C–I) ∪ (C–II)

Question

Q. Find the solution set of inequality, $\log_{x+3}(x^2 - x) < 1$

Sol. $\boxed{\log_{(x+3)}(x^2 - x) < 1}$

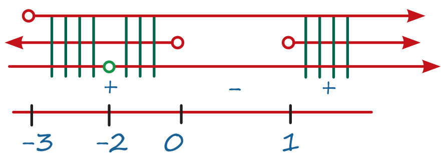
C–I: base > 1	C–II: 0 < base < 1
$\Rightarrow x + 3 > 1$	$\Rightarrow 0 < x + 3 < 1$
$\Rightarrow \boxed{x > -2}$	$\Rightarrow \boxed{-3 < x < -2}$
$\log_{(x+3)}(x^2 - x) < 1$	$\log_{(x+3)}(x^2 - x) < 1$
$\Rightarrow x^2 - x < (x+3)$	$\Rightarrow x^2 - x > (x+3)$
$\Rightarrow x^2 - 2x - 3 < 0$	$\Rightarrow x^2 - 2x - 3 > 0$
$\Rightarrow (x-3)(x+1) < 0$	
	
$x \in (-1, 3)$	$x \in (-3, -2)$

$\Rightarrow$ Solution (C-I U C-II) $\Rightarrow x \in (-3, -2) \cup (-1, 3)$

Defining:

$\boxed{x^2 - x > 0} \cap \boxed{x + 3 > 0} \cap \boxed{x + 3 \neq 1}$

$\Rightarrow x(x-1) > 0 \cap x > -3 \cap x \neq -2$



$x \in (-3\ -2) \cup (-2, 0) \cup (1, \infty)$

fnf: Solution ∩ Defining

$x \in (-3\ -2) \cup (-1, 0) \cup (1, 3)$ fnf.

Question [JEE Main-2023] ★★★

Q. The number of integral solution x of

$$\log_{x+\frac{7}{2}}\left(\frac{x-7}{2x-3}\right)^2 \geq 0 \text{ is}$$

(A) 6 (B) 8

(C) 5 (D) 7

Sol. Case-I: $x + \frac{7}{2} > 1 \Rightarrow \boxed{x > \frac{-5}{2}}$

$$\Rightarrow \log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$$

$$\Rightarrow \left(\frac{x-7}{2x-3}\right)^2 \geq 1$$

$$\Rightarrow \frac{(x-7)^2}{(2x-3)^2} - 1 \geq 0$$

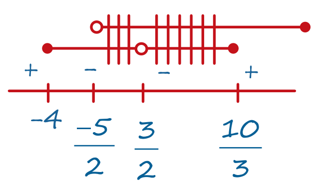
$$\Rightarrow \frac{(3x-10)(x+4)}{(2x-3)^2} \leq 0$$

$$x \in \left(\frac{-5}{2}, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \frac{10}{3}\right]$$

Case-II: $0 < x + \frac{7}{2} < 1 \Rightarrow \left(\frac{x-7}{2x-3}\right)^2 \leq 1 \Rightarrow \phi$

Defining:

$$x + \frac{7}{2} > 0, 0 + \frac{7}{2} \neq 1 \cap \left(\frac{x-7}{2x-3}\right)^2 > 0$$



Defining solution: $x \in \left(\frac{-5}{2}, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \frac{10}{3}\right]$

No. of intergeral solution = 6 fnf.

EXPONENTIAL FUNCTION

Domain, range & graph:

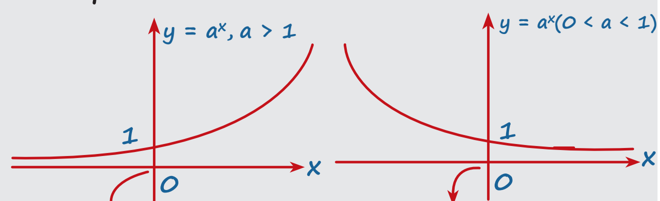
$\boxed{y = a^x}$ $(a > 0, a \neq 1)$

$2^x, 3^x, 4^x, e^x$.....

Domain $(x) \in R$

Range $(y) \in R^+$ or $(0, \infty)$

Graph:



Use this shape for $2^x, 3^x, e^x$......

Use this shape for $\left(\frac{1}{2}\right)^x = 2^{-x}, 3^{-x}, ...$

MOST IMPORTANT NOTEL

$(a)^b \rightarrow$ power (b) बदल रहा hai या variable है by forming an interval $b \in (p, q)$, then $a > 0$.

