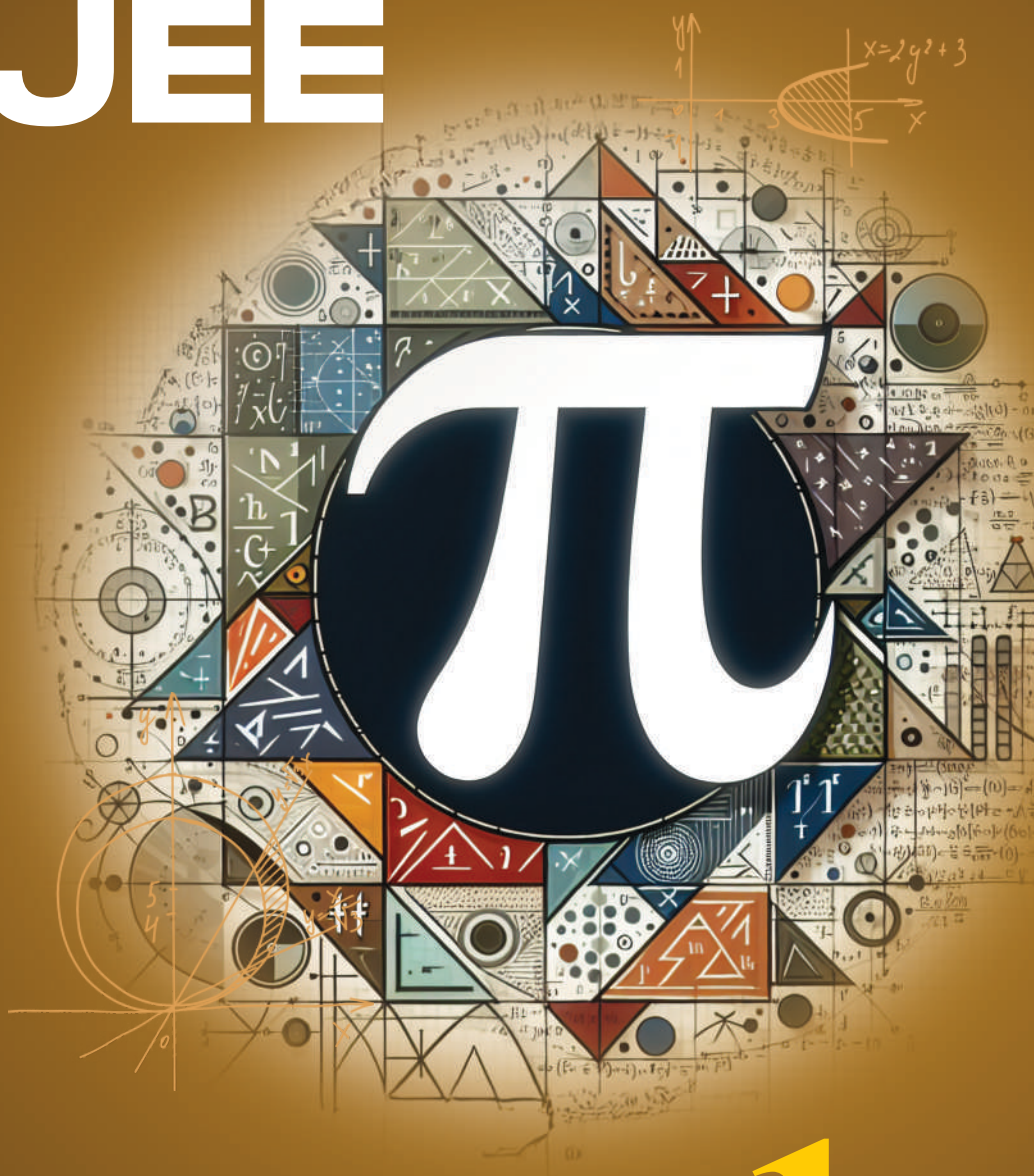
PHYSICS WALLAH

# PRAYAS JEE



- Basic Maths
- Sets
- Quadratic Equations
- Sequences and Series
- Trigonometric Functions

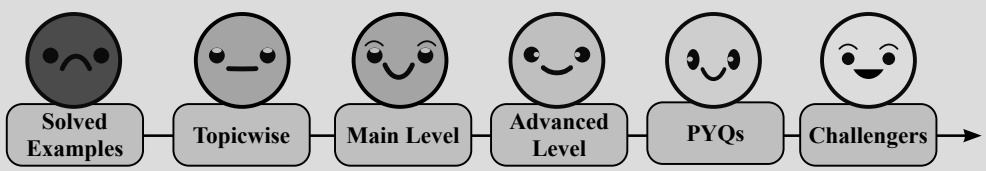
# Mathematics MODULE 1

# 3P PROGRESSOR

## PLANNER

| CHAPTER NAMES | Start Date of Module | End Date of Module | Solve More **PYQs** | Revision Period | Re-revision Period **(Before Test Series)** |
|---|---|---|---|---|---|
| **Basic Maths** | | | | | |
| **Sets** | | | | | |
| **Quadratic Equations** | | | | | |
| **Sequences and Series** | | | | | |
| **Trigonometric Functions** | | | | | |

## PROGRESS TRACKING




| CHAPTER NAMES | | | | | | |
|---|---|---|---|---|---|---|
| **Basic Maths** | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ |
| **Sets** | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ |
| **Quadratic Equations** | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ |
| **Sequences and Series** | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ |
| **Trigonometric Functions** | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ |

## PREP POINTERS

*Note down the topic you want to revise or section you want to re attempt while your preparation.*

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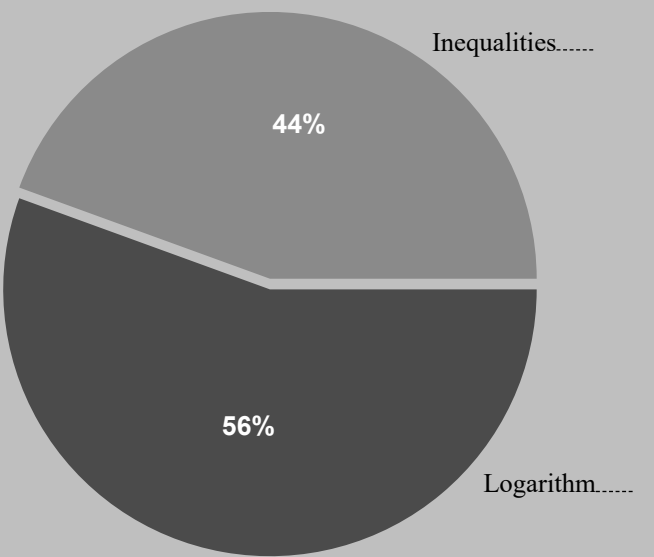
CHAPTER

# 1 Basic Maths

## Topicwise Weightage of JEE Main 6 Years Paper (124 Sets)




**"How's the Josh?" for these Topics:** *Mark your confidence level in the blank space around the topic (Low-L, Medium-M, High-H)*

## NUMBER SYSTEM

(*i*) **Natural numbers:** The counting numbers 1, 2, 3, 4, ... are called Natural Numbers. The set of natural numbers is denoted by *N*.

Thus $N = \{1, 2, 3, 4, ....\}$.

(*ii*) **Whole numbers:** Natural numbers including zero are called whole numbers. The set of whole numbers is denoted by W.

Thus W = {0, 1, 2, .....}

(*iii*) **Integers:** The numbers ... – 3, – 2, – 1, 0, 1, 2, 3 .... are called integers and the set is denoted by I or Z. Thus I (or Z) = {.. – 3, – 2, – 1, 0, 1, 2, 3...}

**Note:** (*a*) Positive integers $I^+ = \{1, 2, 3 ....\} = N$

(*b*) Negative integers $I^- = \{....., –3, –2, –1\}$.

(*c*) Non-negative integers (whole numbers) = {0, 1, 2, ...}.

(*d*) Non-positive integers = {....., –3, –2, –1, 0}.

(*iv*) **Even integers:** Integers which are divisible by 2 are called even integers.

e.g. 0, ± 2, ± 4,.....

(*v*) **Odd integers:** Integers which are not divisible by 2 are called odd integers.

e.g. ± 1, ± 3, ± 5, ± 7.....

(*vi*) **Prime numbers:** Natural numbers which are divisible by 1 and itself only are called prime numbers.

e.g. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, .....

(*vii*) **Composite number:** Let 'a' be a natural number, '*a*' is said to be composite if, it has atleast three distinct factors.

e.g. 4, 6, 8, 9, 10, 12, 14, 15 .....

**Note:** (*a*) 1 is neither a prime number nor a composite number.

(*b*) Numbers which are not prime are composite numbers (except 1).

(*c*) '4' is the smallest composite number.

(*d*) '2' is the only even prime number.

(*viii*) **Co-prime numbers:** Two natural numbers (not necessarily prime) are called coprime, if their H.C.F (Highest common factor) is one.

e.g. (1, 2), (1, 3), (3, 4), (3, 10), (3, 8), (5, 6), (7, 8) (15, 16) etc.

These numbers are also known as **relatively prime** numbers.

**Note:**

(*a*) Two prime numbers are always co-prime but converse need not be true.

(*b*) Consecutive natural numbers are always co-prime numbers.

(*ix*) **Twin prime numbers:** If the difference between two prime numbers is two, then the numbers are called twin prime numbers.

e.g. {3, 5}, {5, 7}, {11, 13}, {17, 19}, {29, 31}

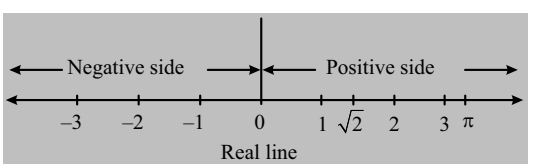
(*x*) **Rational numbers:** All the numbers that can be represented in the form $p/q$, where $p$ and $q$ are integers and $q \neq 0$, are called rational numbers and their set is denoted by $Q$. Thus $Q = \{p/q : p, q \in \text{I and } q \neq 0\}$. It may be noted that every integer is a rational number since it can be written as $p/q$. It may be noted that all recurring decimals are rational numbers.

**Note:** Maximum number of different decimal digits in $\frac{p}{q}$ is equal to $q$, i.e. $\frac{11}{9}$ will have maximum of 9 different decimal digits.

(*xi*) **Irrational numbers:** The numbers which can not be expressed in $p/q$ form where $p, q \in \text{I}$ and $q \neq 0$ i.e. the numbers which are not rational are called irrational numbers and their set is denoted by $Q^c$. (i.e. complementary set of $Q$) e.g. $\sqrt{2}$, $1 + \sqrt{3}$ etc. Irrational numbers can not be expressed as recurring decimals.

**Note:** $e \approx 2.71$ (is called Napier's constant) and $\pi \approx 3.14$ are irrational numbers.

(*xii*) **Real numbers:** Numbers which can be expressed on number line are called real numbers. The complete set of rational and irrational numbers is the set of real numbers and is denoted by $R$. Thus $R = Q \cup Q^C$.




All real numbers follow the order property i.e. if there are two distinct real numbers $a$ and $b$ then either $a < b$ or $a > b$.

**Note:**

(*a*) Integers are rational numbers, but converse need not be true.

(*b*) Negative of an irrational number is an irrational number.

(*c*) Sum of a rational number and an irrational number is always an irrational number

e.g. $2 + \sqrt{3}$

(*d*) The product of a non zero rational number and an irrational number will always be an irrational number.

(*e*) If $a \in Q$ and $b \notin Q$, then $ab$ = rational number, only if $a = 0$.

(*f*) Sum, difference, product and quotient of two irrational numbers need not be a irrational number or we can say, result may be a rational number also.

**ADVANCED LEARNING**

(*xiii*) **Complex number:** A number of the form $a + ib$ is called a complex number, where $a, b \in R$ and $i = \sqrt{-1}$. Complex number is usually denoted by $z$ and the set of complex number is represented by $C$. Thus $C = \{a + ib : a, b \in R \text{ and } i = \sqrt{-1}\}$

**Note:** It may be noted that $N \subset W \subset I \subset Q \subset R \subset C$.

## Train Your Brain

**Example 1:** The value of $1.\overline{285714} \div 1.\overline{714285} =$ _____.

(*a*) $\frac{3}{4}$ (*b*) $\frac{7}{8}$

(*c*) $\frac{7}{12}$ (*d*) $\frac{3}{7}$

**Sol. (*a*)**

$1.\overline{285714}$

$= 1 + 0.\overline{285714}$

$= 1 + \frac{2}{7} = \frac{9}{7}$

$1.\overline{714285}$

$= 1 + \frac{5}{7} = \frac{12}{7}$

$\therefore 1.\overline{285714} \div 1.\overline{714285}$

$= \frac{9}{7} \div \frac{12}{7}$

$= \frac{9}{7} \times \frac{7}{12}$

$= \frac{3}{4}$

**Example 2:** Prove that the difference $10^{25} - 7$ is divisible by 3.

**Sol.** Write the given difference in the form $10^{25} - 7 = (10^{25} - 1) - 6$. The number $10^{25} - 1 = \underbrace{\overline{99..9}}_{25 \text{ digits}}$ is divisible by 3 (and 9). Since the numbers $(10^{25} - 1)$ and 6 are divisible by 3, the number $10^{25} - 7$, being their difference, is also divisible by 3 without a remainder.

## Concept Application

**1.** The product of $1.\overline{142857}$ and $0.\overline{63}$ = _____.

($a$) $\frac{8}{11}$

($b$) $\frac{7}{11}$

($c$) $\frac{11}{7}$

($d$) $\frac{8}{7}$

**2.** If $x = \sqrt{12} - \sqrt{9}, y = \sqrt{13} - \sqrt{10}$, and $z = \sqrt{11} - \sqrt{8}$, then which of the following is true?

($a$) $z > x > y$

($b$) $z > y > x$

($c$) $y > x > z$

($d$) $y > z > x$

## SOME IMPORTANT IDENTITIES

1. $(a + b)^2 = a^2 + 2ab + b^2 = (a - b)^2 + 4ab$
2. $(a - b)^2 = a^2 - 2ab + b^2 = (a + b)^2 - 4ab$
3. $a^2 - b^2 = (a + b)(a - b)$
4. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
5. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
6. $a^3 + b^3 = (a + b)^3 - 3ab(a + b) = (a + b)(a^2 + b^2 - ab)$
7. $a^3 - b^3 = (a - b)^3 + 3ab(a - b) = (a - b)(a^2 + b^2 + ab)$
8. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = a^2 + b^2 + c^2 + 2abc\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$
9. $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2]$
10. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
    $= \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$
    If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$
11. $a^4 - b^4 = (a + b)(a - b)(a^2 + b^2)$
12. $a^4 + a^2 + 1 = (a^2 + 1)^2 - a^2 = (1 + a + a^2)(1 - a + a^2)$

## Train Your Brain

**Example 3:** Show that the expression, $(x^2 - yz)^3 + (y^2 - zx)^3 + (z^2 - xy)^3 - 3(x^2 - yz)(y^2 - zx)(z^2 - xy)$ is a perfect square and find its square root.

**Sol.** $(x^2 - yz)^3 + (y^2 - zx)^3 + (z^2 - xy)^3 - 3(x^2 - yz)(y^2 - zx)(z^2 - xy) = a^3 + b^3 + c^3 - 3abc$

(where $a = x^2 - yz, b = y^2 - zx, c = z^2 - xy$)

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= \frac{1}{2}(a + b + c)((a - b)^2 + (b - c)^2 + (c - a)^2)$

$= \frac{1}{2}(x^2 + y^2 + z^2 - xy - yz - zx)[(x^2 - yz - y^2 + zx)^2 + (y^2 - zx - z^2 + xy)^2 + (z^2 - xy - x^2 + yz)^2]$

$= \frac{1}{2}(x^2 + y^2 + z^2 - xy - yz - zx)[\{x^2 - y^2 + z(x - y)\}^2 + \{y^2 - z^2 + x(y - z)\}^2 + \{z^2 - x^2 + y(z - x)\}^2]$

$= \frac{1}{2}(x^2 + y^2 + z^2 - xy - yz - zx)(x + y + z)^2 [(x - y)^2 + (y - z)^2 + (z - x)^2]$

$= (x + y + z)^2 (x^2 + y^2 + z^2 - xy - yz - zx)^2$

$= (x^3 + y^3 + z^3 - 3xyz)^2$

(which is a perfect square) its square roots are

$\pm (x^3 + y^3 + z^3 - 3xyz)$

**Example 4:** If $x^2 - 4x + 1 = 0$, then what is the value of $x^3 + \frac{1}{x^3}$?

**Sol.** $x^2 - 4x + 1 = 0 \Rightarrow x + \frac{1}{x} = 4$

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) = 4^3 - 3 \times 4 = 52$$

**Example 5:** If $x + \frac{1}{x} = a$, then what is the value of $x^3 + x^2 + \frac{1}{x^3} + \frac{1}{x^2}$?

($a$) $a^3 + a^2$ ($b$) $a^3 + a^2 - 5a$

($c$) $a^3 + a^2 - 3a - 2$ ($d$) $a^3 + a^2 - 4a - 2$

**Sol. ($c$)**

Given, $x + \frac{1}{x} = a$

Now, $x^3 + x^2 + \frac{1}{x^3} + \frac{1}{x^2} = \left(x^3 + \frac{1}{x^3}\right) + \left(x^2 + \frac{1}{x^2}\right)$

$$= \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) + \left(x + \frac{1}{x}\right)^2 - 2$$

$= a^3 - 3a + a^2 - 2 = a^3 + a^2 - 3a - 2.$

**1.** The value of $81^{(1/\log_5 3)} + 27^{\log_9 36} + 3^{4/\log_7 9}$ is equal to

(*a*) 49 (*b*) 625 (*c*) 216 (*d*) 890

**Sol.** $81^{(1/\log_5 3)} + 27^{\log_9 36} + 3^{4/\log_7 9}$

$= 3^{4\log_3 5} + 3^{3.\frac{1}{2}\log_3 36} + 3^{4\log_9 7}$

$= 3^{\log_3 5^4} + 3^{\log_3 36^{3/2}} + 3^{\log_3 7^{4/2}}$

$= 5^4 + 36^{3/2} + 7^2 = 890$

Therefore, option (*d*) is the correct answer.

**2.** The largest integral value of $x$ satisfying

$\sqrt{18^x - 5} \le \sqrt{2(18^x + 12)} - \sqrt{18^x + 5}$ is

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) no integral value of $x$ possible

**Sol.** Let $18^x = p$

$\sqrt{p-5} + \sqrt{p+5} \le \sqrt{2(p+12)}$

$\Rightarrow\ p - 5 + p + 5 + 2\sqrt{p^2 - 25} \le 2p + 24$

$\Rightarrow \sqrt{p^2 - 25} \le 12 \Rightarrow p^2 \le 169 \Rightarrow p \le 13$

Also $p \ge 5$

Thus $5 \le p \le 13 \Rightarrow \log_{18} 5 \le x \le \log_{18} 13$

Therefore, option (*d*) is the correct answer.

**3.** Solve if $|x - 5| + |x + 4| = 9$

(*a*) [–4, 5] (*b*) (–4, 5) (*c*) (–4, 5] (*d*) [–4, 5)

**Sol.** Given equation is of form $|a| + |b| = |a - b|$

It is true for $ab \le 0$

$(x - 5)(x + 4) \le 0$

So $x \in [-4, 5]$

Therefore, option (*a*) is the correct answer.

**4.** Solve $\dfrac{(e - \sin x)(x-2)}{(x+4)} \ge 0.$

(*a*) $(-\infty, -4) \cup [2, \infty)$ (*b*) $(-\infty, -4] \cup (2, \infty)$

(*c*) $(-\infty, -4) \cup (2, \infty)$ (*d*) None of these

**Sol.** Zeros $x = 2$, Pole $x \ne -4$

$e - \sin x > 0$ always positive

$\dfrac{(e - \sin x)(x-2)}{(x+4)} \ge 0$

Final solution $x \in (-\infty, -4) \cup [2, \infty)$

Therefore, option (*a*) is the correct answer.

**5.** Values of $x$ satisfying the equation

$\log_{\frac{5}{2}} x + \log_{5x}\left(\dfrac{5}{x}\right) = 1$ are

(*a*) 1 (*b*) 5

(*c*) $\dfrac{1}{25}$ (*d*) 3

**Sol.** $(\log_5 x)^2 + \log_{5x} \dfrac{5}{x} = 1$

$\Rightarrow (\log_5 x)^2 + \log_{5x} 5 - \log_{5x} x = 1$

$\Rightarrow \left(\log_5 x\right)^2 + \dfrac{\log_5 5}{\log_5 5 + \log_5 x} - \dfrac{\log_5 x}{\log_5 5 + \log_5 x} = 1$

$\Rightarrow (\log_5 x)^2 + \dfrac{1}{1 + \log_5 x} - \dfrac{\log_5 x}{1 + \log_5 x} = 1$

Let $\log_5 x = t$

$\therefore\ t^2 + \dfrac{1}{1+t} - \dfrac{t}{1+t} = 1$

$\Rightarrow\ \dfrac{t^2(1+t) + 1 - t}{1+t} = 1$

$\Rightarrow t^3 + t^2 + 1 - t = 1 + t$

$t^3 + t^2 - 2t = 0$

$t(t^2 + t - 2) = 0$

$t(t - 1)(t + 2) = 0$

$t = 0, 1, -2$

$\therefore \log_5 x = 0, 1, -2$

$\therefore x = 1, 5, \dfrac{1}{25}$

Therefore, option (*a, b, c*) is the correct answers.

**6.** The equation $\log_{x^2} 16 + \log_{2x} 64 = 3$ has

(*a*) One irrational solution

(*b*) No prime solution

(*c*) Two real solutions

(*d*) One integral solution

**Sol.** $\dfrac{4}{2}\log_x 2 + \dfrac{\log_x 64}{\log_x 2x}$

$\Rightarrow\ 2\log_x 2 + \dfrac{6\log_x 2}{1 + \log_x 2} = 3$

Let $\alpha = \log_x 2$

$\therefore 2\alpha + \dfrac{6\alpha}{1+\alpha} = 3$

$\Rightarrow 2\alpha + 2\alpha^2 + 6\alpha - 3 - 3\alpha = 0$

$\Rightarrow 2\alpha^2 + 5\alpha - 3 = 0$

$\Rightarrow (\alpha + 3)(2\alpha - 1) = 0 \Rightarrow a = -3, 1/2$

$\therefore \log_x 2 = -3 \Rightarrow x = 2^{-1/3}$ (Irrational)

or $\log_x 2 = \dfrac{1}{2} \Rightarrow x = 4$ (Integer)

Therefore, option (*a, b, c, d*) is the correct answers.

## BASIC CONCEPTS AND NUMBER SYSTEM

**1.** The number of real roots of the equation $(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$ is:

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

**2.** If $x - a$ is a factor of $x^3 - a^2x + x + 2$, then '$a$' is equal to

(*a*) 0 (*b*) 2 (*c*) –2 (*d*) 1

**3.** If $x, y$ are integral solutions of $2x^2 - 3xy - 2y^2 = 7$, then value of $|x + y|$ is

(*a*) 2 (*b*) 4

(*c*) 6 (*d*) 2 or 4 or 6

**4.** If $a, b, c$ are real, then $a(a-b) + b(b-c) + c(c-a) = 0$, only if

(*a*) $a + b + c = 0$ (*b*) $a = b = c$

(*c*) $a = b$ or $b = c$ or $c = a$ (*d*) $a - b - c = 0$

**5.** If $2x^3 - 5x^2 + x + 2 = (x-2)(ax^2 - bx - 1)$, then $a$ & $b$ are respectively

(*a*) 2, 1 (*b*) 2, – 1 (*c*) 1, 2 (*d*) –1, 1/2

**6.** If $L = \frac{1}{\sqrt{7}-\sqrt{8}} + \frac{1}{\sqrt{7}-\sqrt{6}} + \frac{1}{3-\sqrt{8}} + \frac{1}{\sqrt{5}+2} + \frac{1}{\sqrt{5}-\sqrt{6}}$

$= 1 + 2\sqrt{a} + 2\sqrt{b}$, then $a \times b$ is equal to

(*a*) 30 (*b*) 45 (*c*) 8 (*d*) 0

**7.** If $a$, $b$, $c$ are real and distinct numbers, then the value of $\frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)}$ is

(*a*) 1 (*b*) $a\,b\,c$

(*c*) 2 (*d*) 3

**8.** The remainder obtained when the polynomial $1 + x + x^3 + x^9 + x^{27} + x^{81} + x^{243}$ is divided by $x - 1$ is

(*a*) 3 (*b*) 5 (*c*) 7 (*d*) 11

## LOGARITHM AND ITS PRINCIPLE PROPERTIES

**9.** $\frac{1}{1+\log_b a + \log_b c} + \frac{1}{1+\log_c a + \log_c b} + \frac{1}{1+\log_a b + \log_a c}$ has the value equal to

(*a*) $abc$ (*b*) $\frac{1}{abc}$ (*c*) 0 (*d*) 1

**10.** $\log_7 \log_7 \sqrt{7(\sqrt{7\sqrt{7}})} =$

(*a*) $3 \log_2 7$ (*b*) $1 - 3 \log_3 7$

(*c*) $1 - 3\log_7 2$ (*d*) $1 - 10 \log_2 7$

**11.** $\frac{1}{\log_{\sqrt{bc}} abc} + \frac{1}{\log_{\sqrt{ca}} abc} + \frac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to

(*a*) 1/2 (*b*) 1 (*c*) 2 (*d*) 4

**12.** If $\log_x \log_{18}(\sqrt{2} + \sqrt{8}) = \frac{1}{3}$. Then the value of 1000 $x$ is equal to

(*a*) 8 (*b*) 1/8 (*c*) 1/125 (*d*) 125

**13.** Number of real solutions of the equation $\sqrt{\log_{10}(-x)} = \log_{10}\sqrt{x^2}$ is:

(*a*) none (*b*) exactly 1

(*c*) exactly 2 (*d*) 4

**14.** Greatest integer less than or equal to the number $\log_2 15.\log_{1/6} 2.\log_3 1/6$ is

(*a*) 4 (*b*) 3 (*c*) 2 (*d*) 1

**15.** The ratio $\frac{2^{\log_{\frac{1}{24}} a} - 3^{\log_{27}(a^2+1)^3} - 2a}{7^{4\log_{49} a} - a - 1}$ simplifies to

(*a*) $a^2 - a - 1$ (*b*) $a^2 + a - 1$

(*c*) $a^2 - a + 1$ (*d*) $a^2 + a + 1$

**16.** If $3^{2\log_3 x} - 2x - 3 = 0$, then the number of values of '$x$' satisfying the equation is

(*a*) zero (*b*) 1

(*c*) 2 (*d*) More than 2

**17.** The sum of all the solutions to the equation $2\log x - \log(2x - 75) = 2$:

(*a*) 30 (*b*) 350 (*c*) 75 (*d*) 200

## INEQUALITIES

**18.** If the solution set of the inequality $\log_{\sqrt{0.9}} \log_5 (\sqrt{x^2 + 5 + x}) > 0$ contains '$n$' integral values, then $n$ equals to

(*a*) 7 (*b*) 8 (*c*) 6 (*d*) 10

**19.** If $\log_{0.5} \log_5 (x^2 - 4) > \log_{0.5} 1$, then '$x$' lies in the interval

(*a*) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 3)$

(*b*) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 3\sqrt{5})$

(*c*) $(\sqrt{5}, 3\sqrt{5})$

(*d*) $\phi$

**20.** Solution set of the inequality $2 - \log_2 (x^2 + 3x) \geq 0$ is:

(*a*) $[-4, 1]$ (*b*) $[-4, -3) \cup (0, 1]$

(*c*) $(-\infty, -3) \cup (1, \infty)$ (*d*) $(-\infty, -4) \cup [1, \infty)$

## MODULUS FUNCTION

**21.** Solutions of $|4x + 3| + |3x - 4| = 12$ are

(*a*) $x = -\frac{7}{3}, \frac{3}{7}$ (*b*) $x = -\frac{5}{2}, \frac{2}{5}$

(*c*) $x = -\frac{11}{7}, \frac{13}{7}$ (*d*) $x = -\frac{3}{7}, \frac{7}{5}$

**22.** If $|x^2 - 2x - 8| + |x^2 + x - 2| = 3|x + 2|$, then the set of all real values of $x$ is

($a$) $[1, 4] \cup \{-2\}$ ($b$) $[1, 4]$

($c$) $[-2, 1] \cup [4, \infty)$ ($d$) $(-\infty, -2] \cup [1, 4]$

**23.** The complete set of real '$x$' satisfying $||x - 1| - 1| \le 1$ is:

($a$) $[0, 2]$ ($b$) $[-1, 3]$

($c$) $[-1, 1]$ ($d$) $[1, 3]$

**24.** The number of real roots of the equation $|x|^2 - 3|x| + 2 = 0$ is

($a$) 1 ($b$) 2

($c$) 3 ($d$) 4

**25.** Number of real solution ($x$) of the equation $|x-3|^{3x^2-10x+3} = 1$ is

($a$) exactly four ($b$) exactly three

($c$) exactly two ($d$) exactly one

## MISCELLANEOUS

**26.** Simplify: $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3}$

($a$) 0 ($b$) 1

($c$) 3 ($d$) 4

**27.** The expression $x^2 - y^2 - z^2 + 2yz + x + y - z$ has a factor

($a$) $x + y + z + 1$ ($b$) $-x + y + z$

($c$) $x + y - z + 1$ ($d$) $x - y + z + 1$

**28.** Solve the equation $\frac{3x^4 + x^2 - 2x - 3}{3x^4 - x^2 + 2x + 3} = \frac{5x^4 + 2x^2 - 7x + 3}{5x^4 - 2x^2 + 7x - 3}$

($a$) $x = 5, 2$ ($b$) $x = 4, 1$

($c$) $x = 3, 8$ ($d$) $x = 1, 5$

**29.** If $x$, $y$, $z$ are positive real number and $a$, $b$, $c$ are rational numbers, then the value of $\frac{1}{1 + x^{b-a} + x^{c-a}} + \frac{1}{1 + x^{a-b} + x^{c-b}} + \frac{1}{1 + x^{b-c} + x^{a-c}}$ is

($a$) $-1$ ($b$) 1 ($c$) 0 ($d$) 2

**30.** If $a^x = \sqrt{b}, b^y = \sqrt[3]{c}$ and $c^z = \sqrt{a}$ then the value of $xyz$ is

($a$) $\frac{1}{2}$ ($b$) $\frac{1}{3}$ ($c$) $\frac{1}{6}$ ($d$) $\frac{1}{12}$

**31.** If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, then $a^a \cdot b^b \cdot c^c =$

($a$) 3 ($b$) 1

($c$) 4 ($d$) 2

**32.** The number of prime numbers satisfying the inequality $\frac{x^2 - 1}{2x + 5} < 3$ is

($a$) 1 ($b$) 2 ($c$) 3 ($d$) 4

# Prabal (JEE Main Level)

**1.** If $x^{x\sqrt[3]{x}} = \left(x.\sqrt[3]{x}\right)^x$, then $x =$

($a$) 1 ($b$) $-1$

($c$) 0 ($d$) 2

**2.** The equation $4^{(x^2+2)} - 9.2^{(x^2+2)} + 8 = 0$ has the solution

($a$) $x = \pm 1$ ($b$) $x = 10$

($c$) $x = \pm\sqrt{2}$ ($d$) $x = \sqrt{3}$

**3.** If $x = \log_a(bc)$, $y = \log_b(ca)$, $z = \log_c(ab)$, then which of the following is equal to 1

($a$) $x + y + z$

($b$) $(1 + x)^{-1} + (1 + y)^{-1} + (1 + z)^{-1}$

($c$) $xyz$

($d$) $x + y - z$

**4.** The solution of the equation $\log_7 \log_5 \left(\sqrt{x^2 + 5 + x}\right) = 0$.

($a$) $x = 2$ ($b$) $x = 3$

($c$) $x = 4$ ($d$) $x = -2$

**5.** The value of $(0.05)^{\log_{\sqrt{20}}(0.1+0.01+0.001+.....)}$ is

($a$) 81 ($b$) $\frac{1}{81}$ ($c$) 20 ($d$) $\frac{1}{20}$

**6.** The value of $\log_2 . \log_3 .... \log_{100} 100^{99^{98^{\cdot^{\cdot^{2^1}}}}}$ is

($a$) 0 ($b$) 1 ($c$) 2 ($d$) 100

**7.** The number of solution of $\log_2(x + 5) = 6 - x$ is

($a$) 2 ($b$) 0 ($c$) 3 ($d$) 1

**8.** Exhaustive set of values of $x$ satisfying $\log_{|x|}(x^2 + x + 1) \ge 0$ is

($a$) $(-1, 0)$ ($b$) $(-\infty, -1) \cup (1, \infty)$

($c$) $(-\infty, \infty) - \{-1, 0, 1\}$ ($d$) $(-\infty, -1) \cup (-1, 0) \cup (1, \infty)$

**9.** The set of real values of $x$ satisfying $\log_{1/2}\left(x^2 - 6x + 12\right) \ge -2$ is

($a$) $\left(-\infty, 2\right]$ ($b$) $[2, 4]$

($c$) $\left[4, +\infty\right)$ ($d$) $[3, 8]$

## INTEGER TYPE QUESTIONS

**34.** If $\log_2(\log_3(\log_4(x))) = 0$ and $\log_3(\log_4(\log_2(y))) = 0$ and $\log_4(\log_2(\log_3(z))) = 0$ then the sum of $x$, $y$ and $z$ is

**35.** Let $\log_2 x + \log_4 x + \log_8 x = \log_k x$ for all $x \in R^+$. If $k = \sqrt[b]{a}$ where $a, b \in N$ then find the smallest positive value of $(a + b)$.

**36.** Find the value of the expression
$$6\left(\frac{2}{\log_4(2000)^6} + \frac{3}{\log_5(2000)^6}\right).$$

**37.** If $N = 7^{\log_{49} 900}$, $A = 2^{\log_2 4} + 3^{\log_2 4} + 4^{\log_2 2} - 4^{\log_2 3}$
$D = (\log_5 49)(\log_7 125)$
Find $P = \log_{\left(A-\frac{N}{10}\right)} |N + A + D + 6| - \log_5 2$,

**38.** If $a + b + c = 1$, $a^2 + b^2 + c^2 = 9$, $a^3 + b^3 + c^3 = 1$, then find value of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$.

**39.** If $a + b + c = 10$ and $ab + bc + ac = 20$ then find the value of $a^3 + b^3 + c^3 - 3abc$

**40.** If $(a-b)^3 + (b-c)^3 + (c-a)^3 = p(a-b)(b-c)(c-a)$, then $p =$

**41.** The value of $\sqrt{5\sqrt{5\sqrt{5\sqrt{5.....}}}}$ is

**42.** If $x = \sqrt[3]{7+5\sqrt{2}} - \frac{1}{\sqrt[3]{7+5\sqrt{2}}}$, then the value of $x^3 + 3x - 14$ is equal to

**43.** If $\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{-1/3}\right]^{1/4} = 7^x$ then $-3x =$

**44.** Number of cyphers after decimal before a significant figure starts in $\left(\frac{5}{4}\right)^{-100}$ is equal to [**Use:** $\log_{10} 2 = 0.3010$]

**45.** Number of real solution of $\log_5[2 + \log_3(x + 3)] = 0$ is

**46.** If $4^A + 9^B = 10^C$, where $A = \log_{16} 4$, $B = \log_3 9$ & $C = \log_x 83$, then find $x$.

# Parikshit (JEE Advanced Level)

## SINGLE CORRECT TYPE QUESTIONS

**1.** The solution set of the inequality $\log_{\sin\left(\frac{\pi}{3}\right)}(x^2 - 3x + 2) \geq 2$ is

(*a*) $\left(\frac{1}{2}, 2\right)$ (*b*) $\left(1, \frac{5}{2}\right)$

(*c*) $\left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$ (*d*) (1, 2)

**2.** If $\sqrt{\log_4\{\log_3\{\log_2(x^2 - 2x + a)\}\}}$ is defined $\forall\, x \in R$, then the set of values of '$a$' is

(*a*) $[9, \infty)$ (*b*) $[10, \infty)$ (*c*) $[15, \infty)$ (*d*) $[2, \infty)$

**3.** Number of integers, which satisfy the inequality, $\frac{(16)^{1/x}}{(2^{x+3})} > 1$ is equal to:

(*a*) Infinite (*b*) Zero (*c*) 1 (*d*) 4

**4.** The solution set of inequality $\frac{(3^x - 4^x)\cdot\ln(x+2)}{x^2 - 3x - 4} \leq 0$ is

(*a*) $(-\infty, 0] \cup (4, \infty)$ (*b*) $(-2, 0] \cup (4, \infty)$

(*c*) $(-1, 0] \cup (4, \infty)$ (*d*) $(-2, -1) \cup (-1, 0] \cup (4, \infty)$

**5.** Number of values of $x$ in the interval (0, 5) satisfying the equation $\frac{\ln(\sqrt{\sqrt{x^2+1}+x}) + \ln(\sqrt{\sqrt{x^2+1}-x})}{\ln x} = x$, is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 0

**6.** The number of positive integers not satisfying the inequality $\log_2(4^x - 2.2^x + 17) > 5$.

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 1

**7.** If $a$, $b$, $c$ are positive real numbers such that $a^{\log_3 7} = 27; b^{\log_7 11} = 49$ and $c^{\log_{11} 25} = \sqrt{11}$.
The value of $\left(a^{(\log_3 7)^2} + b^{(\log_7 11)^2} + c^{(\log_{11} 25)^2}\right)$ equals

(*a*) 489 (*b*) 469 (*c*) 464 (*d*) 400

**8.** Find the value of $x$ that satisfies the equation $\log\left(\frac{x^{1/x}}{x^{1/(x+1)}}\right) = \frac{1}{5050}$

(*a*) 1 (*b*) 10 (*c*) 100 (*d*) 1000

**9.** Sum of all the real solutions of the inequality $\frac{(x^2+2)(\sqrt{x^2-16})}{(x^4+2)(x^2-9)} \leq 0$ is

(*a*) 5 (*b*) 4 (*c*) 8 (*d*) 0

**10.** The set of values of $x$ satisfying simultaneously the inequalities $\frac{\sqrt{(x-8)(2-x)}}{\log_{0.3}\left(\frac{10}{7}(\log_2 5 - 1)\right)} \geq 0$ and $2^{x-3} - 31 > 0$ is:

(*a*) singleton set
(*b*) an empty set
(*c*) an infinite set
(*d*) a set consisting of exactly two elements.

**11.** If $|x-3|^2 - 6|x-3| - 23 < 4$, then

(*a*) $x \in (-12, 6)$
(*b*) 17 integers satisfy the inequality
(*c*) 11 non-negative integers satisfy the inequality
(*d*) 6 negative integers satisfy the inequality

## MULTIPLE CORRECT TYPE QUESTIONS

**12.** Indicate all correct alternatives, where base of the log is 2.

The equation $x^{\frac{3}{4}(\log_2 x)^2+\log_2 x-\frac{5}{4}}=\sqrt{2}$ has:

(*a*) At least one real solution

(*b*) Exactly three real solutions

(*c*) Exactly one irrational solution

(*d*) Imaginary roots

**13.** The equation $x^{\left[(\log_3 x)^2-\frac{9}{2}\log_3 x+5\right]}=3\sqrt{3}$ has

(*a*) Exactly three real solution

(*b*) At least one real solution

(*c*) Exactly one irrational solution

(*d*) Complex roots

**14.** Solution set of the inequality

$$(\log_2 x)^4-\left(\log_{1/2}\frac{x^3}{8}\right)^2+9\log_2\left(\frac{32}{x^2}\right)<4(\log_{1/2} x)^2 \text{ is}$$

$(a, b) \cup (c, d)$ then the correct statement is

(*a*) $a = 2b$ and $d = 2c$

(*b*) $b = 2a$ and $d = 2c$

(*c*) $\log_c d = \log_b a$

(*d*) there are 4 integers in $(c, d)$

**15.** Choose the correct from the following

(*a*) $\dfrac{81^{\frac{1}{\log_5 9}}+3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409}\left((\sqrt{7})^{\frac{2}{\log_{25} 7}}-(125)^{\log_{25} 6}\right)=1$

(*b*) $5^{\log_{1/5}\frac{1}{2}}+\log_{\sqrt{2}}\left(\dfrac{4}{\sqrt{7}+\sqrt{3}}\right)+\log_{1/2}\left(\dfrac{1}{(10+2\sqrt{21})}\right)=6$

(*c*) $\sqrt{10^{2+\frac{1}{2}\log(16)}}=20$

(*d*) None of these

**16.** Choose the correct from the following

(*a*) $\log_2(\log_{1/2}(x))<2$, for all $x\in\left(\dfrac{1}{16},1\right)$

(*b*) $\log_{1/2}(\log_3(x))>3$, for all $x \in (1, 3^{1/8})$

(*c*) $(\log_2(x)-1)(\log_3(x)-2)\le 0$, for all $x \in [2, 9]$

(*d*) $(\log_2(x)-1)(\log_{1/2}(x)-2)\le 0$, for all $x\in\left(0,\dfrac{1}{4}\right]\cup[2,\infty)$

**17.** The solution set of the system of equations $\log_3 x + \log_3 y = 2 + \log_3 2$ and $\log_{27}(x + y) = \dfrac{2}{3}$ is

(*a*) (6, 3) (*b*) (3, 6) (*c*) (6, 12) (*d*) (12, 6)

**18.** Consider the quadratic equation, $(\log_{10}8)x^2 - (\log_{10}5)x = 2(\log_2 10)^{-1} - x$. Which of the following quantities are irrational.

(*a*) sum of the roots

(*b*) product of the roots

(*c*) sum of the coefficients

(*d*) discriminant

## COMPREHENSION BASED QUESTIONS

**Comprehension (Q. No. 19 to 21):** Let $\alpha$ and $\beta$ are the solutions of the equation $\left(\sqrt{x}\right)^{\log_5 x-1}=5$ where $\alpha \in I$ and $\beta \in Q$ Then

[**Use:** $\log_{10}2 = 0.3010$, $\log_{10}3 = 0.4771$]

**19.** The number of significant digits before decimal in $(\alpha)^{10}$ is

(*a*) 13 (*b*) 14

(*c*) 15 (*d*) None of these

**20.** Number of zeros after decimal before a significant digit in $(\beta)^{10}$ is

(*a*) 5 (*b*) 7 (*c*) 8 (*d*) 6

**21.** The value of $(\beta)^{\log_{25} 9}$ is

(*a*) $\dfrac{1}{3}$ (*b*) 5 (*c*) $\dfrac{1}{5}$ (*d*) 9

## MATCH THE COLUMN TYPE QUESTIONS

**22.** Match the Column:

| | Column-I | | Column-II |
|---|---|---|---|
| A. | The value(s) of $x$, which does not satisfy the equation $\log_2^2(x^2-x)-4\log_2(x-1)\log_2 x=1$, is (are) | p. | 2 |
| B. | The value of $x$ satisfying the equation $2^{\log_2 e^{\ln 5^{\log_5 7^{\log_7 10^{\log_{10}(8x-3)}}}}}=13$, is | q. | 3 |
| C. | The number $N=\left(\dfrac{1}{\log_2\pi}+\dfrac{1}{\log_6\pi}\right)$ is less than | r. | 4 |
| D. | Let $l = (\log_3 4 + \log_2 9)^2 - (\log_3 4 - \log_2 9)^2$ and $m = (0.8)\left(1+9^{\log_3 8}\right)^{\log_{65} 5}$ then $(l + m)$ is divisible by | s. | 5 |
| | | t. | 6 |

(*a*) A → r, t, s; B → q; C → r, s, q; D → q, r

(*b*) A → q, r, s, t; B → p; C → q, r; D → r, s

(*c*) A → q, r, s, t; B → p; C → q, r, s, t; D → p, r, s

(*d*) A → t; B → s; C → q, t; D → r, s

**23.** Match the column:

| | Column-I | | Column-II |
|---|---|---|---|
| A. | $\log_{\sin 30^\circ}(\cos 60^\circ)+1$ | p. | 3 |
| B. | $\log_{4/3}(1.\overline{3})+3$ | q. | 5 |
| C. | $\log_{2-\sqrt{3}}(2+\sqrt{3})+6$ | r. | 4 |
| D. | $\log_{\tan 20^\circ}\tan 70^\circ+4$ | s. | 2 |
| E. | $\log_{\cot 40^\circ}\tan 50^\circ$ | t. | 0 |
| F. | $\log_{0.125}(8)+8$ | u. | –1 |
| G. | $\log_{1.5}(0.\overline{6})+9$ | v. | 8 |
| H. | $\log_{2.25}(0.\overline{4})$ | w. | 7 |
| I. | $\log_{10}(0.\overline{9})$ | x. | 1 |

(*a*) A → q, B → p, C → s, D → v, E → u, F → u, G → q, H → w, I → x

(*b*) A → s, B → r, C → q, D → p, E → x, F → w, G → v, H → u, I → t

(*c*) A → s, B → v, C → t, D → p, E → t, F → u, G → w, H → x, I → w

(*d*) A → q, B → s, C → r, D → v, E → u, F → v, G → v, H → w, I → x

**24.** Match the following columns:

| Column-I | | Column-II | |
|---|---|---|---|
| A. | If $a = 3\left(\sqrt{8+2\sqrt{7}} - \sqrt{8-2\sqrt{7}}\right)$, $b = \sqrt{(42)(30)+36}$ then the value of $\log_a b$ is equal to | p. | –1 |
| B. | If $a = \sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}}$, $b = \sqrt{(42)(30)+36}$ then the value of $\log_a b$ is equal to | q. | 1 |
| C. | If $a = \sqrt{3+2\sqrt{2}}, b = \sqrt{3-2\sqrt{2}}$ then the value of $\log_a b$ is equal to | r. | 2 |
| D. | If $a = \sqrt{7+\sqrt{7^2-1}}, b = \sqrt{7-\sqrt{7^2-1}}$, then the value of $\log_a b$ is equal to | s. | $2 + 2\log_2 3$ |

(*a*) A → s, B → p, C → q, D → p

(*b*) A → r, B → p, C → r, D → p

(*c*) A → r, B → s, C → p, D → p

(*d*) A → p, B → q, C → p, D → r

**25.** Match the columns:

| Column-I | | Column-II | |
|---|---|---|---|
| A. | If $p = \dfrac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}$ then $\log_{(5+2\sqrt{6})} p$ is | p. | 0 |
| B. | If $r = \dfrac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ then $\log_{9+2\sqrt{15}}(1/r)$ is | q. | 2 |
| C. | If $t = \dfrac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}}$ then $\log_{\sqrt{3}} t^2$ is | r. | –1 |
| D. | If $k = \dfrac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \dfrac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \dfrac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$ then $\log_e(k+1)$ is | s. | 1 |

(*a*) A → t, B → s, C → r, D → q

(*b*) A → s, B → r, C → q, D → p

(*c*) A → r, B → p; C → r, D → s

(*d*) A → t, B → q, C → s, D → r

**26.** Match the values of $x$ given in Column-II satisfying the exponential equation given in Column-I (Do not verify). Remember that for $a > 0$, the terms $a^x$ is always greater than zero $\forall\, x \in R$.

| Column-I | | Column-II | |
|---|---|---|---|
| A. | $5^x - 24 = \dfrac{25}{5^x}$ | p. | –3 |
| B. | $(2^{x+1})(5^x) = 200$ | q. | –2 |
| C. | $4^{2/x} - 5(4^{1/x}) + 4 = 0$ | r. | –1 |
| D. | $2^{2x+1} - 33(2^{x-1}) + 4 = 0$ | s. | 0 |
| E. | $\dfrac{2^{x-1}\cdot 4^{x+1}}{8^{x-1}} = 16$ | t. | 1 |
| F. | $3^{2x+1} + 10\,(3^x) + 3 = 0$ | u. | 2 |
| G. | $64(9^x) - 84(12^x) + 27(16^x) = 0$ | v. | 3 |
| H. | $5^{2x} - 7^x - 5^{2x}(35) + 7^x(35) = 0$ | w. | None |

(*a*) A → u, B → u, C → t, D → q, v, E → p,q,r,s,t,u,v F → w, G → t,u, H → s

(*b*) A → q, B → t, C → u, D → v, E →t,u,v F → w, G → t,H → q

(*c*) A → p, B → u, C → s, D → q, E → p,q, F → s, G → u, H → q

(*d*) A → q, B → s, C → r, D → s, E →q,r, F → q, G → u, H → q

## INTEGER TYPE QUESTIONS

**27.** Find the number of integral solution of the equation $\log_{\sqrt{x}}\left(x + |x-2|\right) = \log_x(5x - 6 + 5|x-2|)$.

**28.** If $a$, $b$ are co-prime numbers and satisfying $\left(2+\sqrt{3}\right)^{\frac{1}{\log_a(2-\sqrt{3})} + \frac{1}{\log_b\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)}} = \dfrac{1}{12}$, then $(a + b)$ is equal to

**29.** The sum of all integral values of $x$ satisfying the equation $2\log_8(2x) + \log_8(x^2 - 2x + 1) = \dfrac{4}{3}$ is.

**30.** If the complete solution set of the inequality $(\log_{10}x)^2 \geq \log_{10}x + 2$ is $(0, a] \cup \left[\dfrac{1}{a^2}, \infty\right)$ then find the value of $10a$.

**31.** If complete solution set of inequality $\log_{1/2}(x+5)^2 > \log_{1/2}(3x-1)^2$ is $(-\infty, p) \cup (q, r) \cup (s, \infty)$ then find $3\left(\dfrac{p^2+q^2+r^2}{s^2}\right)$

**32.** Solve the equation $x^{0.5\log_{\sqrt{x}}(x^2-x)} = 3^{\log_9 4}$.

**33.** If the solution set of $(0.3)^{\log_{\frac{1}{3}}\log_2\frac{3x+6}{x^2+2}} > 1$ is $\left(\dfrac{-1}{\alpha}, \alpha\right)$ then $\alpha =$

**34.** If the solution set of $\log_{0.5}\left(\log_6\dfrac{x^2+x}{x+4}\right) < 0$ is $(\alpha, \beta) \cup (-2\alpha, \infty)$ then $-\alpha + \beta =$

**35.** If the solution set of $\log_3 \frac{|x^2-4x|+3}{x^2+|x-5|} \geq 0$ is $\left(-\infty, -\frac{\alpha}{\beta}\right] \cup \left[\frac{1}{\alpha}, \alpha\right]$, then $\alpha\beta =$

**36.** For the equation

$(0.4)^{\log^2 x+1} = (6.25)^{2-p\log x}$

(base 10)

If $p = 2$, let number of real roots be $m$,

If $p = 3$, let number of real roots be $n$,

Then $m + n =$

**37.** If $p$ is the smallest value of $x$ satisfying the equation $2^x + \frac{15}{2^x} = 8$ then the value of $4^p$ is equal to

**38.** Positive numbers $x$, $y$ and $z$ satisfy $xyz = 10^{81}$ and $(\log_{10}x)(\log_{10}yz) + (\log_{10}y)(\log_{10}z) = 468$.

Find the value of

$(\log_{10} x)^2 + (\log_{10} y)^2 + (\log_{10} z)^2$

**39.** If $(x_1, y_1)$ and $(x_2, y_2)$ are the solution of the system of equation

$\log_{225}(x) + \log_{64}(y) = 4$

$\log_x(225) - \log_y(64) = 1$,

then find the value of $\log_{30}(x_1y_1x_2y_2)$.

**40.** Suppose n be an integer greater than 1. Let $a_n = \frac{1}{\log_n 2002}$.

Suppose $b = a_2 + a_3 + a_4 + a_5$ and $c = a_{10} + a_{11} + a_{12} + a_{13} + a_{14}$. Then find the value of $(c - b)$.

**41.** If $\log_b a . \log_c a + \log_a b . \log_c b + \log_a c . \log_b c = 3$ (where $a, b, c$ are different positive real numbers $\neq 1$), then find the value of $a\,b\,c$.

**42.** If the product of alll solutions of the equation

$\frac{(2009)x}{2010} = (2009)^{\log_x(2010)}$ can be expressed in the lowest form as $\frac{m}{n}$ then the value of $(m - n)$ is

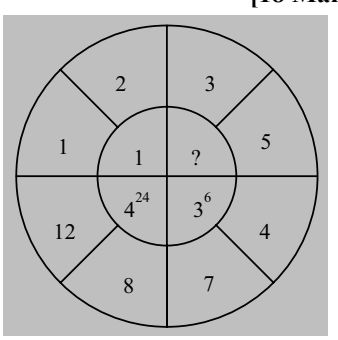
# PYQ's (Past Year Questions)

## INEQUALITIES

**1.** Let the point $(p, p + 1)$ lie inside the region

$E = \left\{(x, y) : 3 - x \leq y \leq \sqrt{9 - x^2}, 0 \leq x \leq 3\right\}$. If the set of all values of $p$ is the interval $(a, b)$. then $b^2 + b - a^2$ is equal to _____ **[6 April, 2023 (Shift-I)]**

**2.** The missing value in the following figure is **[18 Mar, 2021 (Shift-I)]**




Use the logic which gives answer in single digit.

**3.** The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is **[10 April, 2019 (Shift-II)]**

(*a*) 3 (*b*) 2 (*c*) 4 (*d*) 1

## LOGARITHM

**4.** The number of integral solutions $x$ of

$\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$ is **[11 April, 2023 (Shift-I)]**

(*a*) 6 (*b*) 8 (*c*) 5 (*d*) 7

**5.** If the sum of all the roots of the equation $e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$ is $\log_e p$, then $p$ is equal to _____ . **[27 June, 2022 (Shift-I)]**

**6.** The number of solutions of the equation $\log_4(x - 1) = \log_2(x - 3)$ is **[26 Feb, 2021 (Shift-I)]**

**7.** If for $x \in \left(0, \frac{\pi}{2}\right)$ $\log_{10}\sin x + \log_{10}\cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1), n > 0$, then the value of $n$ is equal to: **[16 March, 2021 (Shift-I)]**

(*a*) 16 (*b*) 12 (*c*) 9 (*d*) 20

**8.** The inverse of $y = 5^{\log x}$ is: **[17 March, 2021 (Shift-I)]**

(*a*) $x = y^{\log 5}$ (*b*) $x = y^{\frac{1}{\log 5}}$

(*c*) $x = e^{\log_5 y}$ (*d*) $x = 5^{\frac{1}{\log y}}$

**9.** The sum of the roots of the equation, **[31 Aug, 2021 [Shift-II]**

$x + 1 - 2\log_2\left(3 + 2^x\right) + 2\log_4\left(10 - 2^{-x}\right) = 0$, is:

(*a*) $\log_2 12$ (*b*) $\log_2 13$

(*c*) $\log_2 11$ (*d*) $\log_2 14$

**10.** The number of solutions of the equation $\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x+1)^2 - 4 = 0, x > 0$, is. **[20 July, 2021 (Shift-II)]**

**11.** The number of distinct solutions of the equation, $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|$ in the interval $[0, 2\pi]$, is _____
**[9 Jan, 2020 (Shift-I)]**

**12.** Let $m$ be the minimum possible value of $\log_3(3^{y_1} + 3^{y_2} + 3^{y_3})$, where $y_1, y_2, y_3$ are real numbers for which $y_1 + y_2 + y_3 = 9$. Let $M$ be the maximum possible value of $(\log_3 x_1 + \log_3 x_2 + \log_3 x_3)$, where $x_1, x_2, x_3$ are positive real numbers for which $x_1 + x_2 + x_3 = 9$. Then the value of $\log_2(m^3) + \log_3(M^2)$ is _____.
**[JEE Adv, 2020]**

# PW Challengers

**1.** If $\log_4(x + 2y) + \log_4(x - 2y) = 1$, then the minimum value of $|x| - |y|$ is ____.

**2.** Let $a,b,c,d$ be positive integers and $\log_a b = \frac{3}{2}, \log_c d = \frac{5}{4}$. If $a - c = 9$, then $b - d =$

**3.** Let $x \in N$ such that $2^{1+[\log_2(x-2)]} - x = 20$. ([ · ] is G.I.F.) The smallest value of $x$, is

**4.** If $\sqrt{4+\sqrt{8-\sqrt{32+\sqrt{768}}}} = a\sqrt{2}\cos\left(\frac{11\pi}{b}\right)$, where $a$ and $b$ are natural numbers then find $a + b$.

**5.** Let $r_1, r_2, r_3 \ldots r_n$ be n positive integers, not necessarily distinct, such that $(x + r_1)(x + r_2)(x + r_1)\ldots(x + r_n) = x^n + 56x^{n-1} \ldots + 2009$ then the value of $n$ is equal to

**6.** If $(a + 1)(b + 1)(c + 1)(d + 1) = 1$
$(a + 2)(b + 2)(c + 2)(d + 2) = 2$
$(a + 3)(b + 3)(c + 3)(d + 3) = 3$
$(a + 4)(b + 4)(c + 4)(d + 4) = 4$
Then the value of $(a + 5)(b + 5)(c + 5)(d + 5)$ is equal to.

**7.** Find sum of all possible natural numbers '$n$' for which $\frac{5n^2 - 7n + 84}{n}$ is divisible by 5.

**8.** The value of
$$\left[2008 + \log_{\left(\frac{6561}{256}\right)}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots}}}\right)\right] \text{ is}$$
(where [·] is G.I.F.)

**9.** Let $a$, $b$ and $c$ be distinct non zero real numbers such that $\frac{1-a^3}{a} = \frac{1-b^3}{b} = \frac{1-c^3}{c}$. The value of $10(a^3 + b^3 + c^3)$, is

**10.** Match the Column:

| | Column-I | | Column-II |
|---|---|---|---|
| A. | Number of integral pair of the form $(x, y)$ satisfying $\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}} = \frac{1}{\sqrt{20}}$ is/are equal to | p. | 16 |
| B. | Number of positive integral solutions of the equation $3x + 5y = 1008$ is/are equal to | q. | 2 |
| C. | Number of integers $n$ such that $\sqrt{\frac{3n-5}{n+1}}$ is also an integer, is/are equal to | r. | 0 |
| D. | Number of integers $n$ (positive, negative or 0) such that $n^2 + 73$ is divisible by $(n + 73)$, is/are equal to | s. | 67 |
| | | t. | 3 |

(*a*) A → t; B → s; C → q; D → p
(*b*) A → r; B → p; C → q; D → s
(*c*) A → q; B → p; C → r; D → s
(*d*) A → s; B → p; C → q; D → p

**11.** $\sqrt[3]{20+14\sqrt{2}} + \sqrt[3]{20-14\sqrt{2}} = a$ then find the absolute value of $a - 2023$.

**12.** If the sum of all real numbers $x$ and $y$ such that the following system of inequalities holds:
$$\left\{\begin{array}{l} 4^{-x} + 27^{-y} = \frac{5}{6} \\ \log_{27} y - \log_4 x \geq \frac{1}{6} \\ 27^y - 4^x \leq 1 \end{array}\right\}$$ is k then find 6 $k$.

**13.** Solve the inequality $|\log_2 x - 3| + |2^x - 8| \geq 9$.
(*a*) $x \in (0,1] \cup [4, \infty]$ (*b*) $x \in (0,2] \cup [4, \infty]$
(*c*) $x \in (0,1] \cup [3, \infty]$ (*d*) $x \in (0,1] \cup [2, \infty]$

**14.** Solve the inequality
$\log_2(x^{12} + 3x^{10} + 5x^8 + 3x^6 + 1) < 1 + \log_2(x^4 + 1)$
(*a*) $x \in \left(-\sqrt{\frac{-1+\sqrt{6}}{2}}, \sqrt{\frac{-1+\sqrt{5}}{2}}\right)$
(*b*) $x \in \left(-\sqrt{\frac{-1+\sqrt{5}}{2}}, \sqrt{\frac{-1+\sqrt{5}}{2}}\right)$
(*c*) $x \in \left(-\sqrt{\frac{-1+\sqrt{6}}{2}}, \sqrt{\frac{-1+\sqrt{6}}{2}}\right)$
(*d*) $x \in \left(-\sqrt{\frac{-1+\sqrt{7}}{2}}, \sqrt{\frac{-1+\sqrt{7}}{2}}\right)$

**15.** For what values of $a$, the inequality (for $x$)
$\log_{\frac{1}{a}}\left(\sqrt{x^2 + ax + 5} + 1\right) \cdot \log_5\left(x^2 + ax + 6\right) + \log_a 3 \geq 0$
has exactly one solution?

# ANSWER KEY

## CONCEPT APPLICATION

**1.** (*a*) **2.** (*a*) **3.** (*d*) **4.** (*c*) **5.** (*d*) **6.** (*i*) 14 (*ii*) 52 (*iii*) 194 **9.** [0] **10.** [–224]

**11.** $x^6 - y^6$ **12.** $p = 3/2, q = 1, r = 4/3$ **13.** (*d*) **14.** (*c*) **15.** (*b*) **16.** [60] **17.** [99] **18.** [4]

**19.** (*d*) **20.** [*2ab*] **21.** $\phi$ **22.** $x = \pm\sqrt{\frac{3}{5}}$ **23.** (*c*) **24.** $x \in (-\infty, -3) \cup [-2, 0] \cup [6, \infty)$

**25.** $x \in (-6, 0] \cup [2, 3] \cup (6, \infty) \cup \{4\}$ **26.** [6] **27.** (*d*) **28.** [1, –1] **29.** [$x \in (\sin 4, \sin 3) \cup [\sin 1, \sin 2]$]

**30.** $x \in [1, 2]$ **31.** {9} **32.** $x \in (2, 3]$

**33.** $\left(\frac{1}{5}, \frac{2}{5}\right)$ **34.** (243) **35.** [–5] **36.** [18]

**37.** [–1] **38.** (*i*) $(1, \infty)$ (*ii*) $[1, \infty)$ (*iii*) (0, 1) (*iv*) (0, 1] (*v*) (0, 1) (*vi*) (0, 1] (*vii*) $(1, \infty)$ (*viii*) $[1, \infty)$ (*ix*) $(3, \infty)$ (*x*) $[5/2, \infty)$

**39.** (*i*) $x \in \left(\frac{3}{2}, \frac{19}{2}\right)$ (*ii*) $x \in \left(\frac{2}{3}, \frac{17}{24}\right]$ (*iii*) $x \in (4^{16}, \infty)$ (*iv*) $x \in \left(0, \frac{1}{2}\right)$ **40.** {3, –3} **41.** {–2, 2} **43.** $\frac{12 - 4a}{3 + a}$

**44.** (*i*) $\left\{2^{\pm\sqrt{2}}\right\}$ (*ii*) $x = a^{-\log_5 2}$ (*iii*) {1/32, 2} (*iv*) {1} (*v*) {1}

**47.** [–1] **48.** [44] **49.** (*d*) **50.** (*b*, *c*)

**51.** (*i*) {–1, 5} (*ii*) {–3, –1, 7, 9} (*iii*) (14, –4, 0, 10, 2, 8) **52.** (–6, 8) **53.** (*d*) **54.** $(0, 2) \cup (4, \infty)$ **55.** $x \geq 1$

**56.** (*i*) $x \in (-\infty, 1) \cup (1, \infty)$ (*ii*) $x \in (-\infty, -5] \cup [5, \infty)$ (*iii*) $x \in (-7, 7)$ (*iv*) $x \in [-10, 10]$ (*v*) $x \in R$ (*vi*) $x \in \phi$ (*vii*) $x \in R$ (*viii*) $x \in R$ (*ix*) $x \in \phi$

**57.** (*i*) $x \in (-\infty, 0) \cup (2, \infty)$ (*ii*) $1 < x < 3$ (*iii*) $x \in (-2, -1) \cup (0, 1)$ (*iv*) $x \in [-1, 0] \cup [1, 2]$ (*v*) $-4/3 \leq x \leq 2$ (*vi*) $x \in \phi$

**58.** (*i*) $-1 \leq x \leq 5$ (*ii*) $x \in (-\infty, -3] \cup [9, \infty) \cup [-1, 7]$ (*iii*) $x \in [-4, 0] \cup [2, 8] \cup [10, 14]$

**59.** (*i*) $x \in \left[\frac{1}{5}, \frac{1}{4}\right] \cup \left[\frac{3}{4}, \frac{4}{5}\right]$ (*ii*) $x \in \left[\frac{2}{3}, 2\right]$ (*iii*) $x \in \left[\frac{2}{15}, \frac{2}{5}\right] \cup \left[\frac{6}{5}, \frac{22}{15}\right]$ (*iv*) $x \in (-3, -2) \cup (2, 3)$

**60.** (*i*) $x \in \{-11, -5, -1\}$ (*ii*) $x \in \{-8, -6, -2, -4, 0, 2, 4, 6, 8, 10, 12, 14, 16, 18\}$

**61.** (*i*) $x \in [1, 13]$ (*ii*) $x \in [-8, -6] \cup [-4, 0] \cup [2, 4] \cup [6, 8] \cup [10, 14] \cup [16, 18]$

**62.** (*i*) $x \in (-2, -1] \cup [1, 2)$ (*ii*) $x \in (-\infty, -3] \cup (-2, -1] \cup [3, \infty) \cup [1, 2)$ (*iii*) $x \in (-\infty, -3] \cup [3, \infty)$
(*iv*) $x \in (-\infty, -4] \cup [-1, 5] \cup [6, \infty)$ (*v*) $x \in (-\infty, -4] \cup [6, \infty] \cup \{2\}$

**63.** (*i*) $x \in [1, \infty]$ (*ii*) $x \in (0, \infty) \cup \{-1\}$ (*iii*) $x \in (-1, 0) \cup (0, 3)$ (*iv*) $x \in \phi$ (*v*) $x \in (2, 6)$

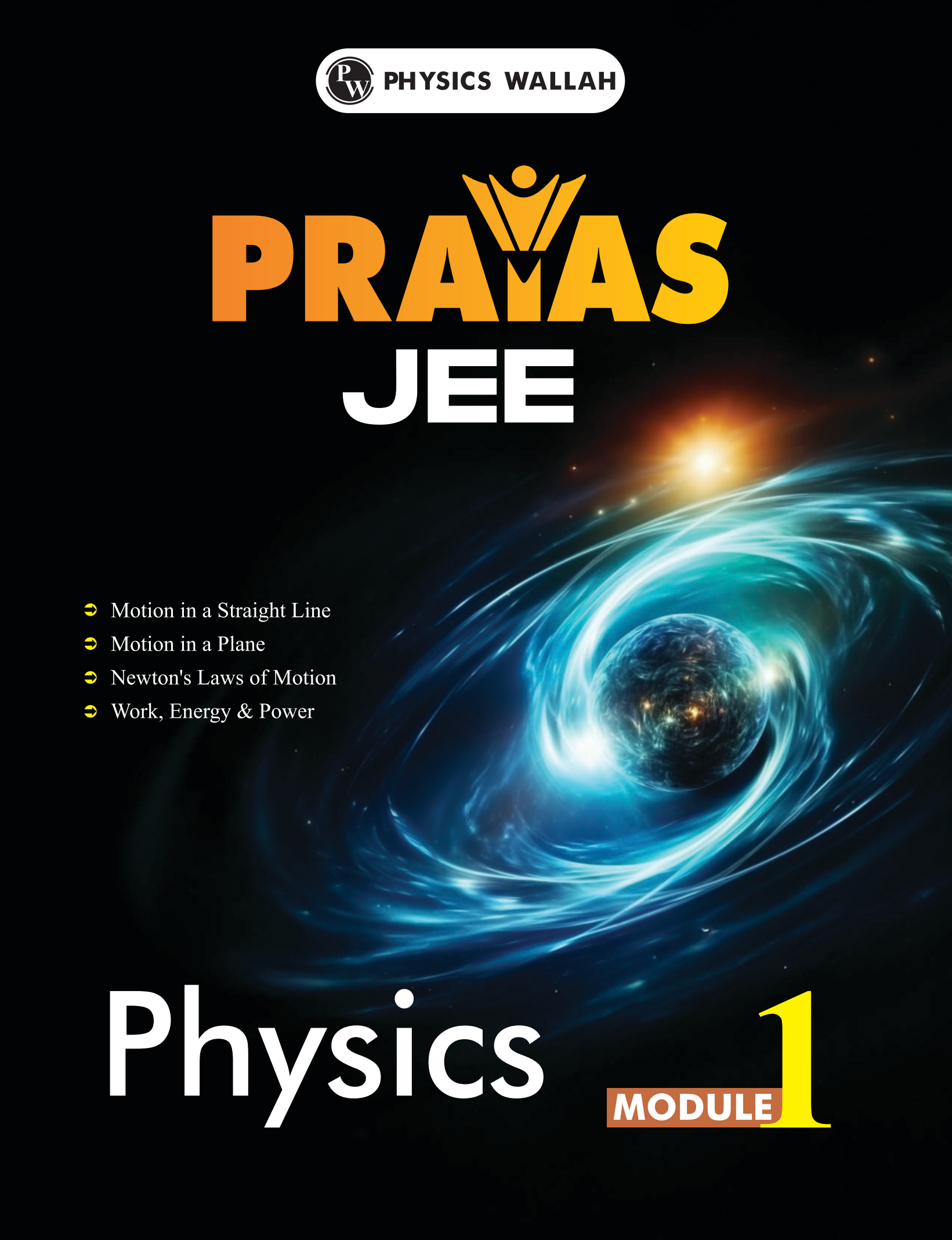
**64.** $[-6, \infty)$

## PRARAMBH (TOPICWISE)

**1.** (*a*) **2.** (*c*) **3.** (*b*) **4.** (*b*) **5.** (*a*) **6.** (*d*) **7.** (*d*) **8.** (*c*) **9.** (*d*) **10.** (*c*)
**11.** (*b*) **12.** (*d*) **13.** (*c*) **14.** (*c*) **15.** (*d*) **16.** (*b*) **17.** (*d*) **18.** (*b*) **19.** (*a*) **20.** (*b*)
**21.** (*c*) **22.** (*a*) **23.** (*b*) **24.** (*d*) **25.** (*b*) **26.** (*a*) **27.** (*d*) **28.** (*c*) **29.** (*b*) **30.** (*d*)
**31.** (*b*) **32.** (*d*)

PHYSICS WALLAH

# PRAYAS JEE



- Motion in a Straight Line
- Motion in a Plane
- Newton's Laws of Motion
- Work, Energy & Power

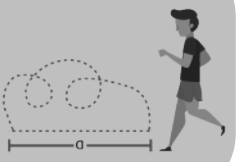
# Physics

MODULE 1

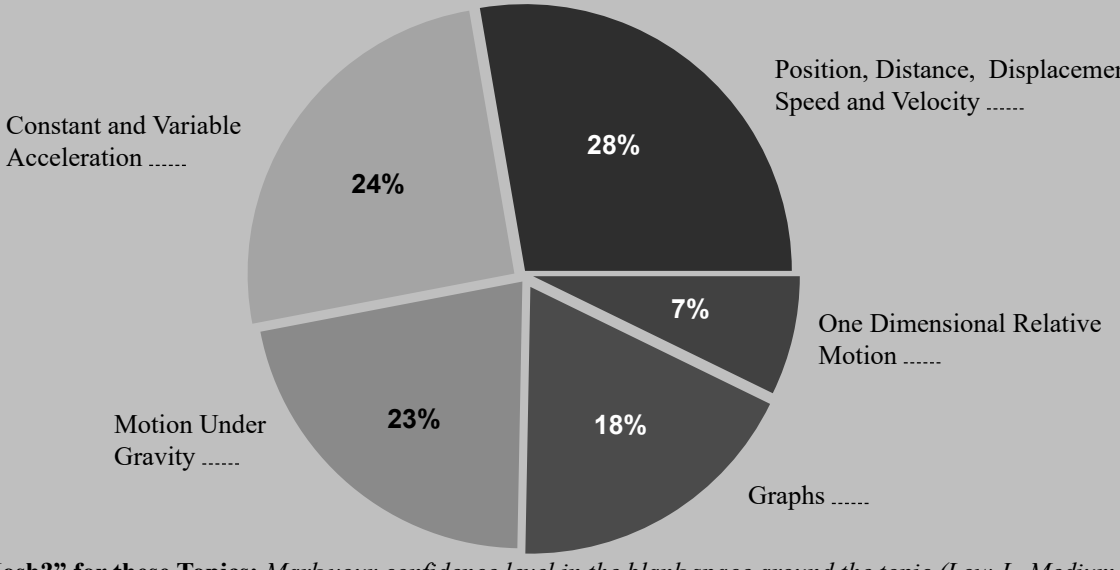
CHAPTER

# 1

# Motion in a Straight Line



## Topicwise Weightage of JEE Main 6 Years Paper (124 Sets)




**"How's the Josh?" for these Topics:** *Mark your confidence level in the blank space around the topic (Low-L, Medium-M, High-H)*

## INTRODUCTION

A body is at rest when it does not change its position with time and is in motion if it changes its position with time in the frame of reference of the observer.

All motion is relative. There is no meaning of rest or motion without reference to the observer.

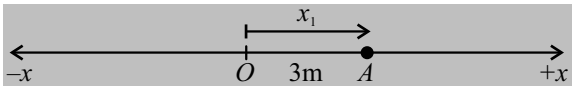
A passenger in a moving train is at rest with respect to another passenger in the same train while both are in motion with respect to observer on the ground. Therefore nothing is at absolute rest or in absolute motion.

To describe the motion of a particle, we introduce four important quantities namely position, displacement, velocity and acceleration. In general motion of a particle in three dimensions these quantities are vectors which have direction as well as magnitude. But for a particle moving in a straight line, there are only two directions, distinguished by designating one as positive and other as negative.

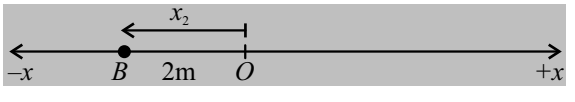
## DISTANCE AND DISPLACEMENT

### Position

The position of a particle is the location of particle measured with respect to some reference point. It is a vector quantity.




Position of $A$ wrt $O$, $x_1 = 3$ m




Position of $B$ wrt $O$, $x_2 = -2$ m

If particle lies towards +ve side of the chosen reference, then its position is also +ve and vice-versa.

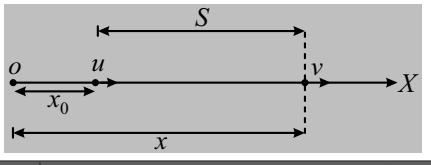
$$\int_u^v v\,dv = a\int_{x_0}^{x} dx$$

$$\frac{v^2}{2}\Big|_u^v = a\,x\Big|_{x_0}^{x}$$

$$\frac{v^2}{2} - \frac{u^2}{2} = a(x - x_0)$$

$$\Rightarrow v^2 - u^2 = 2aS \qquad ...(iii)$$

These relation are very helpful in solving the problems of motion in one dimension. All these relations are given in table below for easy reference.




| Equation | Contains | | |
|---|---|---|---|
| | *s* | *v* | *t* |
| $v = u + at$ | No | Yes | Yes |
| $s = ut + \frac{1}{2}at^2$ | Yes | No | Yes |
| $v^2 - u^2 = 2as$ | Yes | Yes | No |

In simple problems of uniformly accelerated motion, two parameters are given and third is to be found. Depending on convenience one can choose any one of the three relations. The following two relations are also helpful in solving problems.

**Displacement of the Body in the $n^{th}$ Second:**

$$S_n = S(\text{at } t = n) - S(\text{at } t = n-1)$$

$$= \left(un + \frac{1}{2}an^2\right) - \left(u(n-1) + \frac{1}{2}a(n-1)^2\right) = u + \frac{a}{2}(2n-1)$$

**Average velocity:**

$$V_{avg} = \frac{S}{t} = \frac{ut + \frac{1}{2}at^2}{t} = u + \frac{at}{2}$$

$$= u + \frac{v-u}{2} = \frac{u+v}{2}$$

or $$S = \left(\frac{u+v}{2}\right)t$$

This relation is only valid for uniform acceleration.

**Note:**

- These equations can be applied only when acceleration is constant.
- If a body moves with uniform acceleration and velocity changes from $u$ to $v$ in a time interval, then average velocity = $\frac{v+u}{2}$.
- If a body moving with uniform acceleration has velocities $u$ and $v$ at two points in its path, then the velocity at the midpoint of given two points = $\sqrt{\frac{u^2+v^2}{2}}$.
- In position time graph, slope is equal to velocity.
- In velocity time graph area under the curve is displacement and slope is equal to acceleration.
- In acceleration time graph area under the curve is equal to change in velocity.
- **For a body starting from rest and moving with uniform acceleration,**
  - (*a*) The ratio of distances covered in first one sec, two sec, three sec, ... is :
    $1^2 : 2^2 : 3^2 : ...$ , i.e., 1 : 4 : 9 : ...
    Ratio of distances covered in
    1st, 2nd, 3rd sec, ... is 1 : 3 : 5 : ...
  - (*b*) The ratio of velocities after
    1 sec, 2 sec, 3 sec, ... is 1 : 2 : 3 : ...

## Train Your Brain

**Example 4:** The displacement of a particle, moving in a straight line, is given by $S = 2t^2 + 2t + 4$ where $S$ is in metres and $t$ in seconds. The acceleration of the particle is

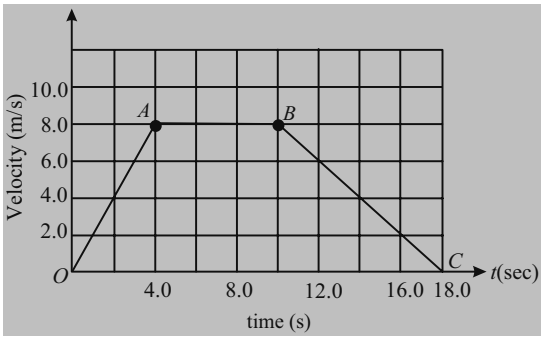
(*a*) 2 m/s$^2$ (*b*) 4 m/s$^2$
(*c*) 6 m/s$^2$ (*d*) 8 m/s$^2$

**Sol.** Given $S = 2t^2 + 2t + 4$

$\therefore$ Velocity $(v) = \frac{dS}{dt} = 4t + 2$

Acceleration $(a) = \frac{dv}{dt} = 4(1) + 0 = 4\text{m/s}^2$

**Example 5:** What is the acceleration for each graph segment in figure? Describe the motion of the object over the total time interval. Also calculate displacement.




**Sol.** Segment $OA$; $a = \frac{8-0}{4-0} = 2$ m/s$^2$

Segment $AB$; graph horizontal i.e., slope zero i.e., $a = 0$

Segment $BC$; $a = \frac{0-8}{18-10} = -1$ m/s$^2$

The graph is trapezium. Its area between $t = 0$ to $t = 18$s is displacement.

Area of $v$-$t$ graph = displacement = $\frac{1}{2}(18+6) \times 8 = 96$ m

Particle accelerates uniformly for first 4 sec., then moves with uniform velocity for next 6 sec. and then retards uniformly to come to rest in next 8 sec.

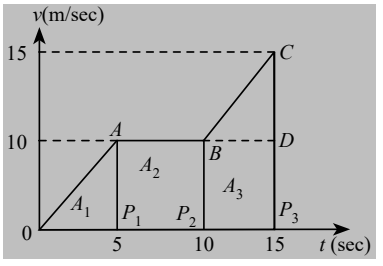
**Example 6:** The motion of a body is described in ($v$–$t$) graph as given under.

Find the following:

($a$) Max and Min acceleration

($b$) Displacement from $t = 10$ to $t = 15$

($c$) Average velocity for the whole journey.




**Sol.** ($a$) We know slope of ($v - t$) graph gives acceleration

$$\text{Slope}_{OA} = \frac{AP_1}{OP_1} = \frac{10}{5} = 2\ \text{m/sec}^2 \text{ (Max-acceleration)}$$

$$\text{Slope}_{AB} = 0\ \text{m/sec}^2 \qquad (\text{min}-\text{acceleration})$$

$$\text{Slope}_{BC} = \frac{CD}{BD} = \frac{5}{5} = 1\ \text{m/sec}^2$$

($b$) Displacement = Area ($v - t$)graph from $t = 10$ to $t = 15$ sec

$$= \frac{1}{2}(10+15)\times 5 = 62.5\ \text{m}$$

($c$) $$\text{Average Velocity} = \frac{\text{Total Displacement}}{\text{Total Time}} = \frac{\text{Area } (v-t)\text{graph}}{t_{total}}$$

$$= \frac{A_1 + A_2 + A_3}{t_{total}} = \frac{25+50+62.5}{15}$$

$$= \frac{137.5}{15} = 9.17\ \text{m/sec}$$

**Example 7:** How long does it take for a particle to travel 100 m if it begins from rest and accelerates at 10 m/s$^2$? What is its velocity when it has travelled 100 m? What is the average velocity during this time?

**Sol.** $u = 0$, $a = 10$ m/s$^2$, $S = 100$ m

Applying $S = ut + \frac{1}{2}at^2$

we get $100 = \frac{1}{2}\times 10 \times t^2$

$\Rightarrow t = \sqrt{20} = 2\sqrt{5}$ s

$v = u + at = 0 + 10 \times 2\sqrt{5} = 20\sqrt{5}$ m/s

$$v_{avg} = \frac{u+v}{2} = \frac{0+20\sqrt{5}}{2} = 10\sqrt{5}\ \text{m/s}$$

**Example 8:** A car travelling with 72 km/hr is 30 m from a barrier when the driver slams the breaks. The car hits barrier 2.0 seconds later.

($a$) What is the car's constant deceleration before impact?

($b$) How fast is car travelling at impact?

**Sol.** ($a$) $u = 72$ km/hr $= 72 \times \frac{5}{18}$ m/s $= 20$ m/s

$S = 30$ m

$t = 2$s

$a = ?$

$$S = ut + \frac{1}{2}at^2$$

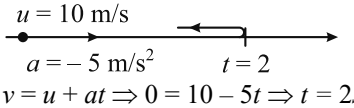
$$30 = 20 \times 2 + \frac{1}{2}\times a \times 2^2$$

$$\Rightarrow a = -\frac{10}{2} = -5\ \text{m/s}^2$$

($b$) $v = u + at = 20 + (-5) \times 2 = 10$ m/s

**Example 9:** A particle moving with initial velocity of 10 m/s towards East has an acceleration of 5 m/s$^2$ towards west. Find the displacement and distance travelled by the particle in first 4 seconds?

**Sol.**




$v = u + at \Rightarrow 0 = 10 - 5t \Rightarrow t = 2s$

The direction of velocity changes after two seconds.

$$S = 10 \times 4 + \frac{1}{2}(-5)\times 4^2 = 0 = \text{displacement}$$

Distance travelled is not equal to displacement because during course of journey, velocity changes direction.

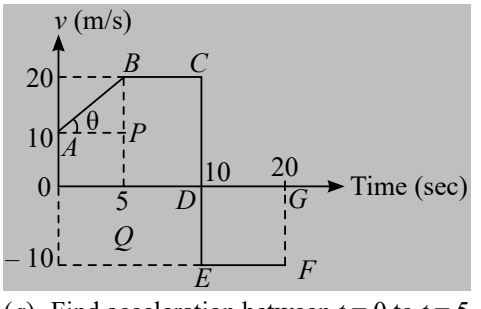
$D = S(\text{at } 2s) + |S(\text{at } 4s) - S(\text{at } 2s)|$

$$= \left(10\times 2 - \frac{1}{2}\times 5\times 2^2\right) + \left|0 - (10\times 2) - \frac{1}{2}\times 5\times 2^2\right|$$

$= 10 + 10 = 20$ m

## Concept Application

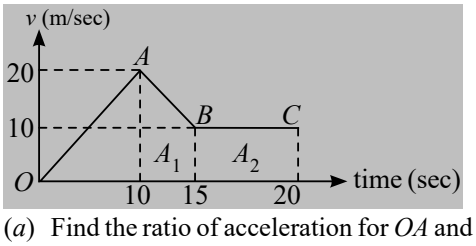
**3.**




($a$) Find acceleration between $t = 0$ to $t = 5$

($b$) Find total displacement between $t = 0$ to $t = 20$ sec.

**4.**




($a$) Find the ratio of acceleration for $OA$ and $AB$ in the graph shown.

($b$) Total distance covered between 10 to 20 sec.

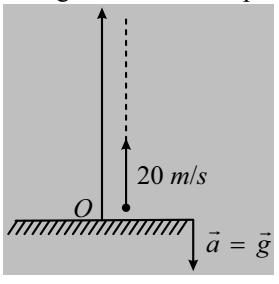
**1.** A stone is thrown vertical upwards from ground level with $u = 20$ m/s.

($a$) Find the maximum height attained by the stone.

($b$) time interval $t$ after which it returns to the point of projection.

($c$) The velocity with which it strikes the ground.

**Sol.** Let us choose our origin $O$ at the point of projection with +ve $X$-axis pointing in the vertical upwards direction.




Note that in this coordinate system, acceleration due to gravity is negative because it points in the downward direction.

Thus $a = -9.8$ m/s$^2$, $u = 20$ m/s

($a$) At the highest point, velocity of the particle will become zero. Let h be the maximum height.

Thus $S = h$.

Using the relation,

$v^2 - u^2 = 2a\,S$

we get $0 - 20^2 = 2 \times (-9.8) \times h$

$$\Rightarrow h = \frac{400}{19.6} = 20.4\,\text{m}$$

Therefore, 20.4 m is the correct answer.

($b$) $S = 0 = 20\text{t} - \frac{1}{2}(9.8)t^2$

$$\Rightarrow t = \frac{40}{9.8} = 4.08\ \text{sec}$$

Therefore, 4.08 s is the correct answer.

($c$) Since, the particle returns to the initial position, $S = 0$.

$$\Rightarrow v = 20 - 9.8 \times \frac{40}{9.8}$$ (we know $t$ from part ($b$))

$= -20$ m/s

Here, minus sign indicates that particle moves in the downward direction.

Therefore, –20 m/s is the correct answer.

**Note:** It returns with same speed with which it was thrown.

**2.** A body is thrown down from the top of a tower of height $h$ with velocity 10 m/s. Simultaneously, another body is projected upward from bottom. They meet at a height $2h/3$ from the ground level. If $h = 60$ m, find the initial velocity of the lower body.

($a$) 19.23 m/s ($b$) 38.27 m/s

($c$) 55.16 m/s ($d$) None of these

**Sol.** Let us choose the origin at the ground level with +ve $y$-axis pointing in the upward direction.

Let us refer lower and upper body as 1 and 2 respectively.

Then,

$a = -g$

$x_{1_i} = 0, x_{1_f} = 2h/3, u_1 = ?$

$x_{2_i} = h, x_{2_f} = 2h/3, u_2 = -10$ m/s

From eqn of motion , we have

$$x_{1_f} = 0 + u_1 t - \frac{1}{2} \times 9.8 \times t^2 \qquad \text{...(i)}$$

$$x_{2_f} = h - 10t - \frac{1}{2} \times 9.8 \times t^2 \qquad \text{...(ii)}$$

But , $x_{1_f} = x_{2_f} = 2h/3$

Hence, equating eqn (i) and (ii) we have

$$u_1 t = h - 10\,t \quad \Rightarrow t = \frac{h}{u_1 + 10}$$

Putting this value in eqn (ii), we get

$$\frac{h}{3} = \frac{10h}{u_1 + 10} + 4.9\left(\frac{h}{u_1 + 10}\right)^2$$

But, $h = 60$ m.

$$\Rightarrow 20 = \frac{600}{u_1 + 10} + 4.9\frac{60^2}{(u_1 + 10)^2}$$

$$\Rightarrow (u_1 + 10)^2 - 30(u_1 + 10) - 882 = 0$$

Solving this quadratic eqn, we find

$$u_1 + 10 = \frac{30 \pm \sqrt{100 + 3528}}{2} \qquad \Rightarrow u_1 = 38.27\ \text{m/s}$$

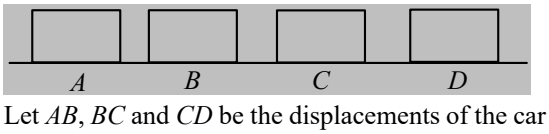
The other value is not possible because body is thrown upwards and is positive in the chosen coordinate system.

Therefore, option ($b$) is the correct answer.

**3.** A car starts moving on a straight road, first with acceleration $a = 5$ m/s$^2$, then moves uniformly, and finally decelerating at the same rate, comes to rest. The total time of motion equals 25 sec. The average velocity during that time is 72 km/hr. How long did the car move uniformly?

($a$) 30 s ($b$) 50 s ($c$) 15 s ($d$) 20 s

**Sol.**




Let $AB$, $BC$ and $CD$ be the displacements of the car when it accelerates, moves with constant velocity and decelerates respectively.

$$< v >= 72\ \text{km/hr} = 72 \times \frac{5}{18}\ \text{m/s} = 20\ \text{m/s}$$

Total distance travelled = $< v > \times$ time = $20 \times 25 = 500$ m.

From $A \rightarrow B$

$$AB = ut_{AB} + \frac{1}{2}a\left(t_{AB}\right)^2$$

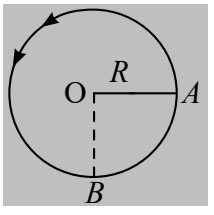
## POSITION, DISTANCE AND DISPLACEMENT

**1.** A body moves 6 m north. 8 m east and 10 m vertically upwards, what is its resultant displacement from initial position?

(*a*) $10\sqrt{2}$ m (*b*) 10 m

(*c*) $\frac{10}{\sqrt{2}}$m (*d*) 20 m

**2.** A body moves in circular path of radius $R$ from $A$ to $B$ as shown. Its displacement and distance covered are




(*a*) $R, \frac{3\pi R}{2}$ (*b*) $\sqrt{2}R, \frac{\pi R}{2}$

(*c*) $\sqrt{2}R, \frac{3\pi R}{2}$ (*d*) None of these

**3.** A particle covers half of the circle of radius $r$. Then the displacement and distance of the particle are respectively

(*a*) $2\pi r, 0$ (*b*) $2r, \pi r$ (*c*) $\frac{\pi r}{2}, 2r$ (*d*) $\pi r, r$

## SPEED AND VELOCITY

**4.** A person travels along a straight road for half the distance with velocity $v_1$ and the remaining half distance with velocity $v_2$. The average velocity is given by

(*a*) $v_1 v_2$ (*b*) $\frac{v_2^2}{v_1^2}$

(*c*) $\frac{v_1 + v_2}{2}$ (*d*) $\frac{2v_1 v_2}{v_1 + v_2}$

**5.** A car travels the first half of a distance between two places at a speed of 30 km/hr and the second half of the distance at 50 km/hr. The average speed of the car for the whole journey is

(*a*) 42.5 km/hr (*b*) 40.0 km/hr

(*c*) 37.5 km/hr (*d*) 35.0 km/hr

**6.** A person travels along a straight road for the first half time with a velocity $v_1$ and the next half time with a velocity $v_2$. The mean velocity $V$ of the man is

(*a*) $\frac{2}{V} = \frac{1}{v_1} + \frac{1}{v_2}$ (*b*) $V = \frac{v_1 + v_2}{2}$

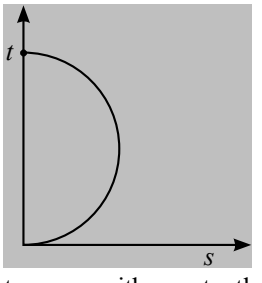
(*c*) $V = \sqrt{v_1 v_2}$ (*d*) $V = \sqrt{\frac{v_1}{v_2}}$

**7.** If a car covers 2/5$^{th}$ of the total distance with $v_1$ speed and 3/5$^{th}$ distance with $v_2$ then average speed is

(*a*) $\frac{1}{2}\sqrt{v_1 v_2}$ (*b*) $\frac{v_1 + v_2}{2}$

(*c*) $\frac{2v_1 v_2}{v_1 + v_2}$ (*d*) $\frac{5v_1 v_2}{3v_1 + 2v_2}$

**8.** Which of the following options is correct for the object having a straight line motion represented by the following graph?




(*a*) The object moves with constantly increasing velocity from $O$ to $A$ and then it moves with constant velocity

(*b*) Velocity of the object increases uniformly

(*c*) Average velocity is zero

(*d*) The graph shown is impossible

## CONSTANT ACCELERATION

**9.** A particle experiences a constant acceleration for 20 sec after starting from rest. If it travels a distance $S_1$ in the first 10 sec and a distance $S_2$ in the next 10 sec, then

(*a*) $S_1 = S_2$ (*b*) $S_1 = S_2/3$

(*c*) $S_1 = S_2/2$ (*d*) $S_1 = S_2/4$

**10.** A body is moving from rest under constant acceleration and let $S_1$ be the displacement in the first $(p - 1)$ sec and $S_2$ be the displacement in the first $p$ sec. The displacement in $(p^2 - p + 1)^{th}$ sec. will be

(*a*) $S_1 + S_2$ (*b*) $S_1 S_2$

(*c*) $S_1 - S_2$ (*d*) $S_1 / S_2$

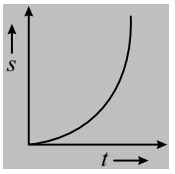
**11.** The displacement of body moving with constant acceleration, in 3rd seconds is 2m and in 5th second is 9m. Find the acceleration of body.

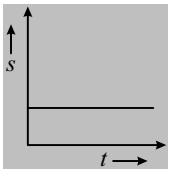
(*a*) $\frac{5}{2}\text{ms}^{-2}$ (*b*) $\frac{7}{2}\text{ms}^{-2}$

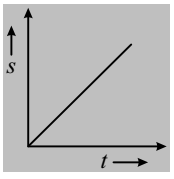
(*c*) $\frac{9}{2}\text{ms}^{-2}$ (*d*) $\frac{11}{2}\text{ms}^{-2}$

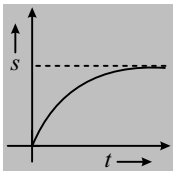
**12.** A point moves with uniform acceleration and $v_1$, $v_2$ and $v_3$ denote the average velocities in the three successive intervals of time $t_1$, $t_2$ and $t_3$. Which of the following relations is correct?

**32.** Which graph must represent non-uniform acceleration (s is displacement)?

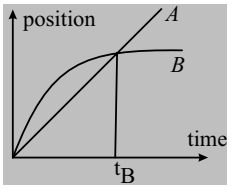
(*a*) 


(*b*) 


(*c*) 


(*d*) 


**33.** The graph shows position as a function of time for two trains running on parallel tracks. Which one of the following statements is true?




(*a*) At time $t_B$, both trains have the same velocity
(*b*) Both trains have the same velocity at some time after $t_B$
(*c*) Both trains have the same velocity at some time before $t_B$
(*d*) Somewhere on the graph, both trains have the same acceleration

## ONE DIMENSIONAL RELATIVE MOTION

**34.** Two trains, each 50 m long are travelling in opposite direction with velocity 10 m/s and 15 m/s. The time of crossing is

(*a*) 2 *s* (*b*) 4 *s* (*c*) $2\sqrt{3}$ *s* (*d*) $4\sqrt{3}$ *s*

**35.** A 210 meter long train is moving due North at a of 25m/s. A small bird is flying due South a little above the train with speed 5m/s. The time taken by the bird to cross the train is

(*a*) 6 *s* (*b*) 7 *s*
(*c*) 9 *s* (*d*) 10 *s*

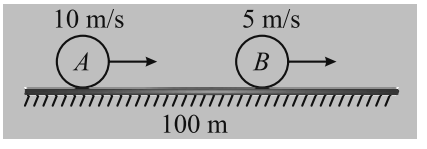
**36.** A stone is dropped from a building, and 2 seconds later another stone is dropped. (Both a are dropped from rest.) How far apart are the two stones by the time the first one has reached a speed of 30 m/s ?

(*a*) 80 m (*b*) 100 m
(*c*) 60 m (*d*) 40 m

**37.** Two trains each of length 50 m are approaching each other on parallel rails. Their velocities are 10 m/sec and 15 m/sec. They will cross each other in

(*a*) 2 sec (*b*) 4 sec
(*c*) 10 sec (*d*) 6 sec

**38.** An object *A* is moving with 10 m/s and *B* is moving with 5 m/s in the same direction of positive *x*-axis. *A* is 100 m behind *B* as shown. Find time taken by *A* to Meet *B*




(*a*) 18 sec (*b*) 16 sec (*c*) 20 sec (*d*) 17 sec

**39.** A thief is running away on a straight road with a speed of 9 $ms^{-1}$. A police man chases him on a jeep moving at a speed of 10 $ms^{-1}$. If the instantaneous separation of the jeep from the motorcycle is 100m, how long will it take for the police man to catch the thief?

(*a*) 1s (*b*) 19s (*c*) 90s (*d*) 100s

## DISTANCE OF NEAREST APPROACH

**40.** A body is projected vertically up at $t = 0$ with a velocity of 98 m/s. Another body is projected from the same point with same velocity after 4 seconds. Both bodies will meet after:

(*a*) 6 *s* (*b*) 8 *s*
(*c*) 10 *s* (*d*) 12 *s*

# Prabal (JEE Main Level)

**1.** A car runs at constant speed on a circular track of radius 100 m taking 62.8 s on each lap. What is the average speed and average velocity on each complete lap?

(*a*) Average velocity 10 m/s, average speed 10 m/s

(*b*) Average velocity zero, average speed 10 m/s

(*c*) Average velocity zero, average speed zero

(*d*) Average velocity 10 m/s, average speed zero

**2.** A body starts from rest and is uniformly accelerated for 30 s. The distance travelled in the first 10s is $x_1$, in next 10 s is $x_2$ and in last 10 s is $x_3$. Then $x_1 : x_2 : x_3$ is

(*a*) 1 : 2 : 4 (*b*) 1 : 2 : 5
(*c*) 1 : 3 : 5 (*d*) 1 : 3 : 9

**3.** A body is thrown upward and reaches its maximum height. At that position

(*a*) Its velocity is zero and its acceleration is also zero

(*b*) Its velocity is zero but its acceleration is maximum

(*c*) Its acceleration is minimum

(*d*) Its velocity is zero and its acceleration is the acceleration due to gravity

**4.** The motion of a body is given by the equation $\frac{dv(t)}{dt} = 6.0 - 3v(t)$, where $v(t)$ is speed in m/s and $t$ in sec.

If body was at rest at $t = 0$ choose the wrong option.

(*a*) The terminal speed is 2.0 m/s

(*b*) The speed varies with the time as $v(t) = 2(1 - e^{-3t})$ m/s

(*c*) The speed is 0.1 m/s when the acceleration is half the initial value

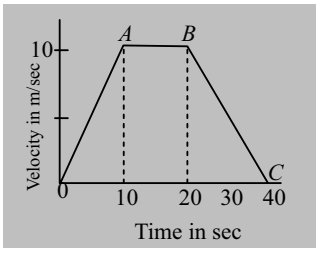
(*d*) The magnitude of the initial acceleration is 6.0 m/s$^2$

**5.** The displacement time graphs of motion of two particles $A$ and $B$ are straight lines making angles of 30º and 60º respectively with the time axis. If the velocity of $A$ is $v_A$ and that of $B$ is $v_B$ then the value of $\frac{v_A}{v_B}$ is

(*a*) 1/2 (*b*) $1/\sqrt{3}$

(*c*) $\sqrt{3}$ (*d*) 1/3

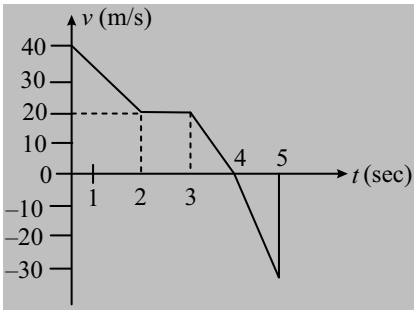
**6.** The curve shown represents the velocity-time graph of a particle, its acceleration values along $OA$, $AB$ and $BC$ in metre/sec$^2$ are respectively




(*a*) 1, 0, –0.5 (*b*) 1, 0, 0.5

(*c*) 1, 1, 0.5 (*d*) 1, 0.5, 0

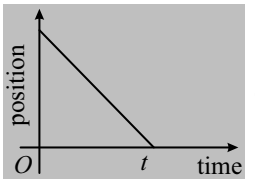
**7.** In the following velocity-time graph of a body, the distance and displacement travelled by the body in 5 second in meters will be

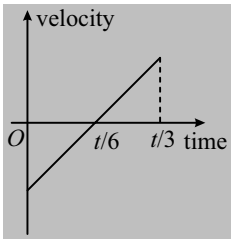


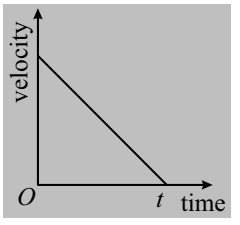

(*a*) 75, 115 (*b*) 105, 75

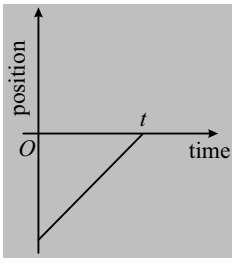
(*c*) 45, 75 (*d*) 95, 55

**8.** For which of the following graphs the average velocity of a particle moving along a straight line for time interval (0, $t$) must be negative?

(*a*)



(*b*)



(*c*)



(*d*)



**9.** Four particles move along $x$-axis. Their coordinates (in meters) as functions of time (in seconds) are given by

Particle 1 : $x(t) = 3.5 - 2.7t^3$

Particle 2 : $x(t) = 3.5 + 2.7t^3$

Particle 3 : $x(t) = 3.5 + 2.7t^2$

Particle 4 : $x(t) = 3.5 - 3.4t - 2.7t^2$

Which of these particles have constant acceleration?

(*a*) All four

(*b*) Only 1 and 2

(*c*) Only 2 and 3

(*d*) Only 3 and 4

**10.** A particle is projected up from ground with initial speed $v_0$. Starting from time $t = 0$ to $t = t_1$,

(*a*) Distance travelled and magnitude of displacement are not equal if $t_1 < \frac{v_0}{g}$

(*b*) Distance travelled and magnitude of displacement are equal if $\frac{v_0}{g} < t_1 < \frac{2v_0}{g}$

(*c*) Distance travelled and magnitude of displacement may not be equal if $0 < t_1 < \frac{2v_0}{g}$

(*d*) The magnitude of displacement is greater than the distance travelled if $\frac{v_0}{g} < t_1 < \frac{2v_0}{g}$

**11.** Two bodies $P$ and $Q$ have to move equal distances starting from rest. $P$ is accelerated with $2a$ for first half distance, then its acceleration becomes $a$ for last half, whereas $Q$ has acceleration $a$ for first half and acceleration $2a$ for last half, then for whole journey.

(*a*) Average speed of $P$ is more than that of $Q$

(*b*) Average speed of both will be same

(*c*) Maximum speed during the journey is more for $P$

(*d*) Maximum speed during the journey is more for $Q$

## MULTIPLE CORRECT TYPE QUESTIONS

**1.** Mark the correct statements for a particle going on a straight line

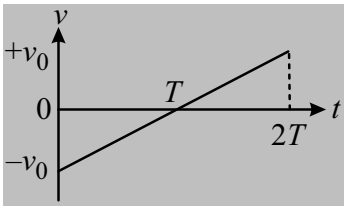
(*a*) If the velocity is zero at any instant, the acceleration should also be zero at that instant.

(*b*) If the velocity is zero for a time interval, the acceleration is zero at any instant within the time interval.

(*c*) If the velocity and acceleration have opposite sign, the object is slowing down.

(*d*) If the position and velocity have opposite sign, the particle is moving towards the origin.

**2.** The figure shows the velocity ($v$) of a particle plotted against time ($t$)




(*a*) The particle changes its direction of motion at some point.

(*b*) The acceleration of the particle remains constant.

(*c*) The displacement of the particle is zero.

(*d*) The initial and final speeds of the particle are the same.

**3.** A particle moves with constant speed $v$ along a regular hexagon $ABCDEF$ in the same order. Then the magnitude of the average velocity for its motion from $A$ to

(*a*) $F$ is $v/5$ (*b*) $D$ is $v/3$

(*c*) $C$ is $\frac{v\sqrt{3}}{2}$ (*d*) $B$ is $v$

**4.** Path of a particle moving in $x$-$y$ plane is $y = 3x + 4$. At some instant suppose $x$-component of velocity is 1 m/s and it is increasing at a rate of 1 m/s$^2$. Then

(*a*) At this instant the speed of particle is $\sqrt{10}$ m/s.

(*b*) At this instant the acceleration of particle is $\sqrt{10}$ m/s$^2$.

(*c*) Velocity time graph is a straight line.

(*d*) Acceleration-time graph is a straight line.

**5.** A particle having a velocity $v = v_0$ at $t = 0$ is brought to rest by decelerating at the rate $|a| = \alpha\sqrt{v}$, where $\alpha$ is a positive constant.

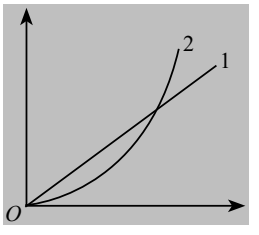
(*a*) The particle comes to rest at $t = \frac{2\sqrt{v_0}}{\alpha}$

(*b*) The particle will come to rest at infinity.

(*c*) The distance travelled by the particle is $\frac{2v_0^{3/2}}{\alpha}$.

(*d*) The distance travelled by the particle is $\frac{2}{3}\frac{v_0^{3/2}}{\alpha}$.

**6.** A particle is resting over a smooth horizontal floor. At $t = 0$, a horizontal force starts acting on it. Magnitude of the force increases with time as $F = kt$, where $k$ is a constant. Two curves are drawn for this particle as shown.




(*a*) Curve-1 shows acceleration versus time.

(*b*) Curve-2 shows velocity versus time.

(*c*) Curve-2 shows velocity versus acceleration.

(*d*) Curve-1 shows velocity versus acceleration.

**7.** The minimum speed with respect to air that a particular jet aircraft must have in order to keep aloft is 300 km/hr. Suppose that as its pilot prepares to take off, the wind blows eastward at a ground speed that can vary between 0 and 30 km/hr. Ignoring any other fact, a safe procedure to follow, consistent with using up as little fuel as possible, is to:

(*a*) Take off eastward at a ground speed of 320 km/hr

(*b*) Take off westward at a ground speed of 320 km/hr

(*c*) Take off westward at a ground speed of 300 km/hr

(*d*) Take off westward at a ground speed of 280 km/hr

**8.** A block is thrown with a velocity of 2 ms$^{-1}$ (relative to ground) on a belt, which is moving with velocity 4 ms$^{-1}$ in opposite direction of the initial velocity of block. If the block stops slipping on the belt after 4 sec of the throwing then choose the correct statements

(*a*) Displacement with respect to ground is zero after 2.66s and magnitude of displacement with respect to ground is 12 m after 4 sec.

(*b*) Magnitude of displacement with respect to ground in 4 sec is 4 m.

(*c*) Magnitude of displacement with respect to belt in 4 sec is 12 m.

(*d*) Displacement with respect to ground is zero in 8/3 sec

**9.** A particle has initial velocity 10 m/s. It moves under constant retarding force along the line of velocity which produces a retardation of 5 m/s$^2$. Then

(*a*) The maximum displacement in the direction of initial velocity is 10 m.

(*b*) The distance travelled in first 3 seconds is 7.5 m.

PHYSICS WALLAH

# PRAYAS JEE

- Some Basic Concepts of Chemistry
- Redox Reaction
- Solutions
- Thermodynamics
- Equilibrium

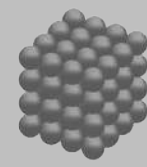
# Chemistry

MODULE 1

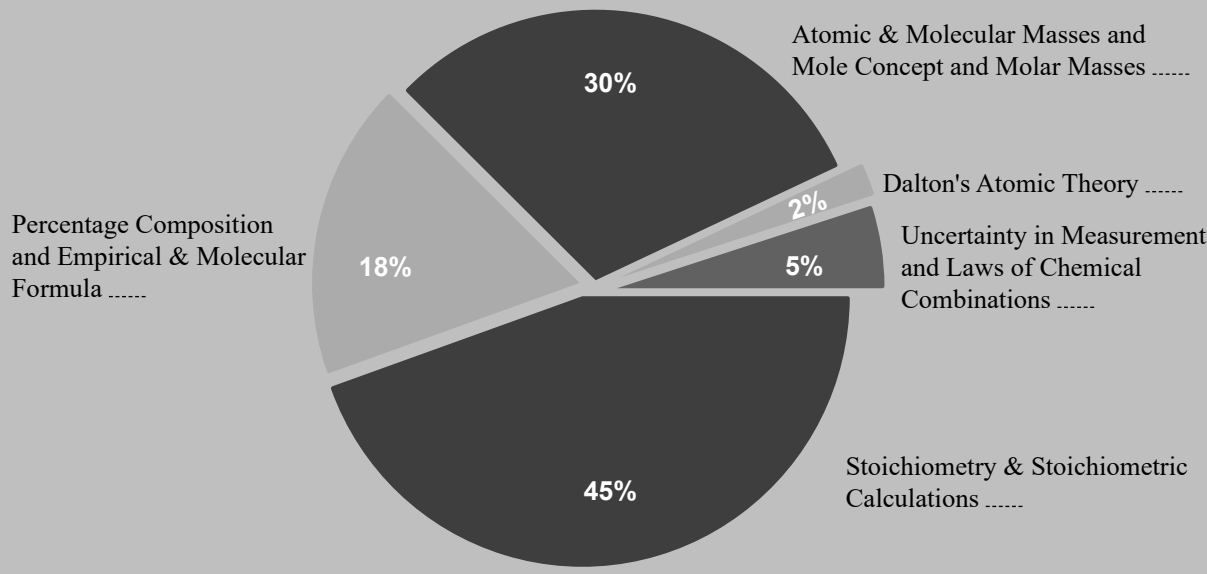
CHAPTER

# 1

# Some Basic Concepts of Chemistry



## Topicwise Weightage of JEE Main 6 Years Paper (124 Sets)




**"How's the Josh?" for these Topics:** *Mark your confidence level in the blank space around the topic (Low-L, Medium-M, High-H)*

## MATTER

A substance which occupies space, possesses mass and can be felt by any one or more of the five senses is called matter.

### Physical Classification of Matter

It is based on physical state under ordinary conditions of temperature and pressure.

(*a*) **Solid:** A substance is said to be solid if it possesses a definite volume and a definite shape, e.g. sugar, iron, gold, wood etc.

(*b*) **Liquid:** A substance is said to be liquid if it possesses a definite volume but not definite shape. They take up the shape of the container, e.g. water, milk, oil, mercury, alcohol etc.

(*c*) **Gas:** A substance is said to be gas if it neither possesses a definite volume nor a definite shape. This is because they fill up the whole container, e.g. Hydrogen ($H_2$), Oxygen ($O_2$), Carbon dioxide ($CO_2$) etc.

### Chemical Classification of Matter

(*a*) **Pure Substance:** A material containing only one type of substance. Pure Substance can not be separated into simpler substance by physical method.

e.g.: Element = Na, Mg, Ca ..... etc.

Compound = HCl, $H_2O$, $CO_2$, $HNO_3$ .....etc.

Pure substance is classified into two types:

(I) Element (II) Compound

(I) **Element:** The pure substance containing only one kind of atoms. It is classified into 3 types

(i) Metal → Zn, Cu, Hg, Ac, Sn, Pb etc.

(ii) Non-metal → $N_2$, $O_2$, $Cl_2$, $Br_2$, $F_2$, $P_4$, $S_8$ etc.

(iii) Metalloids → B, Si, As, Te etc.

(II) **Compound:** It is defined as pure substance containing more than one kind of elements or atoms which are combined together in a fixed proportion by weight and which can be decomposed into simpler substance by the suitable chemical method. The properties of a compound are completely different from those of its constituent element, e.g. HCl, $H_2O$, $H_2SO_4$, $HClO_4$, $HNO_3$ etc.

(*b*) **Mixture:** A material which contain more than one type of substances and which are mixed in any ratio by weight is called as mixture. The property of the mixture is the property of its components. The mixture can be separated by simple physical method.

### Classification of Mixture

(*i*) **Homogeneous mixture:** The mixture, in which all the components are present uniformly is called as

| Oxide | Ratio of weights of nitrogen and oxygen |
|---|---|
| $N_2O$ | 28 : 16 |
| NO | 28 : 32 |
| $N_2O_3$ | 28 : 48 |
| $N_2O_4$ | 28 : 64 |
| $N_2O_5$ | 28 : 80 |

Number of parts by weight of oxygen which combine with 14 parts by weight of nitrogen from the above are 8, 16, 24, 32 and 40 respectively. Their ratio is 1 : 2 : 3 : 4 : 5, which is a simple whole number ratio. Hence, the law of multiple proportion is illustrated.

## ADVANCED LEARNING

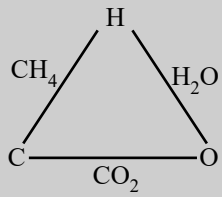
**Law of Reciprocal Proportion:** The ratio of the masses of two elements A and B which combine separately with a fixed mass of the third element C is either the same or some simple multiple of the ratio of the masses in which A and B combine directly with each other.

Like $CH_4$, $CO_2$ and $H_2O$

$CH_4 \rightarrow C : H = 12 : 4$

$CO_2 \rightarrow C : O = 12 : 32$

$H_2O \rightarrow H : O = 2 : 16$




**Gay-Lussac's Law of Combining Volume:** Gases combine in a simple whole number ratio of their volumes provided all measurements should be done at the same temperature and pressure.

$H_2(g)$ + $Cl_2(g)$ → 2HCl

1 vol. 1 vol. 2 vol.

**Avogadro's hypothesis:** Equal volume of all gases have equal number of molecules (not atoms) at same temperature and pressure condition.

**S.T.P.** (Standard Temperature and Pressure) Temperature = 0°C or 273 K, Pressure = 1 atm = 760 mm of Hg.

Volume of one mole of gas at STP is found to be experimentally equal to 22.4 litres which is known as molar volume.

❖ Measuring the volume is equivalent to counting the number of molecules of the gas.

## