PHYSICS WALLAH

LAKSHYA JEE

CLASS-XII

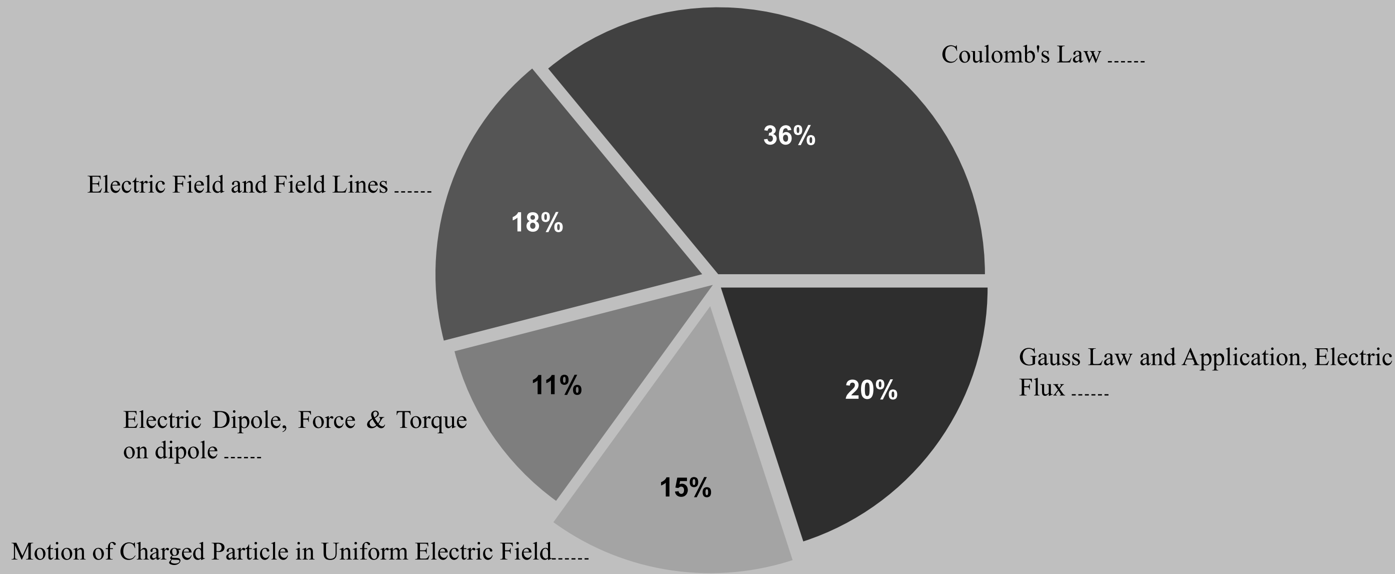
- Electric Charges and Fields
- Electrostatic Potential and Capacitance
- Current Electricity

PHYSICS Module 1

CHAPTER 1

Electric Charges and Fields

Topicwise Weightage of JEE Main 6 Years Paper (124 Sets)



"How's the Josh?" for these Topics: *Mark your confidence level in the blank space around the topic (Low-L, Medium-M, High-H)*

ELECTROSTATICS

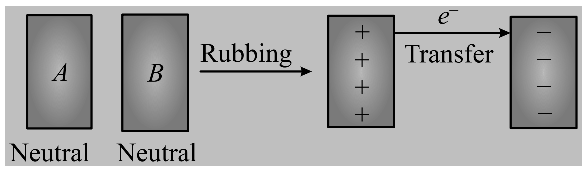
The branch of physics which deals with properties of charges at rest is called electrostatics.

ELECTRIC CHARGE

Charge is scalar physical quantity associated with matter due to which it produces and experiences electrical and magnetic effects. The excess or deficiency of electrons in a body gives it a net charge. A negatively charged body has excess of electrons while a positively charged body has deficiency of electrons.

Properties of Electric Charge

1. Charges interact with each other i.e., they exert force on each other. Like point charges repel each other while unlike point charges attract each other.
2. **Charge is of two kinds:** Positive and negative.
3. **Total charge of an isolated system is conserved** (Conservation of charge).
4. **Charge is quantised:** Charge is an integral multiple of electronic charge i.e., $Q = Ne$, where $e = 1.6 \times 10^{-19}$ C and N is an integer.
5. **Charge can be transferred:** Charge can be transferred from one body to another. This occurs due to transfer of electrons from one body to another. One of the common example of transfer of charge is charging by friction.



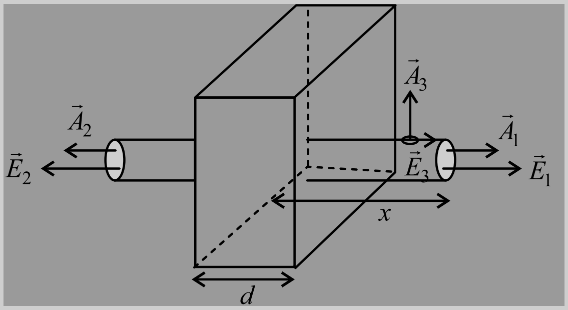
Frictional Electricity: When two bodies are rubbed with each other, they are found to attract each other. This is so because, on rubbing, transfer of electrons takes place from one body to another. One of them acquires a positive charge and other acquires a negative charge.

6. **Charge is invariant:** Charge of a particle is independent of its speed.
7. Charge cannot exist without mass, while mass can exist without charge. e.g., neutron, neutrino, antineutrino all are neutral particles having mass.

   SI Unit: coulomb (C)

   [1 coulomb = 1 ampere × 1 second]

   C.G.S. unit: stat coulomb or franklin



$$\oint \vec{E}\cdot\overrightarrow{dA} = \oint_{A_1} \vec{E}_1\cdot\overrightarrow{dA_1} + \oint_{A_2} \vec{E}_2\cdot\overrightarrow{dA_2} + \oint_{A_3} \vec{E}_3\cdot\overrightarrow{dA_3}$$

$$\frac{q_{enc}}{\varepsilon_0} = E_1A_1 + E_2A_2$$

$$\boxed{E_x = \frac{\rho d}{2\varepsilon_0}}$$

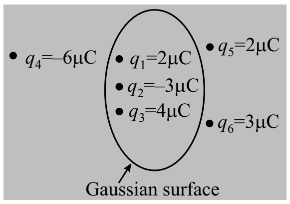
Train Your Brain

Example 18: Find out flux through the given Gaussian surface.

Sol. $\phi = \dfrac{Q_{in}}{\varepsilon_0}$

$= \dfrac{2\mu C - 3\mu C + 4\mu C}{\varepsilon_0}$

$= \dfrac{3\times10^{-6}}{\varepsilon_0}$ Nm²/C



Example 19: If a point charge q is placed at the center of a cube then find out flux through any one surface of cube.

Sol. Flux through 6 surfaces = $\dfrac{q}{\varepsilon_0}$

Since all the surfaces are symmetrical

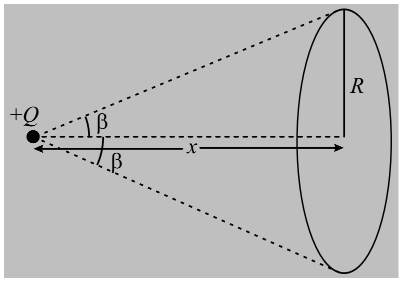
so, flux through one surface = $\dfrac{1}{6}\dfrac{q}{\epsilon_0}$

Example 20: The electric field in a region is given by $\vec{E} = \frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}$ with $E_0 = 2.0\times10^3$ N/C. Find the flux of this field through a rectangular surface of area 0.2 m² parallel to the $Y - Z$ plane.

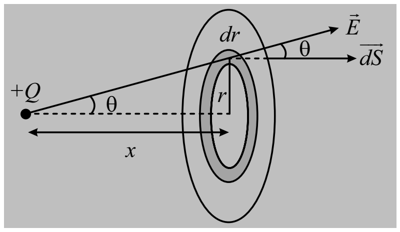
Sol. $\phi = \vec{E}\cdot\vec{S} = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right)\cdot\left(0.2\hat{i}\right)$

$= 240\,\dfrac{\text{N-m}^2}{\text{C}}$

Example 21: Find the electric flux due to a point charge 'Q' through the circular region of radius R if the charge is placed on the axis at a distance x as shown in figure.



Sol. We can divide the circular region into small rings.



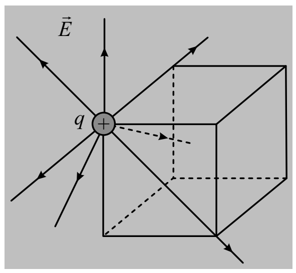
Let us take a ring of radius r and width dr. Flux through this small element $d\phi = E$ ds cos θ

$$\therefore\ \phi_{net} = \int Eds\cos\theta = \int_{r=0}^{r=R} \frac{kQ}{\left(x^2+r^2\right)}\left(2\pi r dr\right)\left(\frac{x}{\sqrt{x^2+r^2}}\right)$$

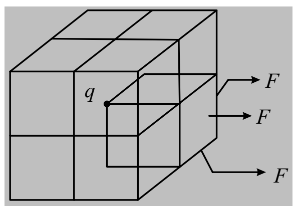
$$= \frac{Q}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right] = \frac{Q}{2\varepsilon_0}[1-\cos\beta]$$

Example 22: Consider a point charge q = 1mC placed at a corner of a cube of side 10 cm. Determine the electric flux through each face of the cube.

Sol. Here we will use the symmetry of the situation, which involves the faces joining at the corner at which the charge resides.



(*a*) A charge q is placed at the corner of a cube



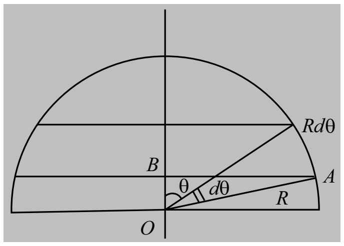
(*b*) By surrounding the charge with a series of cubes such that the charge is at the center of a larger cube, we have created a symmetric arrangement.

The total flux is $\frac{q}{\varepsilon_0}$. So that the flux through each of the sides of the large cube is $\frac{q}{6\varepsilon_0}$ and one quarter of that $\frac{q}{24\varepsilon_0}$ goes through each of the far sides of the small cube (because the faces of small cube which touch the charge have electric field parallel to them). Numerical evaluation gives

$$\phi = \frac{q}{24\varepsilon_0} = \frac{1\times10^{-3}\,\text{C}}{24\left(8.85\times10^{-12}\,\text{C}^2/\text{Nm}^2\right)} = 5\times10^6\,\text{Nm}^2/\text{C}$$

Example 23: Find by direct integration the electric field at the center of a hemispherical surface of charge having uniform surface density σ.

Sol.



Consider a ring-shaped element on the surface of the hemisphere at an angle θ as shown.

AB = the radius of the ring = $R \sin\theta$

Distance of center of ring B from $O = R\cos\theta$

Charge on the element = $dq = \sigma$ (area)

$dq = \sigma\,(2\pi R\sin\theta)\,Rd\theta$

Field at O due to this infinitesimal element = dE

$$dE = \frac{(dq)(R\cos\theta)}{4\pi\varepsilon_0\left(R^2\sin^2\theta + R^2\cos^2\theta\right)^{3/2}}$$

(using the result for the field at axis of a ring)

$$\Rightarrow E = \int dE = \int_0^{\pi/2} \frac{R\sin\theta\left(2\pi\sigma R^2\right)\cos\theta}{4\pi\varepsilon_0 R^3}\,d\theta$$

$$= \frac{\sigma}{2\varepsilon_0}\int_0^{\pi/2}\sin\theta\cos\theta\,d\theta = \frac{\sigma}{2\varepsilon_0}\left|\frac{\sin^2\theta}{2}\right|_0^{\pi/2}$$

$$= \frac{\sigma}{4\varepsilon_0}$$

Example 24: A system consists of a ball of radius R carrying a spherically symmetric charge and the surrounding space is filled with a charge of volume density $\rho = a/r$ where a is a constant, r is the distance from the center of ball. Find the ball's charge for which the magnitude of the electric field is independent of r outside the ball. How high is this strength?

Sol. Let us consider a spherical surface of radius $r(r > R)$ concentric with the ball and apply Gauss's Law.

$$\oint \vec{E}.\vec{dA} = \frac{q}{\varepsilon_0}$$

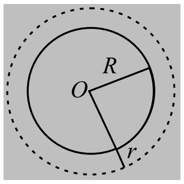
Let Q = Total charge on the ball

$$\varepsilon_0 E\left(4\pi r^2\right) = Q + \int_R^r \rho\, 4\pi x^2 dx$$

$$\varepsilon_0 E\left(4\pi r^2\right) = Q + 4\pi\int_R^r \frac{a}{x}x^2 dx$$

$$\varepsilon_0 E\left(4\pi r^2\right) = Q + 2\pi a\left(r^2 - R^2\right)$$

$$\Rightarrow E = \left(\frac{Q - 2\pi a R^2}{4\pi\varepsilon_0}\right)\frac{1}{r^2} + \frac{2\pi a}{4\pi\varepsilon_0}$$



For E to be independent of r,

$Q = 2\pi a R^2$

$$E = \frac{a}{2\varepsilon_0}$$

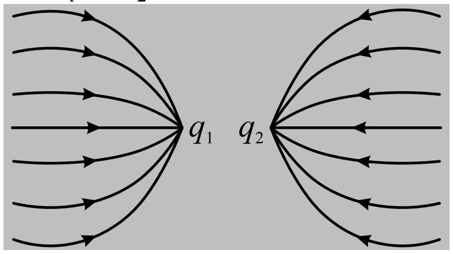
Concept Application

10. If an insulated non-conducting sphere of radius R has charge density ρ. The electric field at a distance r from the center of sphere ($r < R$) will be

(*a*) $\frac{\rho R}{3\varepsilon_0}$ (*b*) $\frac{\rho r}{\varepsilon_0}$

(*c*) $\frac{\rho r}{3\varepsilon_0}$ (*d*) $\frac{3\rho R}{\varepsilon_0}$

11. The given figure gives electric lines of force due to two charges q_1 and q_2. What are the signs of the two charges?



(*a*) Both are negative

(*b*) Both are positive

(*c*) q_1 is positive but q_2 is negative

(*d*) q_1 is negative but q_2 is positive

12. If electric field is uniform, then the electric lines of forces are

(*a*) Divergent

(*b*) Convergent

(*c*) Circular

(*d*) Parallel

13. A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its center.

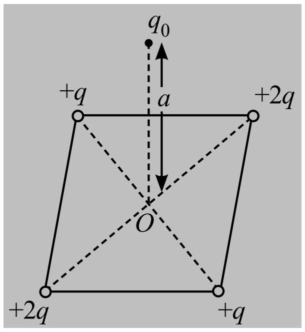
(*a*) Increases as r increases, for $r \le R$

(*b*) Decreases as r increases, for $0 < r < \infty$.

(*c*) Decreases as r increases, for $R < r < \infty$.

(*d*) Is discontinuous at $r = R$

1. Four point charges are fixed at the corners of a square of side length a. A positive charge q_0 is placed at a distance a from center of square perpendicular to the plane of square. If point charge q_0 is in equilibrium then its mass m is $\left(a=\sqrt{\frac{3}{2}}\text{m}\right)$.



(*a*) $\frac{2qq_0}{9\pi\varepsilon_0 g}\sqrt{\frac{3}{2}}$ (*b*) $\frac{4qq_0}{9\pi\varepsilon_0 g}\sqrt{\frac{3}{2}}$

(*c*) $\frac{5qq_0}{9\pi\varepsilon_0 g}\sqrt{\frac{3}{2}}$ (*d*) $\frac{8}{9}\frac{qq_0}{\pi\varepsilon_0 g}\sqrt{\frac{3}{2}}$

Sol. $F_E = mg$

$$\frac{2(k)(2q)q_0}{\left(a^2+\frac{a^2}{2}\right)}\frac{a}{\sqrt{a^2+\frac{a^2}{2}}}+\frac{2kqq_0}{\left(a^2+\frac{a^2}{2}\right)}\frac{a}{\sqrt{a^2+\frac{a^2}{2}}}=mg$$

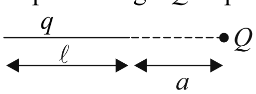
$$\frac{2kqq_0 a}{\left(\frac{3a^2}{2}\right)^{3/2}}(3)=mg$$

$$m=\frac{(2kqq_0)(a)8}{9g}=\left(\frac{2\times 8}{4\pi\varepsilon_0}\right)\frac{(qq_0)}{9g}\sqrt{\frac{3}{2}}$$

$$m=\frac{4qq_0}{9\pi\varepsilon_0 g}\sqrt{\frac{3}{2}}$$

Therefore, option (*b*) is the correct answer.

2. A thin straight rod of length l carrying a uniformly distributed charge q is located in vacuum. Find the magnitude of the electric force on a point charge 'Q' kept as shown in the figure.

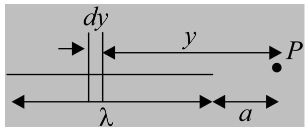


(*a*) $\frac{KQq}{(a+\ell)^2}$ (*b*) $\frac{KQq}{a\ell}$ (*c*) $\frac{KQq}{a(a+\ell)}$ (*d*) $\frac{KQq}{a(a-\ell)}$

Sol. As the charge on the rod is not point charge, therefore, first we have to find force on charge Q due to charge over a very small part on the length of the rod. This part called element of length, dy can be considered as point charge.

Charge on element, $dq=\lambda dy=\frac{q}{l}dy$

Electric force on 'Q' due to element



$$dF=\frac{k.dq.Q}{y^2}=\frac{k.Q.q.dy}{y^2 l}$$

All forces are along the same direction,

$\therefore\ F=\sum dF$. This sum can be calculated using integration,

therefore $F=\int_a^{a+l}\frac{KQqdy}{y^2 l}=\frac{KqQ}{l}=\left[-\frac{1}{y}\right]_a^{a+l}$

$$=\frac{KQ.q}{l}\left[\frac{1}{a}-\frac{1}{a+l}\right]=\frac{KQq}{a(a+l)}$$

NOTE: (1) The total charge of the rod cannot be considered to be placed at the center of the rod as we do in mechanics for mass in many problems.

(2) If $a >> l$ then, $F=\frac{KQq}{a^2}$

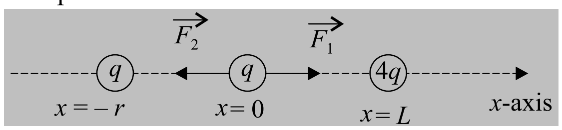
i.e., behavior of the rod is just like a point charge for very far away points.

Therefore, option (*c*) is the correct answer.

3. Two charges q and $4q$ are placed at $x = 0$ and $x = L$ along x-axis. Where should another particle of charge q, be placed on x-axis, so that net force on charge at $x = 0$ becomes zero?

(*a*) $x=\frac{L}{2}$ (*b*) $x = 2L$ (*c*) $x = -2L$ (*d*) $x=-\frac{L}{2}$

Sol. The charge $4q$ repels the charge q towards negative x-axis. So the charge q has to be placed on the left of charge q. Let it be placed at $x = -r$.



Let $\vec{F}_1$ be the force on q (at $x = 0$) due to charge at $x = -r$, given as

$$\vec{F}_1=\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q^2}{r^2}\hat{i}$$

Let $\vec{F}_2$ be the force on q (at $x = 0$) due to charge $4q$ at $x = L$, given as

$$\vec{F}_2=-\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q\times 4q}{L^2}\hat{i}$$

According to the condition given,

Net force $\vec{F}_1+\vec{F}_2 \Rightarrow \frac{q^2}{r^2}=\frac{4q^2}{L^2} \Rightarrow r=\frac{L}{2}$

$\therefore$ The charge q should be placed at $x=\frac{-L}{2}$.

Therefore, option (*d*) is the correct answer.

Board Level Problems

SINGLE CORRECT TYPE QUESTIONS

1. If a body has positive charge on it, then it means it has
   (*a*) Gained some protons
   (*b*) Lost some protons
   (*c*) Gained some electrons
   (*d*) Lost some electrons

2. Sure check for presence of electric charge is
   (*a*) Process of induction
   (*b*) Repulsion between bodies
   (*c*) Attraction between bodies
   (*d*) Frictional force between bodies

3. If a solid and a hollow conducting sphere have same radius then
   (*a*) Hollow sphere will hold more maximum charge
   (*b*) Solid sphere will hold more maximum charge
   (*c*) Both the spheres will hold same maximum charge
   (*d*) Both the sphere can't hold charge

4. When a conducting soap bubble is negatively charged then
   (*a*) Its size starts varying arbitrarily
   (*b*) It expands
   (*c*) It contracts
   (*d*) No change in its size takes place

5. Consider three-point charges P, Q and R. R and Q repel each other, while P and R attract. What is the nature of force between P and Q?
   (*a*) Repulsive force (*b*) Attractive force
   (*c*) No force (*d*) None of these

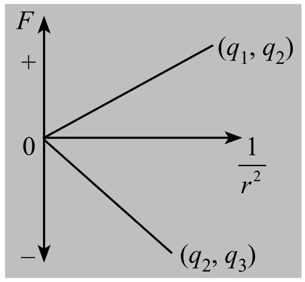
6. Which of the following process involves the principle of electrostatic induction?
   (*a*) Pollination (*b*) Chocolate making
   (*c*) Xerox copying (*d*) All of these

7. The electric field intensity at a point in vacuum is equal to
   (*a*) Zero
   (*b*) Force a proton would experience there
   (*c*) Force an electron would experience there
   (*d*) Force a unit positive charge would experience there

8. A sphere of radius R has electric charge uniformly distributed in its entire volume. At a distance d, from the centre inside the sphere, the electric field intensity is directly proportional to
   (*a*) $\frac{1}{d}$ (*b*) $\frac{1}{d^2}$ (*c*) d (*d*) d^2

9. Two identical small conducting balls B_1 and B_2 are given $-7\ pC$ and $+4\ pC$ charges respectively. They are brought in contact with a third identical ball B_3 and then separated. If the final charge on each ball is $-2pC$. the initial charge on B_3 was
   (*a*) $-2\ pC$ (*b*) $-3\ pC$ (*c*) $-5\ pC$ (*d*) $-15\ pC$

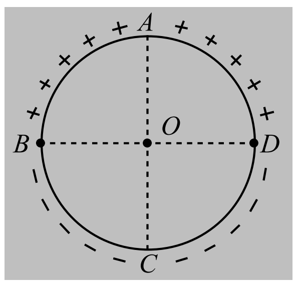
10. The Coulomb force (F) versus ($1/r^2$) graphs for two pairs of point charges (q_1 and q_2) and (q_2 and q_3) are shown in figure. The charge q_2 is positive and has least magnitude. Then



   (*a*) $q_1 > q_2 > q_3$ (*b*) $q_1 > q_3 > q_2$
   (*c*) $q_3 > q_2 > q_1$ (*d*) $q_3 > q_1 > q_2$

Directions: These questions consist of two statements each, printed as Assertion (A) and Reason (R). While answering these questions, you are required to choose any one of the following four responses.
(*a*) Both Assertion (A) and Reason (R) are True and the Reason (R) is a correct explanation of the Assertion (A).
(*b*) Both Assertion (A) and Reason (R) are True but Reason (R) is not a correct explanation of the Assertion (A).
(*c*) Assertion (A) is True but the Reason (R) is False.
(*d*) Assertion (A) is False but Reason (R) is True.

11. **Assertion (A):** Equal amount of positive and negative charges are distributed uniformly on two halves of a thin circular ring as shown in figure. The resultant electric field at the centre O of the ring is along OC.
    Reason (R): It is so because the net potential at O is not zero.



12. **Assertion (A):** Gauss' theorem is applicable on any closed surface.
    Reason (R): In order to find the value of electric field due to a charge distribution, Gauss' theorem should be applied on a symmetrical closed surface.

COULOMB FORCE

1. When 10^{14} electrons are removed from a neutral metal sphere, the net charge that appears on the sphere is

(*a*) 16 μC (*b*) –16 μC

(*c*) 32 μC (*d*) –32 μC

2. Number of electrons in one coulomb of charge will be ($1e = 1.6 \times 10^{-19}$ C)

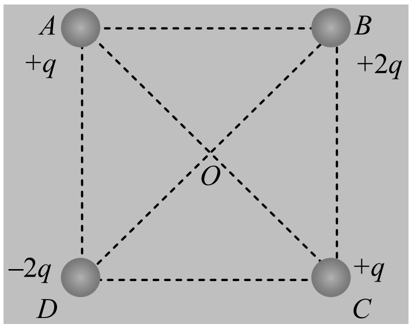
(*a*) 5.46×10^{29} (*b*) 6.25×10^{18}

(*c*) 1.6×10^{19} (*d*) 9×10^{11}

3. The ratio of the forces between two small spheres with constant charge in air to that in a medium of dielectric constant K is

(*a*) $1 : K$ (*b*) $K : 1$ (*c*) $1 : K^2$ (*d*) $K^2 : 1$

4. Four charges are arranged at the corners of a square $ABCD$, as shown in the adjoining figure. The force on a charge kept at the center O is



(*a*) Zero (*b*) Along the diagonal AC

(*c*) Along the diagonal BD (*d*) Perpendicular to side AB

5. A total charge Q is broken in two parts Q_1 and Q_2 and they are placed at a distance R from each other. The maximum force of repulsion between them will occur, when

(*a*) $Q_2 = \frac{Q}{R}, Q_1 = Q - \frac{Q}{R}$ (*b*) $Q_2 = \frac{Q}{4}, Q_1 = Q - \frac{2Q}{3}$

(*c*) $Q_2 = \frac{Q}{4}, Q_1 = \frac{3Q}{4}$ (*d*) $Q_1 = \frac{Q}{2}, Q_2 = \frac{Q}{2}$

6. Two charged spheres separated by a distance R exert a force F on each other. If they are immersed in a liquid of dielectric constant 5, then what is the new force between them?

(*a*) $\frac{F}{5}$ (*b*) F (*c*) $5F$ (*d*) $\frac{F}{2}$

7. A charge q is placed at the center of the line joining two equal charges Q. The system of the three charges will be in equilibrium, if q is equal to

(*a*) $-\frac{Q}{2}$ (*b*) $-\frac{Q}{4}$

(*c*) $+\frac{Q}{4}$ (*d*) $+\frac{Q}{2}$

8. Two charged particles having charge 2×10^{-8} C each are joined by an insulating string of length 1m and the system is kept on a smooth horizontal table, what is the tension in the string?

(*a*) 3.6×10^{-6}N (*c*) 3.4×10^{-6}N

(*b*) 4×10^{-7}N (*d*) 3×10^{-4}N

9. Point charges $+4q$, $-q$ and $+4q$ are kept on the X-axis at point $x = 0$, $x = a$ and $x = 2a$ respectively.

(*a*) Only $-q$ is in stable equilibrium

(*b*) All the charges are in stable equilibrium

(*c*) All of the charges are in unstable equilibrium

(*d*) None of the charges is in equilibrium

10. Charges on two spheres are +10μC and –5μC respectively. They experience a force F. If each of them is given an additional charge +2μC then new force between them, if kept at the same distance, is

(*a*) $18F$ (*b*) $25F$ (*c*) $\frac{18F}{25}$ (*d*) $\frac{25}{18}F$

11. Two charges each of 1μC are at $P\left(2\hat{i}+3\hat{j}+\hat{k}\right)$m and $Q\left(\hat{i}+\hat{j}-\hat{k}\right)$m. Then the force acting on any one of them is

(*a*) 50N (*b*) 10N (*c*) 10^4 dyne (*d*) 100 dyne

12. A charged particle q_1 is at position (2, – 1, 3). The electrostatic force on another charged particle q_2 at (0, 0, 0) is

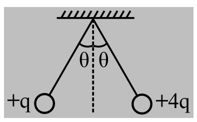
(*a*) $\frac{q_1 q_2}{56\pi \epsilon_0}(2\hat{i} - \hat{j} + 3\hat{k})$

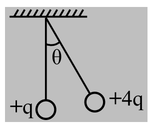
(*b*) $\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{\left(\sqrt{14}\right)^3}(-2\hat{i} + \hat{j} - 3\hat{k})$

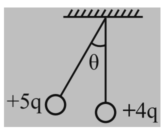
(*c*) $\frac{q_1 q_2}{56\pi \epsilon_0}(\hat{j} - 2\hat{i} - 3\hat{k})$

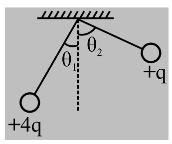
(*d*) $\frac{q_1 q_2}{56\sqrt{14}\,\pi \epsilon_0}(2\hat{i} - \hat{j} + 3\hat{k})$

13. Two point sized metal spheres of same mass are suspended from a common point by two light insulating strings. The length of each string is same. The spheres are given electric charges $+q$ on one of them and $+4q$ on the other. Which of the following diagrams best shows the resulting positions of spheres?

(*a*) 

(*b*) 

(*c*) 

(*d*) 

49. A charged particle having mass m and charge q is released from rest in a uniform electric field E. The kinetic energy of the charged particle moving on a horizontal plane after 't' seconds is

(*a*) $\frac{Eq^2m}{2t^2}$ (*b*) $\frac{2E^2t^2}{3mq}$ (*c*) $\frac{E^2q^2t^2}{2m}$ (*d*) $\frac{Eqm}{t}$

50. A particle having charge q and mass m is projected with velocity $\vec{v} = 2\hat{i} - 3\hat{j}$ in a uniform electric field $\vec{E} = E_0\hat{j}$. What is the magnitude of change in momentum $|\Delta\vec{P}|$ during any time interval t?

(*a*) $\sqrt{13}\,m$ (*b*) qE_0t (*c*) $\frac{qE_0t}{3m}$ (*d*) Zero

51. The bob of a simple pendulum is hanging vertically down from a fixed identical bob by means of a string of length 'l'. If both bobs are charged with a charge 'q' each, time period of the pendulum is (ignore the radii of the bobs).

(*a*) $2\pi\sqrt{\frac{l}{g+\left(\frac{Kq^2}{l^2\,m}\right)}}$ (*b*) $2\pi\sqrt{\frac{l}{g-\left(\frac{Kq^2}{l^2\,m}\right)}}$

(*c*) $2\pi\sqrt{\frac{l}{g}}$ (*d*) $2\pi\sqrt{\frac{l}{g-\left(\frac{Kq^2}{lm^2}\right)}}$

Prabal (JEE Main Level)

1. Two small balls having equal positive charge Q (Coulomb) on each are suspended by two insulating strings of equal length 'L' metre, from a hook fixed to a stand. The whole setup is taken in a satellite into space where there is no gravity (state of weightlessness). Then the angle (θ) between the two strings is

(*a*) 0º (*b*) 90º

(*c*) 180º (*d*) $0º < \theta < 180º$

2. Two charges $4q$ and q are placed 30 cm apart. At what point the value of electric field will be zero?

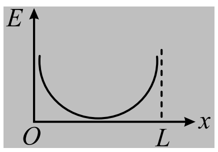
(*a*) 10 cm away from q and between the charges

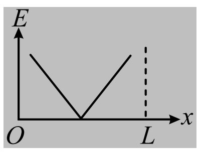
(*b*) 20 cm away from q and between the charges

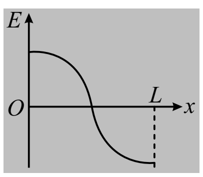
(*c*) 10 cm away from q and outside the line joining the charges

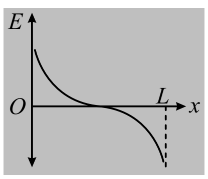
(*d*) 10 cm away from $4q$ and outside the line joining the charges

3. Two identical point charges are placed at a separation of L. P is a point on the line joining the charges, at a distance x from any one charge. The field at P is E. E is plotted against x for values of x from close to zero to slightly less than L. Which of the following best represents the resulting curve?

(*a*)


(*b*)


(*c*)


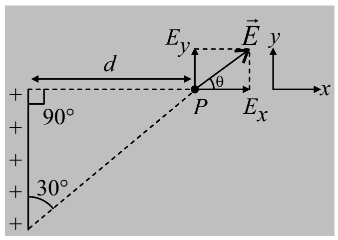
(*d*)


4. The maximum electric field intensity on the axis of a uniformly charged ring of charge q and radius R will be

(*a*) $\frac{1}{4\pi\varepsilon_0}\frac{q}{3\sqrt{3}R^2}$ (*b*) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{3R^2}$

(*c*) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{3\sqrt{3}R^2}$ (*d*) $\frac{1}{4\pi\varepsilon_0}\frac{3q}{2\sqrt{3}R^2}$

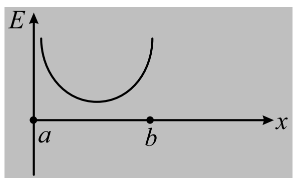
5. The direction (θ) of $\vec{E}$ at point P due to the uniformly charged finite rod will be



(*a*) At 30° from x-axis (*b*) At 45° from x-axis

(*c*) At 60° from x-axis (*d*) None of these

6. Two point charges a and b, whose magnitudes are same, are positioned at a certain distance from each other with a at origin. Graph is drawn between electric field strength E at points between a and b and distance x from a. E is taken positive if it is along the line joining from a to b. From the graph, it can be decided that



(*a*) a is positive, b is negative

(*b*) a and b both are positive

(*c*) a and b both are negative

(*d*) a is negative, b is positive

7. If $\vec{E} = \hat{i} + \sqrt{2}\,\hat{j} + \sqrt{3}\,\hat{k}$ then electric flux through a surface of area 100 m^2 lying in the xy plane is (in V-m)

(*a*) 100 (*b*) 141.4

(*c*) 173.2 (*d*) 200

COULOMB FORCE

1. Force between two point charges q_1 and q_2 placed in vacuum at 'r' cm apart is F. Force between them when placed in a medium having dielectric constant = 5 at 'r/5' cm apart will be: **[31 Jan, 2024 (Shift-II)]**

(*a*) $F/25$ (*b*) $5F$ (*c*) $F/5$ (*d*) $25F$

2. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle θ with each other. When suspended in water the angle remains the same. If density of the material of the sphere is 1.5 g/cc, the dielectric constant of water will be ________ **[30 Jan, 2024 (Shift-I)]**

3. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 37° with each other. When suspended in a liquid of density 0.7 g/cm^3, the angle remains same. If density of material of the sphere is 1.4 g/cm^3, the dielectric constant of the liquid is ________. $\left(\tan 37° = \frac{3}{4}\right)$

[30 Jan, 2024 (Shift-II)]

4. A thin metallic wire having cross sectional area of 10^{-4} m^2 is used to make a ring of radius 30 cm. A positive charge of 2π C is uniformly distributed over the ring, while another positive charge of 30 pC is kept at the centre of the ring. The tension in the ring is ______ N; provided that the ring does not get deformed (neglect the influence of gravity).

(given $\frac{1}{4\pi \in_0}$ = 9×10^9 SI units)

[27 Jan, 2024 (Shift-I)]

5. Suppose a uniformly charged wall provides a uniform electric field of 2×10^4 N/C normally. A charged particle of mass 2 g being suspended through a silk thread of length 20 cm and remain stayed at a distance of 10 cm from the wall. Then the charge on the particle will be $\frac{1}{\sqrt{x}}\mu C$ where x =______. [use g = 10 m/s^2]

[1 Feb, 2024 (Shift-II)]

6. In hydrogen like system the ratio of coulombian force and gravitational force between an electron and a proton is of the order of: **[05 April, 2024 (Shift-I)]**

(*a*) 10^{39} (*b*) 10^{19}

(*c*) 10^{29} (*d*) 10^{36}

7. Two identical conducting spheres P and S with charge Q on each, repel each other with a force 16N. A third identical uncharged conducting sphere R is successively brought in contact with the two spheres. The new force of repulsion between P and S is: **[06 April, 2024 (Shift-II)]**

(*a*) 4 N (*b*) 6 N

(*c*) 1 N (*d*) 12 N

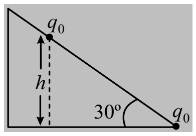
8. If two charges q_1 and q_2 are separated with distance 'd' and placed in a medium of dielectric constant K. What will be the equivalent distance between charges in air for the same electrostatic force? **[24 Jan, 2023 (Shift-I)]**

(*a*) $d\sqrt{k}$ (*b*) $k\sqrt{d}$ (*c*) $1.5d\sqrt{k}$ (*d*) $2d\sqrt{k}$

9. As shown in the figure. a configuration of two equal point charges (q_0= +2μ C) is placed on an inclined plane. Mass of each point charge is 20 g. Assume that there is no friction between charge and plane. For the system of two point charges to be in equilibrium (at rest) the height $h = x \times 10^{-3}$ m The value of x is ______ mm.

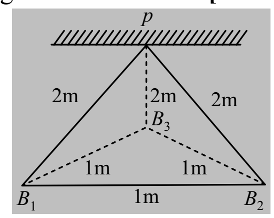
(Take $\frac{1}{4\pi\varepsilon_0} = 9\times10^9\ Nm^2C^{-2},\ g = 10\,\text{ms}^{-1}$)

[11 April, 2023 (Shift-I)]



10. Three identical charged balls each of charge 2C are suspended from a common point P by silk threads of 2 m each (as shown in figure). They form an equilateral triangle of side 1 m.

The ratio of net force on a charged ball to the force between any two charged balls will be **[27 June, 2022 (Shift-II)]**



(*a*) 1 : 1 (*b*) 1 : 4 (*c*) $\sqrt{3}:2$ (*d*) $\sqrt{3}:1$

11. A disk of radius R with uniform positive charge density σ is placed on the xy plane with its center at the origin. The Coulomb potential along the z-axis is

$$V(z) = \frac{\sigma}{2\in_0}\left(\sqrt{R^2+z^2} - z\right)$$

A particle of positive charge q is placed initially at rest at a point on the z axis with $z = z_0$ and $z_0 > 0$. In addition to the Coulomb force, the particle experiences a vertical force $\vec{F} = -c\hat{k}$ with $c > 0$. Let $\beta = \frac{2c\in_0}{q\sigma}$. Which of the following statements is (are) correct? **[JEE Adv, 2022]**

(*a*) For $\beta = \frac{1}{4}$ and $z_0 = \frac{25}{7}R$, the particle reaches the origin

(*b*) For $\beta = \frac{1}{4}$ and $z_0 = \frac{3}{7}R$, the particle reaches the origin

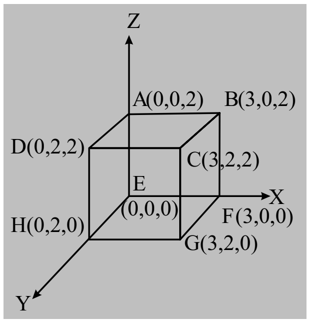
(*c*) For $\beta = \frac{1}{4}$ and $z_0 = \frac{R}{\sqrt{3}}$, the particle returns back to $z = z_0$

(*d*) For $\beta > 1$ and $z_0 > 0$, the particle always reaches the origin

48. A charge q is surrounded by a closed surface consisting of an inverted cone of height h and base radius R, and a hemisphere of radius R as shown in the figure. The electric flux through the conical surface is $\frac{nq}{6\epsilon_0}$ (in SI units). The value of n is _______. **[JEE Adv, 2022]**

49. The electric field in a region is given by $\vec{E} = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right)\frac{N}{c}$. The ratio of flux of reported field through the rectangular surface of area 0.2 m^2 (parallel to $y-z$ plane) to that of the surface of area 0.3 m^2 (parallel to $x-z$ plane) is a:b, where a = __________ (round off to nearest integer) [Here $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axes respectively] **[25 Feb, 2021 (Shift-I)]**

50. An electric field $\vec{E} = 4x\hat{i} - (y^2+1)\hat{j}$ N/C passes through the box shown in figure. The flux of the electric field through surface $ABCD$ and $BCGF$ and marked as ϕ_I and ϕ_{II} respectively. The difference between $(\phi_I - \phi_{II})$ is (in Nm^2/C) **[9 Jan, 2020 (Shift-II)]**



51. A circular disc of radius R carries surface charge density $\sigma(r) = \sigma_0\left(1 - \frac{r}{R}\right)$, where σ_0 is a constant and r is the distance from the center of the disc. Electric flux through a large spherical surface that encloses the charged disc completely is ϕ_0. Electric flux through another spherical surface of radius $\frac{R}{4}$ and concentric with the disc is ϕ. Then the ratio $\frac{\phi_0}{\phi}$ is ..... **[JEE Adv, 2020]**

52. A charged shell of radius R carries a total charge Q. Given ϕ as the flux of electric field through a closed cylindrical surface of height h, radius r and with its center same as that of the shell. Here, center of the cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Which of the following option(s) is/are correct? [ε_0 is the permittivity of free space] **[JEE Adv, 2019]**

(*a*) If $h > 2R$ and $r > R$ then $\phi = \frac{Q}{\varepsilon_0}$

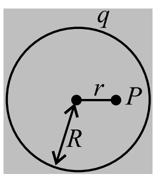
(*b*) If $h < \frac{8R}{5}$ and $r = \frac{3R}{5}$ then $\phi = 0$

(*c*) If $h > 2R$ and $r = \frac{4R}{5}$ then $\phi = \frac{Q}{5\varepsilon_0}$

(*d*) If $h > 2R$ and $r = \frac{3R}{5}$ then $\phi = \frac{Q}{5\varepsilon_0}$

PW Challengers

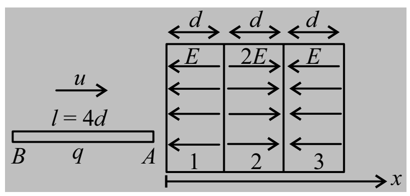
1. Charge q is uniformly distributed over the circumference of a ring of radius R. The electric field at a distance r from the center of ring and in the plane of ring is ($r << R$) [Hint: You may use Gauss's law]



(*a*) $\frac{qr}{4\pi\varepsilon_0 R^3}$ (*b*) $\frac{3qr}{4\pi\varepsilon_0 R^3}$

(*c*) $\frac{qr}{8\pi\varepsilon_0 R^3}$ (*d*) $\frac{3qr}{8\pi\varepsilon_0 R^3}$

2. A thin insulator rod of mass m and length $l = 4d$ carrying a uniform positive charge q, moving with velocity u enters a three section region of electric field. All the three sections are of equal thickness d each. Electric field in section 1 and section 3 has magnitude E and is opposite to the direction of initial velocity while it is in direction of initial velocity and of magnitude $2E$ in the second section. Assume charge distribution of the rod to be uniform. The minimum velocity u with which rod should be projected so that it passes through all the three sections is:



(*a*) $\sqrt{\frac{3qE}{2ml}}d$ (*b*) $\sqrt{\frac{5qE}{2ml}}d$

(*c*) $\sqrt{\frac{7qE}{2ml}}d$ (*d*) $\sqrt{\frac{9qE}{2ml}}d$

ANSWER KEY

CONCEPT APPLICATION

1. (*b*)	**2.** (*d*)	**3.** (*d*)	**4.** (*b*, *c*)	**5.** (*b*)	**6.** (*a*)	**7.** (*d*)	**8.** (*a*)	**9.** (*c*)	**10.** (*c*)
11. (*a*)	**12.** (*d*)	**13.** (*a*, *c*)	**14.** (*b*)	**15.** (*c*)	**16.** (*c*)				

BOARD LEVEL PROBLEMS

1. (*d*)	**2.** (*b*)	**3.** (*c*)	**4.** (*b*)	**5.** (*b*)	**6.** (*c*)	**7.** (*d*)	**8.** (*c*)	**9.** (*b*)	**10.** (*d*)
11. (*c*)	**12.** (*b*)	**24.** (*i*)-(*b*), (*ii*)-(*b*)		**25.** (*i*)-(*a*), (*ii*)-(*a*)					

PRARAMBH (TOPICWISE)

1. (*a*)	**2.** (*b*)	**3.** (*b*)	**4.** (*c*)	**5.** (*d*)	**6.** (*a*)	**7.** (*b*)	**8.** (*a*)	**9.** (*c*)	**10.** (*c*)
11. (*d*)	**12.** (*b*)	**13.** (*a*)	**14.** (*c*)	**15.** (*a*)	**16.** (*c*)	**17.** (*b*)	**18.** (*a*)	**19.** (*b*)	**20.** (*c*)
21. (*a*)	**22.** (*c*)	**23.** (*b*)	**24.** (*b*)	**25.** (*d*)	**26.** (*d*)	**27.** (*c*)	**28.** (*c*)	**29.** (*b*)	**30.** (*c*)
31. (*b*)	**32.** (*b*)	**33.** (*b*)	**34.** (*c*)	**35.** (*c*)	**36.** (*b*)	**37.** (*a*)	**38.** (*c*)	**39.** (*c*)	**40.** (*b*)
41. (*d*)	**42.** (*b*)	**43.** (*b*)	**44.** (*a*)	**45.** (*d*)	**46.** (*a*)	**47.** (*d*)	**48.** (*d*)	**49.** (*c*)	**50.** (*b*)
51. (*c*)									

PRABAL (JEE MAIN LEVEL)

1. (*c*)	**2.** (*a*)	**3.** (*d*)	**4.** (*c*)	**5.** (*a*)	**6.** (*a*)	**7.** (*c*)	**8.** (*d*)	**9.** (*d*)	**10.** (*a*)
11. (*a*)	**12.** (*a*)	**13.** (*b*)	**14.** (*d*)	**15.** (*a*)	**16.** (*d*)	**17.** (*a*)	**18.** (*a*)	**19.** (*b*)	**20.** (*a, b*)
21. (*a*)	**22.** (*d*)	**23.** (*b*)	**24.** (*d*)	**25.** (*a*)	**26.** (*b*)	**27.** (*c*)	**28.** (*a*)	**29.** (*a*)	**30.** [9]
31. [3]	**32.** [72]	**33.** [9]	**34.** [8]	**35.** [2]	**36.** [148]	**37.** [9]	**38.** [208]		

PARIKSHIT (JEE ADVANCED LEVEL)

1. (*b*,*d*)	**2.** (*a*,*d*)	**3.** (*a*,*b*,*c*,*d*)	**4.** (*b*,*c*)	**5.** (*a*,*b*,*c*)	**6.** (*a*, *c*)	**7.** (*a*,*b*,*c*,*d*)	**8.** (*b*,*c*)	**9.** (*a*,*b*,*d*)	**10.** (*b*,*c*)
11. (*a*,*b*,*c*)	**12.** (*a*,*c*)	**13.** (*c*,*d*)	**14.** (*b*,*c*)	**15.** (*a*,*c*)	**16.** (*a*,*c*)	**17.** (*a*,*c*)	**18.** (*c*)	**19.** (*a*)	**20.** (*b*)
21. (*a*)	**22.** (*d*)	**23.** (*b*)	**24.** (*c*)	**25.** (*a*)	**26.** (*b*)	**27.** [0.33]	**28.** [208.33]	**29.** [4.25]	**30.** [0.50]
31. [9.30]	**32.** [2.08]	**33.** [0.65]	**34.** [100]	**35.** [24]	**36.** [5]	**37.** [7]	**38.** [10]	**39.** [4]	**40.** [3]
41. [648]	**42.** (*b*)	**43.** (*c*)	**44.** (*c*)	**45.** (*a*)	**46.** (*c*)	**47.** (*a*)	**48.** (*b*)	**49.** (*b*)	**50.** (*c*)
51. (*c*)	**52.** (*a*)	**53.** (*a*)	**54.** (*a*)	**55.** (*a*)					

PYQ's (PAST YEAR QUESTIONS)

1. (*b*)	**2.** [3]	**3.** [2]	**4.** [3]	**5.** [3]	**6.** (a)	**7.** (*b*)	**8.** (*a*)	**9.** [300]	**10.** (*d*)
11. (*a*,*c*,*d*)	**12.** (*b*)	**13.** (*a*,*c*)	**14.** (*c*)	**15.** (*c*)	**16.** [5]	**17.** (*b*)	**18.** (*c*)	**19.** (*a*)	**20.** (*c*)
21. (*d*)	**22.** [2]	**23.** [16]	**24.** (*c*)	**25.** [6]	**26.** (*d*)	**27.** (*a*)	**28.** (*d*)	**29.** (*b*)	**30.** [8]
31. (*c*)	**32.** [8]	**33.** (*b*)	**34.** [1]	**35.** (*b, c*)	**36.** [6]	**37.** (*a*)	**38.** (*c*)	**39.** (*b*)	**40.** (*b*)
41. (16)	**42.** (*b*)	**43.** (*c*)	**44.** [12]	**45.** (*b*)	**46.** (16)	**47.** [640]	**48.** [3]	**49.** [1]	**50.** [–48]
51. [6.40]	**52.** (*a, b, d*)								

PW CHALLENGERS

1. (*c*)	**2.** (*a*)	**3.** (*d*)	**4.** (*a*)	**5.** (*b*)	**6.** (*a*,*b*)	**7.** (*a*)	**8.** (*c*)	**9.** (*b*)	**10.** $\sqrt{\frac{\eta^3}{ab}}t_0$

PHYSICS WALLAH

LAKSHYA

JEE

CLASS-XII

- Matrices and Determinants
- Relations and Functions
- Inverse Trigonometric Functions

MATHEMATICS

Module 1

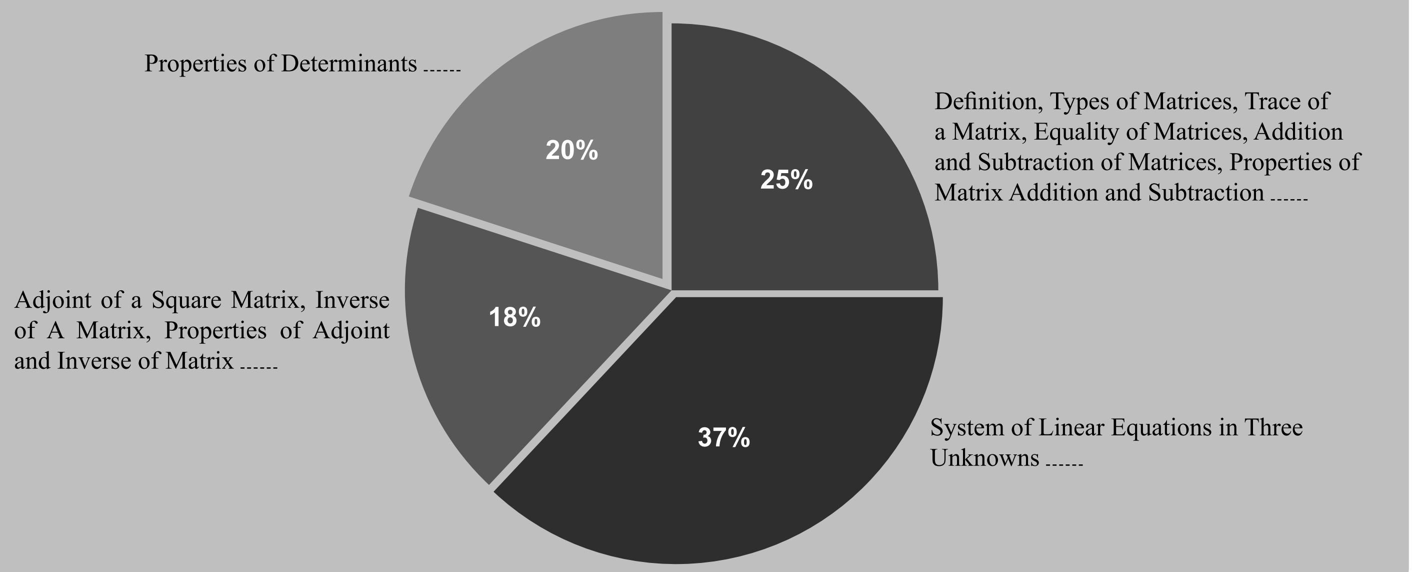
CHAPTER 1

Matrices and Determinants

$M = \begin{bmatrix} m & n \\ o & p \end{bmatrix}$

$|M| = mp - no$

Topicwise Weightage of JEE Main 6 Years Paper (124 Sets)



"How's the Josh?" for these Topics: *Mark your confidence level in the blank space around the topic (Low-L, Medium-M, High-H)*

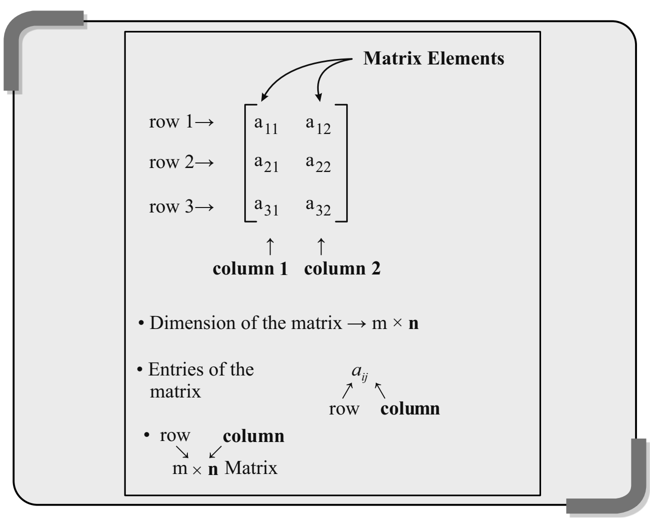
INTRODUCTION

A matrix is a definite collection of quantities like numbers, symbols, or expressions, arranged in a tabular form of rows and columns. Basically it is ordered arrangement of data.

The order of a matrix is written as the number of rows by the number of columns.

For e.g. a 2 × 2 matrix consists of 2 rows and 2 columns. It has a total of 4 elements.

If a matrix has m rows and n columns then the **order** of matrix is written as $m \times n$ and we call it as order m by n



The general $m \times n$ matrix is

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots\ldots & a_{1j} & \ldots\ldots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \ldots\ldots & a_{2j} & \ldots\ldots & a_{2n} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{i1} & a_{i2} & a_{i3} & \ldots\ldots & a_{ij} & \ldots\ldots & a_{in} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{m1} & a_{m2} & a_{m3} & \ldots\ldots & a_{mj} & \ldots\ldots & a_{mn} \end{bmatrix}$$

where a_{ij} denote the element of i^{th} row and j^{th} column. The above matrix is usually denoted as $[a_{ij}]_{m\times n}$.

Now any matrix of order $m \times n$ will have the notation $[a_{ij}]_{m\times n}$.

i.e., $A = [a_{ij}]_{m\times n}$ or $(a_{ij})_{m\times n}$ or $\|a_{ij}\|_{m\times n}$

it is trivial that $1 \le i \le m$ and $1 \le j \le n$

e.g., $A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}, B = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix},$

$$C = \begin{bmatrix} -1 & 3 & c \\ 2 & 9 & 4 \end{bmatrix}, D = [1, 4, 9]$$

PTR (Points to Remember):

1. Generally capital letters of English alphabets are used to denote matrices.
2. The matrix is not a number. It has no numerical value.

Train Your Brain

Example 1: Consider a 3 × 4 matrix $A = [a_{ij}]$, whose elements are given by $a_{ij} = 2i + 3j$, then A =

(a) $\begin{bmatrix} 5 & 8 & 11 & 14 \\ 7 & 10 & 13 & 16 \\ 9 & 12 & 15 & 18 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 8 & 11 & 14 \\ 14 & 10 & 13 & 16 \\ 9 & 12 & 15 & 18 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 8 & 11 & 14 \\ 14 & 10 & 8 & 16 \\ 9 & 12 & 15 & 18 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 8 & 11 & 14 \\ 14 & 10 & 8 & 5 \\ 9 & 12 & 15 & 18 \end{bmatrix}$

Sol. In this problem, i and j are the number of rows and columns respectively. By substituting the respective values of rows and columns in $a_{ij} = 2i + 3j$ we can construct the required matrix.

We have $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$;

$a_{11} = 2 \times 1 + 3 \times 1 = 5$; $a_{12} = 2 \times 1 + 3 \times 2 = 8$

Similarly, $a_{13} = 11$, $a_{14} = 14$, $a_{21} = 7$, $a_{22} = 10$, $a_{23} = 13$, $a_{24} = 16$, $a_{31} = 9$, $a_{32} = 12$, $a_{33} = 15$, $a_{34} = 18$

$$\therefore\ A = \begin{bmatrix} 5 & 8 & 11 & 14 \\ 7 & 10 & 13 & 16 \\ 9 & 12 & 15 & 18 \end{bmatrix}$$

Example 2: Construct a 3 × 4 matrix, whose elements are given by: $a_{ij} = \frac{1}{2}|-3i + j|$

Sol. $A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

Concept Application

1. The order of $\begin{bmatrix} 3 & -1 & 5 \\ 6 & 2 & -7 \end{bmatrix}$ matrix is ______ .

2. The number of different possible orders of matrices having 18 identical elements are ______ .

TYPES OF MATRIX

Row Matrix

A matrix having only one row is called as row matrix (or row vector). General form of row matrix is

$A = [a_{11}\ a_{12}\ a_{13} \ldots a_{1n}]$

This is a matrix of order "$1 \times n$" (or a row matrix of order n).

Column Matrix

A matrix having only one column is called *a* column matrix (or column vector).

Column matrix is of the form $A = \begin{bmatrix} a_{11} \\ a_{21} \\ \ldots \\ a_{m1} \end{bmatrix}$.

This is a matrix of order "$m \times 1$" (or a column matrix of order m).

Zero or Null Matrix

$A = [a_{ij}]_{m \times n}$ is called a zero matrix, if $a_{ij} = 0\ \forall\ i$ and j.

e.g., (i) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Singleton Matrix

If in a matrix there is only one element then it is called singleton matrix. Thus, $A = [a_{ij}]_{m \times n}$ is a singleton matrix if $m = n = 1$. E.g. [2], [3], [a], [–3] are singleton matrices.

Horizontal Matrix

A matrix of order $m \times n$ is a horizontal matrix if $n > m$; E.g. $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 1 & 1 \end{bmatrix}$

Vertical Matrix

A matrix of order $m \times n$ is a vertical matrix if $m > n$; E.g. $\begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 3 & 6 \\ 2 & 4 \end{bmatrix}$

Square Matrix

A matrix in which number of rows and columns are equal is called a square matrix. The general form of a square matrix is

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots\ldots & a_{1n} \\ a_{21} & a_{22} & \ldots\ldots & a_{2n} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{n1} & a_{n2} & \ldots\ldots & a_{nn} \end{bmatrix}$$

which we denote as $A = [a_{ij}]_{n \times n}$

This is a matrix of order "$n \times n$" (or a square matrix of order n).

Diagonal Elements: An element of a matrix $A = [a_{ij}]$ is said to be diagonal element if $i = j$. Thus an element whose row suffix equals to the column suffix is a diagonal element e.g., $a_{11}, a_{22}, a_{33}, \ldots$ are all diagonal element.

3. $k(A + B) = kA + kB$ here k is any scalar.

4. $A + O = O + A = A$, here O (null matrix) will be additive identity.

5. If A be a given matrix then the matrix $-A$ is the additive inverse of A i.e. $A + (-A) = O$ (null matrix).

6. If A, B and C be three matrices of the same type

then $A + B = A + C \Rightarrow B = C$ (Left Cancellation Law)

and $B + A = C + A \Rightarrow B = C$ (Right Cancellation Law)

Train Your Brain

Example 6: If $X = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ and $3X - \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

then a is equal to

(*a*) 1 (*b*) 2

(*c*) 0 (*d*) –2

Sol. $3X = \begin{bmatrix} 3 & 3a \\ 0 & 3 \end{bmatrix}$

$\Rightarrow$ L.H.S. $= \begin{bmatrix} 3-2 & 3a-3 \\ 0-0 & 3-2 \end{bmatrix} = \begin{bmatrix} 1 & 3a-3 \\ 0 & 1 \end{bmatrix}$

Now by equality of two matrices, we have $3a - 3 = 3$ $\Rightarrow a = 2$.

Example 7: Find the value of x and y if

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Sol. Using the method of multiplication and addition of matrices, then equating the corresponding elements of L.H.S. and R.H.S, we can easily get the required values of x and y,

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+y & 6+0 \\ 0+1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Equating the corresponding elements, a_{11} and a_{22}, we get

$2 + y = 5 \Rightarrow y = 3$; $2x + 2 = 8$

Hence $x = 3$ and $y = 3$.

Concept Application

8. If $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$, then the sum of the elements of the matrix $3X - 4Y$ is equal to

9. If $3A - 2B + X = 0$, where

$A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$; $B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$, than find x.

(*a*) $\begin{bmatrix} -16 & -4 \\ 1 & 3 \end{bmatrix}$ (*b*) $\begin{bmatrix} -16 & -4 \\ 3 & -5 \end{bmatrix}$

(*c*) $\begin{bmatrix} -4 & -4 \\ 3 & -5 \end{bmatrix}$ (*d*) $\begin{bmatrix} -4 & -4 \\ 3 & -8 \end{bmatrix}$

Multiplication of Matrices

Let A and B be two matrices such that the number of columns of A is same as number of rows of B i.e.,

$A = [a_{ij}]_{m\times p}$ & $B = [b_{ij}]_{p\times n}$,

Then $AB = [c_{ij}]_{m\times n}$ where $c_{ij} = \sum_{k=1}^{p} a_{ik}b_{kj}$, which is the dot product of i^{th} row vector of A and j^{th} column vector of B.

Notes: The product AB is defined iff the number of columns of A is equal to the number of rows of B. A is called as premultiplier and B is called as post multiplier. AB is defined $\not\Rightarrow$ BA is defined.

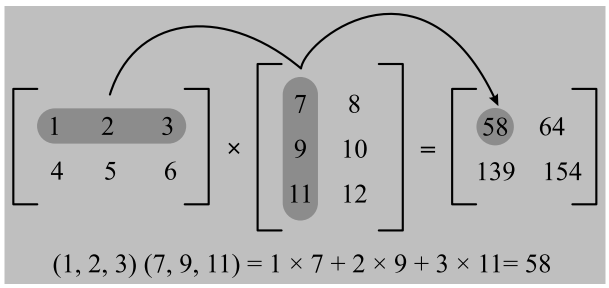


Fig. 1

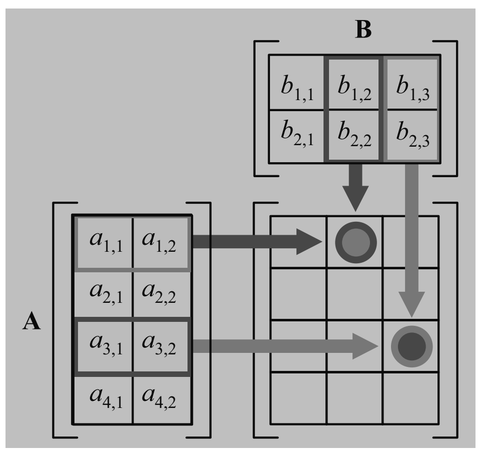


Fig. 2

Aarambh (Solved Examples)

1. If $A = [a_{ij}]_{3\times 3}$, such that $a_{ij} = \begin{cases} 2, \text{when } i = j \\ 0, \text{when } i \neq j \end{cases}$, then $\log_{1/2}\det(\text{adj}\ (\text{adj}\ (A)) =$

(*a*) –12 (*b*) –10
(*c*) –13 (*d*) –11

Sol. $A = [a_{ij}]$

$a_{ij} = 2, i = j = 0\ i \neq j$

$|A| = 8$

Now, $\det(\text{adj}\ (\text{adj}\ (A)) = (8)^4 = 2^{12}$

$\log_{1/2}(2^{12}) = -12$

Therefore, option (*a*) is the correct answer.

2. Find $c^2 + x^2 + y^2$ if the matrix A given by

$$A = \begin{bmatrix} a & 2/3 & 2/3 \\ 2/3 & 1/3 & b \\ c & x & y \end{bmatrix} \text{- is orthogonal.}$$

(*a*) 1 (*b*) 2 (*c*) 0 (*d*) 3

Sol. It is given that the matrix A is orthogonal. Therefore, $AA' = I$

$$\begin{bmatrix} a & 2/3 & 2/3 \\ 2/3 & 1/3 & b \\ c & x & y \end{bmatrix}\begin{bmatrix} a & 2/3 & c \\ 2/3 & 1/3 & x \\ 2/3 & b & y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Comparing the element in the 3rd column of 3rd row, we get $c^2 + x^2 + y^2 = 1$.

Therefore, option (*a*) is the correct answer.

3. Let t be the trace of the matrix

$$A = \begin{bmatrix} \frac{|x+y|}{|x|+|y|} & \alpha_1 & \beta_1 \\ \alpha_2 & \frac{|y+z|}{|y|+|z|} & \beta_2 \\ \alpha_3 & \beta_3 & \frac{|z+x|}{|z|+|x|} \end{bmatrix}, \text{ then}$$

(*a*) $0 \le t \le 3$ (*b*) $1 \le t \le 2$
(*c*) $1 \le t \le 3$ (*d*) $-1 \le t \le 1$

Sol. Now, $|x + y| \le |x| + |y|$

$$\Rightarrow \frac{|x+y|}{|x|+|y|} \le 1$$

Hence, $t \le 3$

Also among the diagonal elements, at least one of the element must be 1.

Therefore, option (*c*) is the correct answer.

4. If $P = \begin{bmatrix} \cos(\pi/6) & \sin(\pi/6) \\ -\sin(\pi/6) & \cos(\pi/6) \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP'$ then $P'Q^{2009}P$ is equal to

(*a*) $\begin{bmatrix} 1 & \sqrt{3}/2 \\ 0 & 2009 \end{bmatrix}$ (*b*) $\begin{bmatrix} 1 & 2009 \\ 0 & 1 \end{bmatrix}$

(*c*) $\begin{bmatrix} \sqrt{3}/2 & 2009 \\ 0 & 1 \end{bmatrix}$ (*d*) $\begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1 & 2009 \end{bmatrix}$

Sol. Now, $P'P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

$$\Rightarrow P'P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \Rightarrow P'P = I \Rightarrow P' = P^{-1}$$

Since $Q = PAP'$

$\therefore P'Q^{2009}P$

$= P'[(PAP')\ (PAP')\ \ldots\ 2009 \text{ times}]P$

$$= \underbrace{(P'P)A(P'P)A(P'P)A\ldots(P'P)A(P'P)}_{2009 \text{ times}}$$

$= IA^{2009} = A^{2009}$

$$\therefore A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \Rightarrow = A^{2009} = (I + B)^{2009}$$

where $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. As $B^2 = O$, we get $B^r = O$ $\forall\ r \ge 2$.

Thus, by binomial theorem

$$A^{2009} = \begin{bmatrix} 1 & 2009 \\ 0 & 1 \end{bmatrix}$$

Therefore, option (*b*) is the correct answer.

5. If T_p, T_q, T_r are the p^{th}, q^{th} and r^{th} terms of an A.P., then $\begin{vmatrix} T_p & T_q & T_r \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ equals

(*a*) 1 (*b*) –1
(*c*) 0 (*d*) $p + q + r$

Board Level Problems

SINGLE CORRECT TYPE QUESTIONS

1. If a matrix has 36 elements, the number of possible orders it can have, is:

(*a*) 13 (*b*) 3

(*c*) 5 (*d*) 9

2. If $\begin{bmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{bmatrix} = kabc$, then the value of k is:

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 4

3. If for the matrix A = $\begin{bmatrix} \tan x & 1 \\ -1 & \tan x \end{bmatrix}$, $A + A' = 2\sqrt{3}I$, then the value of $x \in \left[0, \frac{\pi}{2}\right]$ is:

(*a*) 0 (*b*) $\frac{\pi}{4}$

(*c*) $\frac{\pi}{3}$ (*d*) $\frac{\pi}{6}$

4. If A = [a_{ij}] is an identity matrix, then which of the following is true?

(*a*) $a_{ij} = \begin{cases} 0, & \text{if } i = j \\ 1, & \text{if } i \neq j \end{cases}$ (*b*) $a_{ij} = 1, \forall i, j$

(*c*) $a_{ij} = 0, \forall i, j$ (*d*) $a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$

5. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix such that adj $A = A$. Then, $(a + b + c + d)$ is equal to:

(*a*) $2a$ (*b*) $2b$

(*c*) $2c$ (*d*) 0

6. If $\begin{vmatrix} 1 & 3 & 1 \\ k & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \pm 6$, then the value of k is:

(*a*) 2 (*b*) –2

(*c*) ±2 (*d*) ∓2

7. If $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix, then the value of $a + 2b + 3c + 4d$ is:

(*a*) 0 (*b*) 5

(*c*) 10 (*d*) 25

8. The equations $x + 2y + 3z = 1$, $x - y + 4z = 0$, $2x + y + 7z = 1$ have

(*a*) Only one solution (*b*) Two solutions

(*c*) No solution (*d*) Infinitely many solutions

9. If the system of equations $x + ay = 0$, $az + y = 0$ and $ax + z = 0$ has infinite solutions, then the value of a, is

(*a*) –1 (*b*) 1

(*c*) 0 (*d*) No real values

VERY SHORT ANSWER TYPE QUESTIONS

10. If a matrix has 28 elements, what are the possible orders it can have? What if it has 2027 elements?

11. If $A = \begin{bmatrix} 1 & \cot x \\ -\cot x & 1 \end{bmatrix}$, show that

$$A'A^{-1} = \begin{bmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{bmatrix}$$

12. Find values of x, if $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

13. Find |AB|, if $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$.

14. Verify that $A^2 = I$ when $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$.

15. If $A = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$, find a matrix C such that $3A + 5B + 2C$ is a null matrix.

16. Evaluate $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$

LONG ANSWER TYPE QUESTIONS

17. If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations:

$x - 2y = 10$, $2x - y - z = 8$, $-2y + z = 7$

18. Solve the following system of equations, using matrices:

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

where $x, y, z \neq 0$

19. Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.

20. If $A = \begin{bmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}$, find A^{-1} and hence solve the following system of equaitons:

$2x + y - 3z = 13$
$3x + 2y + z = 4$
$x + 2y - z = 8$

21. Consider the matrix $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and find k such that $A^2 = kA - 7I$.

CASE STUDY BASED QUESTIONS

CASE STUDY-I

If A and B are square matrix of order 3 given by

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 4 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

On the basis of above information, answer the following questions:

22. |adj (adj A)| is equal to

(*a*) 10^2 (*b*) 100^3
(*c*) 10^4 (*d*) None of these

23. |adj (AB) | is equal to

(*a*) 100
(*b*) 1000
(*c*) 10^4
(*d*) None of these

CASE STUDY-II

Let A and B are two matrices of same order i.e. 3×3 where

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & k & 5 \\ 4 & 2 & 1 \end{bmatrix} B = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 4 \\ 3 & 3 & 5 \end{bmatrix}$$

On the basis of above information, answer the following questions:

24. If $k = 2$ then tr(AB) + tr(BA) is equal to

(*a*) 66 (*b*) 42
(*c*) 84 (*d*) 63

25. If $C = A + 3B$ and tr(C) = 0, then k is equal to

(*a*) –10 (*b*) –20
(*c*) –29 (*d*) –39

Prarambh (Topicwise)

TYPES OF MATRICES, ADDITION, SUBTRACTION AND EQUALITY OF MATRICES

1. The number of different possible orders of matrices having 12 identical elements is

(*a*) 3 (*b*) 1 (*c*) 6 (*d*) 2

2. In an upper triangular matrix $A = [a_{ij}]_{n \times n}$ the elements $a_{ij} = 0$ for

(*a*) $i < j$ (*b*) $i = j$ (*c*) $i > j$ (*d*) $i \leq j$

3. $\begin{bmatrix} x^2+x & x \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ -x+1 & x \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 5 & 1 \end{bmatrix}$

then x is equal to

(*a*) – 1 (*b*) 2
(*c*) 1 (*d*) No value of x

4. Which of the following is a diagonal matrix

(*a*) $\begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ (*b*) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(*c*) $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (*d*) None of these

TRACE OF MATRIX

5. If trace of matrix A= $\begin{bmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ -5 & 1 & x \end{bmatrix}$ is 5 then x is

(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4

MULTIPLICATION OF MATRICES AND IDENTITY MATRIX

6. If $\begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix} X = \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$, then $X =$

(*a*) $\begin{bmatrix} -3 & 4 \\ 14 & -13 \end{bmatrix}$ (*b*) $\begin{bmatrix} 3 & -4 \\ -14 & 13 \end{bmatrix}$

(*c*) $\begin{bmatrix} 3 & 4 \\ 14 & 13 \end{bmatrix}$ (*d*) $\begin{bmatrix} -3 & 4 \\ -14 & 13 \end{bmatrix}$

7. If $M = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $M^2 - \lambda M - I_2 = O$, then $\lambda =$

(*a*) –2 (*b*) 2

(*c*) –4 (*d*) 4

8. If $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ and A^2 is the identity matrix, then $x =$

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 0

9. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ and $B = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$, then the correct relation is

(*a*) $A^2 = B^2$

(*b*) A + B = B – A

(*c*) AB = BA

(*d*) AB = 0

10. If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ and I is the identity matrix of order 2, then (A – 2I) (A – 3I) =

(*a*) I (*b*) O

(*c*) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ (*d*) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

11. If A and B are two matrices such that AB = B and BA = A, then $A^2 + B^2 =$

(*a*) 2AB (*b*) 2BA

(*c*) A + B (*d*) AB

12. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$, then

(*a*) $A^3 + 3A^2 + A - 9I_3 = 0$

(*b*) $A^3 - 3A^2 + A + 9I_3 = 0$

(*c*) $A^3 + 3A^2 - A + 9I_3 = 0$

(*d*) $A^3 - 3A^2 - A + 9I_3 = 0$

TRANSPOSE OF MATRICES AND OTHER PROPERTIES

13. If $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 5 \\ 2 & -5 & 0 \end{bmatrix}$, then

(*a*) A′ = A (*b*) A′ = –A

(*c*) A′ = 2A (*d*) A′ = –2A

14. For the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & \lambda & 5 \end{bmatrix}$ to be non-singular, λ should not be equal to

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

15. If $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$, then $(AB)^T =$

(*a*) $\begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$ (*b*) $\begin{bmatrix} -3 & 10 \\ -2 & 7 \end{bmatrix}$

(*c*) $\begin{bmatrix} -3 & 10 \\ 7 & -2 \end{bmatrix}$ (*d*) $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

16. Which of the following is incorrect?

(*a*) $A^2 - B^2 = (A+B)(A-B)$

(*b*) $(A^T)^T = A$

(*c*) $(AB)^n = A^n B^n$, where *A*, *B* commute

(*d*) $(A-I)(I+A) = O \Leftrightarrow A^2 = I$

SYMMETRIC & SKEW-SYMMETRIC MATRIX

17. If *A* is a skew- symmetric matrix of odd order, then trace of *A* is

(*a*) 1 (*b*) –1 (*c*) |*A*| (*d*) 2

18. For what value of *x*, is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetric matrix?

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

19. Out of the following a skew-symmetric matrix is

(*a*) $\begin{bmatrix} 0 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & 0 \end{bmatrix}$ (*b*) $\begin{bmatrix} 1 & 4 & 5 \\ -4 & 1 & -6 \\ -5 & 6 & 1 \end{bmatrix}$

(*c*) $\begin{bmatrix} 1 & 4 & 5 \\ -4 & 2 & -6 \\ -5 & 6 & 3 \end{bmatrix}$ (*d*) $\begin{bmatrix} i+1 & 4 & 5 \\ -4 & i & -6 \\ -5 & 6 & i \end{bmatrix}$

62. If $\Delta(x)=\begin{vmatrix}\sin^2 x & \log\cos x & \log\tan x\\ n^2 & 2n-1 & 2n+1\\ 1 & -2\log 2 & 0\end{vmatrix}$ then evaluate $\int_0^{\pi/2}\Delta(x)dx$

(*a*) –1 (*b*) 0 (*c*) 1 (*d*) 2

63. Let $\lim_{x\to\infty}\frac{f(x)}{x^2}$ then find

$f(x)=\begin{vmatrix}x(x+1) & x^2-1 & x^2+5x+6\\ 1 & 2 & 3\\ 1 & -1 & 1\end{vmatrix}$

(*a*) –1 (*b*) 0 (*c*) 1 (*d*) 4

64. The determinant $\begin{vmatrix}\cos(\theta+\phi) & -\sin(\theta+\phi) & \cos 2\phi\\ \sin\theta & \cos\theta & \sin\phi\\ -\cos\theta & \sin\theta & \cos\phi\end{vmatrix}$ is

(*a*) 0

(*b*) independent of θ

(*c*) independent of ϕ

(*d*) independent of θ and ϕ both

65. If $l_i^2+m_i^2+n_i^2=1$ and $l_il_j+m_im_j+n_in_j=0\ \forall\ i,j\in\{1,2,3\}$, $i\neq j$ and $\Delta=\begin{vmatrix}l_1 & m_1 & n_1\\ l_2 & m_2 & n_2\\ l_3 & m_3 & n_3\end{vmatrix}$ then

(*a*) $|\Delta|=3$ (*b*) $|\Delta|=2$

(*c*) $|\Delta|=1$ (*d*) $\Delta=0$

66. If $f_r(x)$, $g_r(x)$, $h_r(x)$, $r=1,2,3$ are polynomials in x such that $f_r(a)=g_r(a)=h_r(a)$, $r=1,2,3$ and $F(x)=\begin{vmatrix}f_1(x) & f_2(x) & f_3(x)\\ g_1(x) & g_2(x) & g_3(x)\\ h_1(x) & h_2(x) & h_3(x)\end{vmatrix}$ then value of $F'(x)$ at $x=a$ is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 0

CRAMER'S RULE: SYSTEM OF LINEAR EQUATIONS

67. $x+ky-z=0$, $3x-ky-z=0$ and $x-3y+z=0$ has non-zero solution for $k=$

(*a*) –1 (*b*) 0 (*c*) 1 (*d*) 2

68. The number of solutions of equations $x+4y-z=0$, $3x-4y-z-0$, $x-3y+z=0$ is

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) Infinite

69. If the system of equations, $x+2y-3z=1$, $(k+3)z=3$, $(2k+1)x+z=0$ is inconsistent, then the value of k is

(*a*) –3 (*b*) 1/2

(*c*) 0 (*d*) 2

70. If the system of equation $3x-2y+z=0$, $\lambda x-14y+15z=0$, $x+2y+3z=0$ have a non-trivial solution, then $\lambda=$

(*a*) 5 (*b*) –5

(*c*) –29 (*d*) 29

71. The existence of the unique solution of the system $x+y+z=\lambda$, $5x-y+\mu z=10$, $2x+3y-z=6$ depends on

(*a*) μ only (*b*) λ only

(*c*) λ and μ both (*d*) Neither λ nor μ

Prabal (JEE Main Level)

1. If α and β are the roots of the equation

$$[1\ \ 5]\begin{bmatrix}1 & 3\\ -4 & 7\end{bmatrix}^2\begin{bmatrix}\frac{7}{19} & -\frac{3}{19}\\ \frac{4}{19} & \frac{1}{19}\end{bmatrix}^4\begin{bmatrix}1 & 3\\ -4 & 7\end{bmatrix}^2\begin{bmatrix}x^2-5x+5\\ -3\end{bmatrix}=[-4]$$

then the value of $(2-\alpha)(2-\beta)$ is

(*a*) 51 (*b*) –12

(*c*) 12 (*d*) –7

2. Let $S=\left\{\begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix}: a_{ij}\in\{-1,0,1\}\right\}$

then the number of symmetric matrices with trace equals zero, is

(*a*) 729 (*b*) 189

(*c*) 162 (*d*) 27

3. If $A_1, A_3, \ldots\ldots\ldots A_{2n-1}$ are n skew-symmetric matrices of same order, then $B=\sum_{r=1}^{n}(2r-1)(A_{2r-1})^{2r-1}$ will be

(*a*) Symmetric

(*b*) Skew-symmetric

(*c*) Neither sysmmetric nor skew-symmetric

(*d*) Data is insufficient

4. A skew-symmetric matrix A satisfies the relation $A^2+I=O$, where I is a unit matrix. Then A is

(*a*) Idempotent matrix (*b*) Orthogonal matrix

(*c*) Nilpotent matrix (*d*) Non-periodic matrix

PHYSICS WALLAH

LAKSHYA JEE

CLASS-XII

- Solutions
- Electrochemistry
- Chemical Kinetics

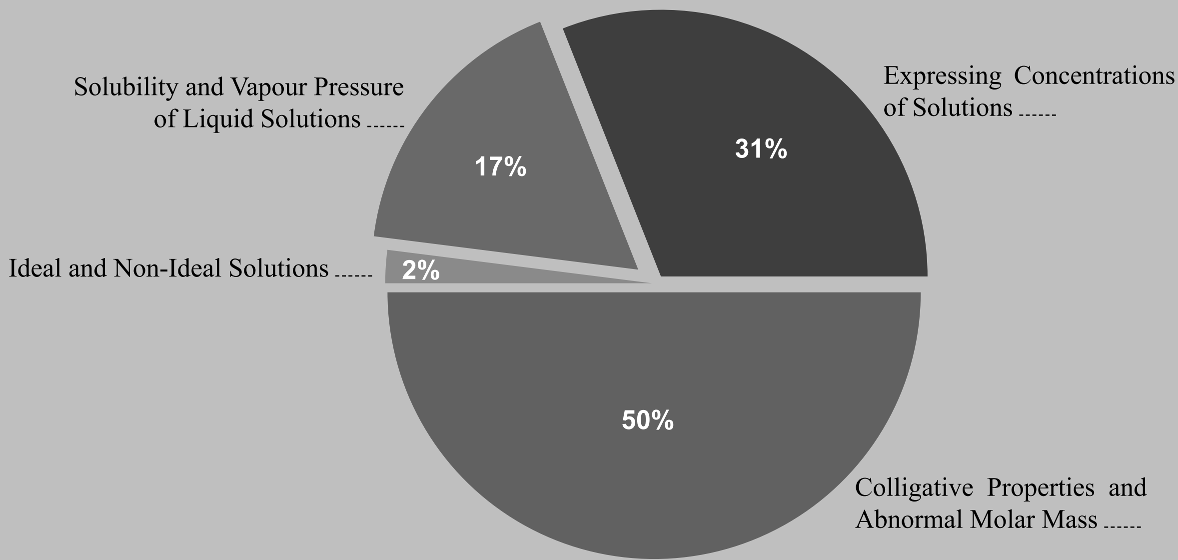
CHEMISTRY

Module 1

CHAPTER 1

Solutions

Topicwise Weightage of JEE Main 6 Years Paper (124 Sets)



"How's the Josh?" for these Topics: *Mark your confidence level in the blank space around the topic (Low-L, Medium-M, High-H)*

INTRODUCTION

A solution may be defined as a homogeneous mixture of a single phase containing two or more of the chemical species dispersed on a molecular scale. The characteristics of any one section of the homogeneous solution will be completely identical to those of any other section of the solution. Depending upon the number of total constituents present in the solution, it is called binary solution (two constituents), ternary solution (three constituents), quaternary solution (four constituents) and so on.

(*i*) **Solvent:** The liquid or medium of dissolution which allows the solute to dissolve in it so as to form a solution is called a solvent.

(*ii*) **Solute:** The substance which dissolves or disappears in the solvent to form a solution is called solute.

Solute + Solvent = Solution.

(*iii*) The component which has the same physical state in pure form as the solution is called solvent and the other is called solute. Example, in case of solution of sugar and water, sugar is the solute and water is solvent.

(*iv*) If both the components have same state as the solution, the one component which is in excess is called solvent and the other is called solute. Example, alcohol in water, benzene in toluene etc.

Types of Solution

S.No.	Solvent	Solute	Examples
1.	Gas	Gas	Mixture of gases, air.
2.	Gas	Liquid	Water vapour in air, mist.
3.	Gas	Solid	Sublimation of a solid into a gas, smoke storms.
4.	Liquid	Gas	CO_2 gas dissolve in water (aerated drink), soda water.
5.	Liquid	Liquid	Mixture of miscible liquids e.g. alcohol in water.
6.	Liquid	Solid	Salt in water, sugar in water.
7.	Solid	Gas	Adsorption of gases over metals, hydrogen over palladium.
8.	Solid	Liquid	Mercury in zinc, mercury in gold i.e. all amalgams.
9.	Solid	Solid	Homogeneous mixture of two or more metals (i.e. alloys) e.g. copper In gold. zinc In copper.

2. **Molality and mole fraction:** Consider a binary solution consisting of two components A (Solute) and B (Solvent). Let X_A & X_B are the mole fraction of A & B respectively.

$$X_A = \frac{n_A}{n_A + n_B}, \quad X_B = \frac{n_B}{n_A + n_B}$$

If molality of solution be m then:

$$m = \frac{n_A}{\text{mass of solvent}} \times 1000 = \frac{n_A}{n_B \times M_B} \times 1000$$

where M_B is the molecular wt. of the solvent B.

$$m = \frac{X_A}{X_B} \times \frac{1000}{M_B} \Rightarrow m = \frac{\text{mole fraction of A}}{\text{mole fraction of B}} \times \frac{1000}{M_B}$$

$$\boxed{m = \frac{\text{mole fraction of solute}}{\text{mole fraction of solvent}} \times \frac{1000}{\text{molecular wt. of solvent}}}$$

3. **Mole fraction of solute into molarity of solution $M = \frac{X_2 d \times 1000}{X_1M_1 + M_2X_2}$**

Mole fraction of solvent and solute are X_1 and X_2 so $X_1 + X_2 = 1$
Suppose total mole of solution is = 1 then mole of solute and solute and solvent are X_2 & X_1 respectively
weight of solute = X_2M_2, weight of solvent = X_1M_1
& total wt. of solution = $X_1M_1 + X_2M_2$

$$\text{volume of solution} = \frac{X_1M_1 + X_2M_2}{d}\text{ ml} = \frac{X_1M_1 + X_2M_2}{d \times 1000}\text{ L}$$

$$\text{molarity (M)} = \frac{X_2 \times d \times 1000}{X_1M_1 + X_2M_2}$$

4. **Molarity into mole fraction X_2 = 1000M/ [1000d– MM_2]**

Molarity = M moles solute in 1000 ml of solution
So, moles of solute = M & mass of solution = d × 1000
wt. of solute = MM_2 & wt. of solvent = (1000d – MM_2)
Where M_2 is molar mass of solute
mole fraction of solute = 1000M / [1000d – MM_2]

5. **Molality into mole fraction $X_2 = \frac{mM_1}{1000 + mM_1}$**

Molality = moles of solute in 1000 gm of solvent = m

moles of solvent = $\frac{1000}{M_1}$ where M_1 is molar mass of solvent

$$\text{mole fraction } X_2 = \frac{m}{\frac{1000}{M_1} + m} = \frac{mM_1}{1000 + mM_1}$$

6. **Molality into molarity $M = \frac{md \times 1000}{1000 + mM_2}$**

Molality = m moles of solute in 1000 gm of solvent
mole of solute = m & weight of solute = mM_2
Weight of solution = 1000 + mM_2

$$\text{volume of solution} = \frac{1000 + mM_2}{d}\text{ mL} = \frac{1000 + mM_2}{d \times 1000}\text{ L}$$

$$\text{molarity} = \frac{m \times d \times 1000}{1000 + mM_2}$$

7. **Molarity into Molality $m = \frac{M \times 1000}{1000d - MM_2}$**

M_1 and M_2 are molar masses of solvent and solute.
Molarity = M mole of solute in 1000 ml of solution
moles of solute = M & weight of solute = MM_2
weight of solution = 1000d
mass of solvent = 1000d – MM_2

$$\text{molality} = \frac{M \times 1000}{1000d - MM_2}$$

on simplifying $\boxed{d = M\left[\frac{1}{m} + \frac{M_2}{1000}\right]}$

Dilution & Mixing of two Liquids

- Upon dilution no. of moles of solute remains constant. If a particular solution having volume V_1 mL and molarity M_1 is diluted upto volume V_2 mL.

  $M_1V_1 = M_2V_2$

  M_2 : final molarity
- If a solution having volume V_1 and molarity M_1 is mixed with another solution of same solute having volume V_2 & molarity M_2 then $M_1V_1 + M_2V_2 = MR\,(V_1 + V_2)$

  $$MR = \text{Resultant molarity} = \frac{M_1V_1 + M_2V_2}{V_1 + V_2}$$

Train Your Brain

Example 1: If we have 6% w/w urea solution with density 1.060 g/mL, then calculate its strength in g/L.

Sol. 6 g urea is present in 100 gm solution.

$$6\,g \text{ in } \frac{100}{1.060}\text{ mL}\left(\text{density} = \frac{\text{mass}}{\text{volume}}\right)$$

$$\frac{100}{1.060}\text{ mL} \rightarrow 6\text{ gm}$$

$$\therefore\ 1000\,\text{mL} = \frac{6}{100} \times 1.060 \times 1000$$

$$= 10.6 \times 6 = 63.6\,\text{g/L}$$

Example 2:

(i) Which type of solution is milk?
(ii) Name a solid - solid type solution.
(iii) Which type of solution is smoke?

Identify the correct matching			
	(i)	(ii)	(iii)
(*a*)	Liquid in solid	Atmosphere	Gas in liquid
(*b*)	Liquid in gas	Alloys	Liquid in solid
(*c*)	Liquid in liquid	Rubber	Solid in gas
(*d*)	Gas in liquid	Rubber	Liquid in liquid

Sol. (*c*) Liquid in liquid – Rubber – Solid in gas

Example 3: A storage battery contains a solution of H_2SO_4 30% by weight. Find

(*i*) Molality (*ii*) Molarity

(*iii*) Normality (*iv*) Mole fraction of H_2SO_4

(Given density of solution = 1.2 gm/cm^3)

Sol. 30% by weight $\Rightarrow W_{solute} = 30$ gm, $W_{solution} = 100$ gm, $W_{solvent} = 70$ gm

$$V_{solution} = \frac{100}{1.2} \text{mL}$$

$$n_{solute} = \frac{30}{98} = 0.306 \qquad n_{solvent} = \frac{70}{18} = 3.889$$

(*i*) $$\text{Molality} = \frac{n_{solute} \times 1000}{W_{solvent}} = \frac{0.306 \times 1000}{70} = 4.37 \text{ m}$$

(*ii*) $$\text{Molarity} = \frac{n_{solute} \times 1000}{V_{solution}} = \frac{0.306 \times 1000}{\frac{100}{1.2}} = 3.67 \text{ M}$$

(*iii*) $$\text{Normality} = \frac{W_{solute} \times 1000}{E_{solute} \times V_{solution}} = \frac{30 \times 1000}{49 \times \frac{100}{1.2}} = 7.34 \text{ N}$$

Alternatively

Normality = n-factor × Molarity

= 2 × 3.67 = 7.34

(*iv*) $$X_{H_2SO_4} = \frac{X_{H_2SO_4}}{n_{H_2SO_4} + n_{H_2O}} = \frac{0.306}{0.306 + 3.889} = 0.073$$

Concept Application

1. A sample of H_2SO_4 (density 1.8 g/mL) is labelled as 74.66% by weight. What is the molarity of acid?

(*a*) 15.2 M (*b*) 16.3 M

(*c*) 17.5 M (*d*) 13.7 M

SOLUTIONS OF SOLID/ LIQUID IN LIQUID

Solubility of a solid in liquid

(*i*) Introduction: Different solutes dissolve to a different extent in the same mass of a solvent i.e., they have different solubilities. Solubility is thus the ability of a solute to dissolve in a particular solvent.

(*ii*) Definition: The solubility of a particular solute in a solvent is the maximum amount of solute that will dissolve in 100 g solvent.

$$\text{Solubility} = \frac{\text{Wt. of solute}}{\text{Wt. of solvent}} \times 100$$

(*iii*) Factors affecting solubility:

(*a*) Nature of solute and solvent: Like dissolves like. Polar solutes dissolves in polar solvents and non polar solutes dissolve in non polar solvents. NaCl dissolves in water because NaCl and water both are polar. CS_2 dissolves in benzene because CS_2 and C_6H_6 both are non polar.

(*b*) Size of solute particles: Dissolution is a surface phenomenon like the evaporation. So, increase of surface area of the solute increases the rate of dissolution. The surface area of a solid solute can be increased by converting it into powder. The powdered solute dissolves more easily than the large crystals, as in the former case, larger surface area is in contact with the solvent.

(*c*) Effect of Temperature: Solubility may increase or decrease with increase of temperature. This depend on the enthalpy change of the solution. If the solute dissolves with the evolution of heat, solubility decreases with increase of temperature. On the other hand if the solute dissolves with the absorption of heat, solubility increases with rise in temperature.

Example: When ammonium chloride or silver nitrate is dissolved in water, the solution gets cooled.

$$NH_4Cl(s) + H_2O(\ell) \longrightarrow NH_4Cl(s) - \text{heat}$$

For such solutions, solubility increases on increase of temperature.

Example: When calcium oxide or lithium carbonate is placed in water the solution gets heated.

$$CaO(s) + H_2O \longrightarrow Ca(OH)_2(aq) + \text{Heat}$$

For such solutions, solubility decreases on increase of temperature.

Key Note

The amount of heat change during the formation of a solution depends mainly on two factors:

- **Lattice energy:** It is the amount of heat required to separate one mole of the ionic substances into its component positive and negative ions. This is an endothermic process.
- **Heat of hydration:** These ions get hydrated. The ions hold the water molecules by ion dipole attraction. Heat is liberated during the process of hydration. Thus it is an exothermic process.

(*a*) If lattice energy > hydration energy, the system cools down.

Examples: $NaNO_3$, KNO_3, KCl, NH_4Cl etc, (solubility increases with rise in temperature are).

(*b*) If lattice energy < hydration energy, the system heats up.

Examples: NaOH, Na_2CO_3, Na_2SO_4, all gases, etc, (solubility decreases with rise in temperature are).

Aarambh (Solved Examples)

1. If 20 mL of 0.5 M Na_2SO_4 is mixed with 50 mL of 0.2 M H_2SO_4 and 30 mL of 0.4 M $Al_2(SO_4)_3$ solution. Calculate $[Na^+]$, $[H^+]$, $[Al^{3+}]$, and $[SO_4^{2-}]$. [Assuming 100% dissociation]

Sol. Molarity = $\frac{\text{moles}}{\text{volume}}$

(*i*) $Na_2SO_4 \rightarrow 2Na^+ + SO_4^{2-}$

10 m mol

— 20 10

Total volume = 100 mL

$\therefore [Na^+] = \frac{20}{100} = 0.2$ M

(*ii*) $H_2SO_4 \rightarrow 2H^+ + SO_4^{2-}$

10 m mol

— 20 10

Total volume = 100 mL

$[H^+] = \frac{20}{100} = 0.2$ M

(*iii*) $Al_2(SO_4)_3 \rightarrow 2Al^{3+} + 3SO_4^{2-}$

12 m mol

— 24 36

Total volume = 100 mL

$[Al^{3+}] = \frac{24}{100} = 0.24$ M

(*iv*) Total m mol of SO_4^{2-} = m.m of SO_4^{2-} from $(Na_2SO_4 + H_2SO_4 + Al_2(SO_4)_3) = (10 + 10 + 36) = 56$

Total volume = 100 mL

$[SO_4^{2-}] = \frac{10+10+36}{100} = \frac{56}{100} = 0.56$ M

2. Calculate molality of 1.2 M H_2SO_4 solution. If its density (ρ) = 1.4 g/mL?

(*a*) 0.936 (*b*) 0.562
(*c*) 0.386 (*d*) 0.425

Sol. Molality = $\frac{1.2 \times 1000}{1000 \times 1.4 - 1.2 \times 98} = 0.936$

Therefore, option (*a*) is the correct answer.

3. A 6.90 M solution of KOH in water has 30% by weight of KOH. Calculate density of solution (in g/mL).

(*a*) 9.345 g/mL
(*b*) 2.445 g/mL
(*c*) 5.852 g/mL
(*d*) 1.288 g/mL

Sol. Let V = 1 lt, then moles of solute = 6.9

wt. of solute = 6.9 × 56 gm

% by mass = $\frac{M_{solute}}{M_{solution}} \times 100$

So, = $\frac{6.9 \times 56}{d \times 1000} \times 100 = 30$

d = 1.288 g/mL

Therefore, option (*d*) is the correct answer.

4. The density of 3M solution of $Na_2S_2O_3$ is 1.25 g mL^{-1}. Calculate.

(*a*) The % by weight of $Na_2S_2O_3$
(*b*) Mole fraction of $Na_2S_2O_3$

Sol. (*a*) mass of solution = 1000 × 1.25 = 1250 gm

mass of $Na_2S_2O_3$ = 3 × 158 = 474

% (w/w) = $\frac{474}{1250} \times 100 = 37.92$

Therefore, 37.92 is the correct answer.

(*b*) $X_{Na_2S_2O_3} = \frac{3}{3 + 43.11} = 0.065.$

Mole of water solvent = $\frac{1250 - 474}{18} = 43.11$

Therefore, 43.11 is the correct answer.

5. Calculate molality of aqueous urea solution which has $X_{urea} = 0.2$.

(*a*) 15.99 (*b*) 13.88
(*c*) 10.99 (*d*) 19.76

Sol. $X_u + X_w = 1$

$X_w = 1 – 0.2 = 0.8$

Let total moles = 1

$n_u = 0.2$

$n_w = 0.8$

$w_{water} = 0.8 \times 18 = 14.4$

Molality (m) = $\frac{\text{Number of moles of solute}}{\text{Weight of solvent (kg)}}$

$= \frac{0.2}{14.4/1000} = 13.88$

Therefore, option (*b*) is the correct answer.

6. 10 gm of a solute is dissolved in 80 gm of acetone. Vapour pressure of this solution = 271 mm of Hg. If vapour pressure of pure acetone is 283 mm of Hg, then calculate molar mass of solute?

(*a*) 173 g/mol (*b*) 163 g/mol
(*c*) 143 g/mol (*d*) 183 g/mol

Board Level Problems

SINGLE CORRECT TYPE QUESTIONS

1. A 1% solution of solute 'X' is isotonic with a 6% solution of sucrose (molar mass = 342 g mol^{-1}). The molar mass of solute 'X' is:

(*a*) 34.2 g mol^{-1} (*b*) 57 g mol^{-1}
(*c*) 114 g mol^{-1} (*d*) 3.42 g mol^{-1}

2. Low concentration of oxygen in the blood and tissues of people living at high altitudes is due to:

(*a*) Low temperature
(*b*) Low atmospheric pressure
(*c*) High atmospheric pressure
(*d*) High temperature

3. Which of the following solutions would have the highest osmotic pressure:

(*a*) $\frac{M}{10}$ NaCl (*b*) $\frac{M}{10}$ Urea
(*c*) $\frac{M}{10}$ $BaCl_2$ (*d*) $\frac{M}{10}$ Glucose

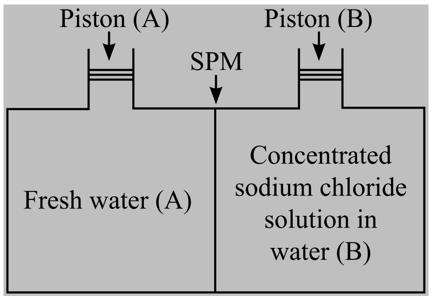
4. For an electrolyte undergoing association in a solvent, the van't Hoff factor:

(*a*) is always greater than one
(*b*) has negative value
(*c*) has zero value
(*d*) is always less than one

5. Henry's law constant K of CO_2 in water at 25°C is 3×10^{-2} mol/L atm^{-1}. Calculation the mass of CO_2 present in 100 L of soft drink bottled with a partial pressure of CO_2 of 4 atm at the same temperature.

(*a*) 5.28 g (*b*) 12.0 g (*c*) 428 g (*d*) 528 g

6. Consider the following figure and mark the correct option.



(*a*) Water will move from side (A) to side (B) if a pressure lower than osmotic pressure is applied on piston (B).
(*b*) Water will move from side (B) to side (A) if a pressure greater than osmotic pressure is applied on piston (B).
(*c*) Water will move from side (B) to side (A) if a pressure equal to osmotic pressure is applied on piston (B).
(*d*) Water will move from side (A) to side (B) if pressure equal to osmotic pressure is applied on piston (A).

7. Van't Hoff factor for $Na_2SO_4.10H_2O$ solution, assuming complete ionization is

(*a*) 1 (*b*) 3 (*c*) 13 (*d*) 2

8. We have three aqueous solutions of NaCl labelled as 'A', 'B' and 'C' with concentrations 0.1M, 0.01M and 0.001M, respectively. The value of van't Hoff factor for these solutions will be in the order______.

(*a*) i A < i B < i C (*b*) i A > iB > iC
(*c*) i A = i B = i C (*d*) i A < i B > i C

9. The relative lowering of vapour pressure of an aqueous solution containing non-volatile solute is 0.0225. The mole fraction of the non-volatile solute is:

(*a*) 0.80 (*b*) 0.725 (*c*) 0.15 (*d*) 0.0225

10. Which of the following statements is false?

(*a*) Two different solutions of sucrose of same molality prepared in different solvents will have the same depression in freezing point.
(*b*) The osmotic pressure of a solution is given by the equation $\Pi = CRT$ (where C is the molarity of the solution).
(*c*) Decreasing order of osmotic pressure for 0.01 M aqueous solutions of barium chloride, potassium chloride, acetic acid and sucrose is $BaCl_2 > KCl > CH_3COOH >$ sucrose.
(*d*) According to Raoult's law, the vapour pressure exerted by a volatile component of a solution is directly proportional to its mole fraction in the solution.

ASSERTION AND REASON TYPE QUESTIONS

11. **Assertion :** Addition of ethylene glycol to water lowers its freezing point.

Reason : Ethylene glycol is insoluble in water due to lack of its ability to form hydrogen bonds with water molecules.

(*a*) Both Assertion and Reason are true and Reason is correct explanation of Assertion.
(*b*) Both Assertion and Reason are true but Reason is not a correct explanation of Assertion.
(*c*) Assertion is true but Reason is false.
(*d*) Assertion is false but Reason is true.

12. **Assertion:** When NaCl is added to water a depression in freezing point is observed.

Reason: The lowering of vapour pressure of a solution causes depression in the freezing point.

(*a*) Both Assertion and Reason are true and Reason is correct explanation of Assertion.
(*b*) Both Assertion and Reason are true but Reason is not a correct explanation of Assertion.
(*c*) Assertion is true but Reason is false.
(*d*) Assertion is false but Reason is true.

several applications such as freeze concentration of liquid food and to find the molar mass of an unknown solute in the solution. Freeze concentration is a high quality liquid food concentration method where water is removed by forming ice crystals. This is done by cooling the liquid food below the freezing point of the solution. The freezing point depression is referred as a colligative property and it is proportional to the molar concentration of the solution (m), along with vapour pressure lowering, boiling point elevation, and osmotic pressure. These are physical characteristics of solutions that depend only on the identity of the solvent and the concentration of the solute. The characters are not depending on the solute's identity. (Jayawardena, J. A. E. C., Vanniarachchi, M. P. G. & Wansapala, M. A. J. (2017). Freezing point depression of different Sucrose solutions and coconut water.)

I. When a non volatile solid is added to pure water it will:

(*a*) Boil above 100°C and freeze above 0°C
(*b*) Boil below 100°C and freeze above 0°C
(*c*) Boil below 100°C and freeze below 0°C
(*d*) Boil above 100°C and freeze below 0°C

II. Colligative properties are:

(*a*) Dependent only on the concentration of the solute and independent of the solvent's and solute's identity.
(*b*) Dependent only on the identity of the solute and the concentration of the solute and independent of the solvent's identity.
(*c*) Dependent on the identity of the solvent and solute and thus on the concentration of the solute.
(*d*) Dependent only on the identity of the solvent and the concentration of the solute and independent of the solute's identity.

III. Assume three samples of juices A, B and C have glucose as the only sugar present in them. The concentration of sample A, B and C are 0.1M, .5M and 0.2 M respectively. Freezing point will be highest for the fruit juice:

(*a*) A
(*b*) B
(*c*) C
(*d*) All have same freezing point

IV. Identify which of the following is a colligative property:

(*a*) Freezing point (*b*) Boiling point
(*c*) Osmotic pressure (*d*) All of the above

Prarambh (Topicwise)

CONCENTRATION TERMS (REVISION OF MOLE)

1. The amount of anhydrous Na_2CO_3 present in 250 mL of 0.25 M solution is

(*a*) 225 g (*b*) 66.25 g
(*c*) 6.0 g (*d*) 6.625 g

2. 2.0 molar solution is obtained, when 0.5 mole solute is dissolved in

(*a*) 250 mL solvent (*b*) 250 g solvent
(*c*) 250 mL solution (*d*) 1000 mL solvent

3. 36 g of water and 828 g of ethyl alcohol form an ideal solution. The mole fraction of water in it, is

(*a*) 1.0 (*b*) 0.7
(*c*) 0.4 (*d*) 0.1

4. An X molal solution of a compound in benzene has mole fraction of solute equal to 0.2. The value of X is

(*a*) 14 (*b*) 3.2
(*c*) 4 (*d*) 2

5. 4.0 gm of NaOH is contained in one decilitre of solution. Its molarity would be

(*a*) 4 M (*b*) 2 M
(*c*) 1 M (*d*) 1.5 M

HENRY'S LAW AND BASICS OF VAPOUR PRESSURE

6. Calculate the solubility of gaseous oxygen in water at a temperature of 293 K when the partial pressure exerted by O_2 is 1 bar. (Given: K_H for O_2 34840 bar L mol^{-1})

(*a*) 2.87×10^{-5} mol/L (*b*) 4×10^{-4} mol/L
(*c*) 2.87×10^{-4} mol/L (*d*) 5×10^{-4} mol/L

7. Solubility of gas in water is x g/cm^3 at 300 K temperature. When temperature increases to 400 K, then solubility of gas

(*a*) Increases (*b*) Decreases
(*c*) Remain same (*d*) $\frac{1}{2}$x

8. Which one of the following gases has the lowest value of Henry's law constant?

(*a*) N_2 (*b*) He
(*c*) H_2 (*d*) CO_2

9. The vapour pressure of water depends upon

(*a*) Surface area of container
(*b*) Volume of container
(*c*) Temperature
(*d*) All of these

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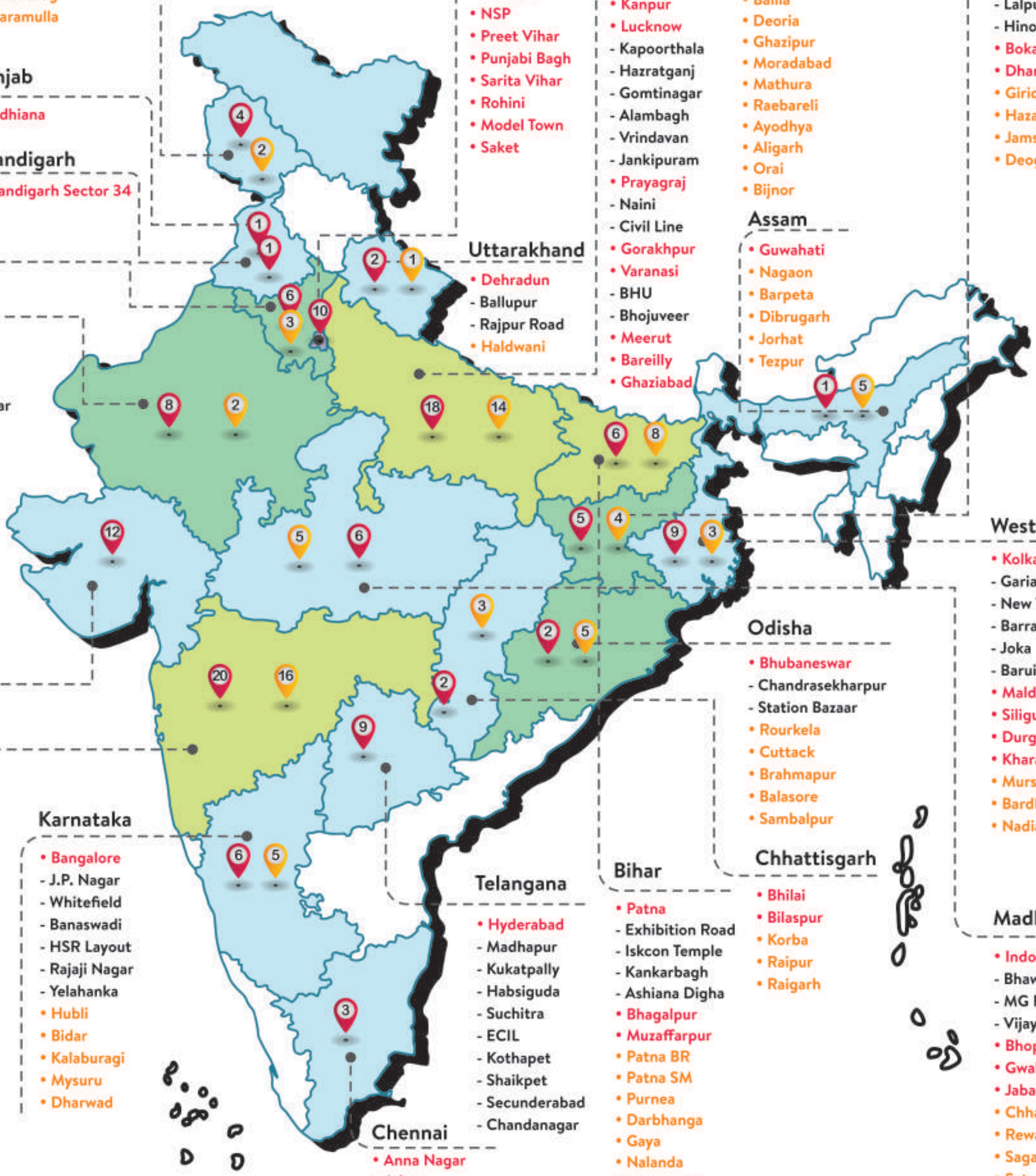

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  - Barzulla
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- Rohtak
- Gurugram
  - South City
  - Sector 16
- Narnaul
- Sonipat
- Sirsa

Punjab
- Ludhiana

Chandigarh
- Chandigarh Sector 34

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  - Dahod
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- Vadodara
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- Darbhanga
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