LAW OF INDICES (POWERS)

1. $a^0 = 1, (a \neq 0)$	6. $\sqrt[p]{\sqrt[q]{a}} = a^{\frac{1}{pq}} = \sqrt[q]{\sqrt[p]{a}}$
2. $a^{-m} = \frac{1}{a^m}, (a \neq 0)$	7. $a^{\frac{1}{p}} = \sqrt[p]{a}$
3. $a^m.a^n = a^{m+n}$	8. $a^m b^m = (ab)^m$
4. $\frac{a^m}{a^n} = a^{m-n}$	9. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
5. $(a^m)^n = a^{mn} = (a^n)^m$	10. $a^{b^c} = (a)^{(b^c)}$

Question

Q. If $a^x = \sqrt{b}, b^y = \sqrt[3]{c}$ and $c^z = \sqrt{a}$, then the value of xyz is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{1}{6}$ (D) $\frac{1}{12}$

Sol. $a^x = b^{\frac{1}{2}}$, $b^y = c^{\frac{1}{3}}$, $c^z = a^{\frac{1}{2}}$

$c^{2zx} = c^{\frac{1}{6y}}$

$2zx = \dfrac{1}{6y}$

fnf: $\boxed{xyz = \dfrac{1}{12}}$

$\Downarrow$

$b = c^{\frac{1}{3y}}$

$b^{\frac{1}{2}} = \left(c^{\frac{1}{3y}}\right)^{\frac{1}{2}}$

$\Downarrow$

SBS:

$c^{2z} = a$

$c^{2zx} = a^x$

EXPONENTIAL EQUATION

1. $a^x = a^y \Rightarrow \boxed{x = y}$
2. $\boxed{a^x = b^y}$ (Take log both side & proceed)

Question

Q. Solve: $4^{\sqrt{x^2-2}+x} - 5 \cdot 2^{x-1+\sqrt{x^2-2}} = 6$

Sol. $4^{\sqrt{x^2-2}+x} - 5 \cdot 2^{x-1+\sqrt{x^2-2}} = 6$

$$4^{\sqrt{x^2-2}+x} - 5 \cdot \frac{2^{x+\sqrt{x^2-2}}}{2} = 6$$

मान लो: $2^{x+\sqrt{x^2-2}} = t$

$$t^2 - \frac{5}{2}t - 6 = 0$$

$$2t^2 - 5t - 12 = 0 \Rightarrow (2t+3)(t-4) = 0$$

rejected

$$t = -\frac{3}{2}, 4 \qquad \Rightarrow 2^{x+\sqrt{x^2-2}} = 2^2$$

$$x + \sqrt{x^2-2} = 2 \Rightarrow \sqrt{x^2-2} = 2 - x$$

$$\Rightarrow x = \frac{3}{2} \text{ fnf}$$

EXPONENTIAL INEQUALITY

If $\boxed{a^x \geq a^y}$ $\begin{cases} \Rightarrow x \geq y, a > 1 \\ \Rightarrow x \leq y, 0 < a < 1 \end{cases}$

Aap dono traf log apply krke bhi proceed kr skte ho.

Question

A. Solve: $\left(\dfrac{3}{4}\right)^{6x+10-x^2} < \dfrac{27}{64}$

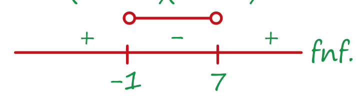
B. Solve: $2(25)^x - 5(10^x) + 2(4^x) \geq 0$ ★★★

Sol. (A) $\left(\dfrac{3}{4}\right)^{6x+10-x^2} < \left(\dfrac{3}{4}\right)^3$

$\Rightarrow 6x + 10 - x^2 > 3$

$\Rightarrow 0 > x^2 - 6x - 7$

$\Rightarrow 0 > (x-7)(x+1)$



(B) $2(25)^x - 5(10^x) + 2(4^x) \geq 0$

$\Rightarrow 2(5)^{2x} - 5.5^x.2^x + 2.2^{2x} \geq 0$

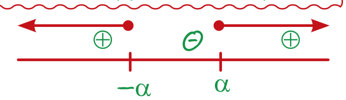
put: $5^x = m$, $2^x = n$

$2m^2 - 5mn + 2n^2 \geq 0$

$\Rightarrow 2m^2 - 4mn - mn + 2n^2 \geq 0$

$\Rightarrow (m-2n)(2m-n) \geq 0$

$\boxed{(5^x - 2.2^x)(2.5^x - 2^x) \geq 0}$



$x \in (-\infty, -\alpha] \cup [\alpha, \infty)$ where $\boxed{\alpha = \dfrac{\log_5 2}{1 - \log_5 2}}$

Question [JEE Main-2019, IIT-JEE-1997]

Q. Find the set of all solutions of the equation $2^{|y|} - |2^{y-1} - 1| = 2^{y-1} + 1$.

Sol. $\boxed{2^{|y|} = |2^{y-1} + 1| + |2^{y-1} - 1|}$

$\Rightarrow$ C-I: $y \geq 0$

$\boxed{2^{|y|} = |2^{y-1} - 1| + |2^{y-1} + 1|}$

$|\alpha + \beta| = |\alpha| + |\beta|$

Solution is $\alpha\beta \geq 0$

$(2^{y-1} + 1)(2^{y-1} - 1) \geq 0$

$2^{y-1} \geq 1 \Rightarrow y - 1 \geq 0 \Rightarrow \boxed{y \geq 1}$

$\boxed{\text{C-I: } y \geq 0 \cap y \geq 1} \Rightarrow \boxed{y \geq 1}$

C-II: $\boxed{y < 0}$

$2^{-y} = |2^{y-1} + 1| + |2^{y-1} - 1|$

$2^{-y} = 2^{y-1} + 1 + 1 - 2^{y-1}$

$2^{-y} = 2^1$

$-y = 1 \Rightarrow y = -1$

C-I U C-II $\Rightarrow \boxed{y \in [1, \infty) \cup \{-1\}}$ fnf

IMPORTANT POINT TO NOTE

JEE wale पुछते hai kyunki ismein cases सोचने pdte hai

$(v_1)^{v_2} = 1$		
Case-I: $v_1 = 1$ & $v_2 \in R$ solve: $v_1 = 1$	Case-II: $v_1 = -1$ & v_2 = aisa value jo LHS ko '1' bna de solve: $v_1 = -1$ & check!	Case-III: $v_2 = 0$ & $v_1 \neq 0$ solve: $v_2 = 0$ & check!

Critical Thinking Question [JEE Main-2016]

Q. The sum of all real values of x satisfying

$(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is

(A) 5 (B) 3

(C) −4 (D) 6

Sol. Case-I: $v_1 = 1$

$x^2 - 5x + 5 = 1$

$(x - 1)(x - 4) = 0$

$\boxed{x = 1, 4}$

Case-II: $v_2 = 0$

$x^2 + 4x - 60 = 1$

$(x - 6)(x + 10) = 0$

$\boxed{x = -10, 6}$

Check: $x = -10$

LHS $= (155)^0 = 1$

$x = 6$

LHS $= (11)^0 = 1$

Case-III: $v_1 = -1$

$x^2 - 5x + 5 = -1 \Rightarrow x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0 \Rightarrow \boxed{x = 2, 3}$

Check: $x = 2 \Rightarrow \text{LHS} = (-1)^{-48} = \dfrac{1}{(-1)^{48}} = 1$ ✓

$x = 3 \Rightarrow \text{LHS} = (-1)^{-39} = \dfrac{1}{(-1)^{39}} = \dfrac{1}{-1} = -1$ ✗

$\Rightarrow x = 1, 4, -10, 6, 2$

$\Rightarrow$ Sum of all values = 3 fnf.

RATIO'S

Note-01

If $a : b = c : d \Rightarrow \boxed{\dfrac{a}{b} = \dfrac{c}{d}}$

Note-02

Componendo & Dividendo (C & D)

If $\dfrac{a}{b} = \dfrac{c}{d} \Rightarrow \boxed{\dfrac{a+b}{a-b} = \dfrac{c+d}{c-d}}$

Note-03

If $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f}, \ldots$ then

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots\ldots = \frac{a+c+e+\ldots}{b+d+f+\ldots}$$

Note-04

If $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f}, \ldots$ then

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots\ldots = \frac{xa + yc + ze + \ldots}{xb + yd + zf + \ldots}$$

Note-05

If $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f}, \ldots$

then each $= \left(\dfrac{xa^n + yc^n + ze^n}{xb^n + yd^n + zf^n}\right)^{1/n}$

CHALLENGER QUESTION New to me

Q. Let a, b and c be distinct non zero real number such that $\dfrac{1-a^3}{a} = \dfrac{1-b^3}{b} = \dfrac{1-c^3}{c}$.

The value of $10(a^3 + b^3 + c^3)$ is

Sol. मान लो: $\dfrac{1-a^3}{a} = \dfrac{1-b^3}{b} = \dfrac{1-c^3}{c} = k$

$\boxed{\dfrac{1-a^3}{a} = k} \Rightarrow 1 - a^3 = ak$

Similarly

$\boxed{a^3 + ak - 1 = 0}$

$\boxed{b^3 + bk - 1 = 0}$

$\boxed{c^3 + ck - 1 = 0}$

Combine

Cubic

$\boxed{x^3 + kx - 1 = 0}$ → a, b, c

$x^3 + 0x^2 + kx - 1 = 0$

$\boxed{\begin{array}{l} a + b + c = 0 \\ abc = 1 \end{array}}$

$\Rightarrow 10(a^3 + b^3 + c^3) = 10(3abc) = 30$ fnf

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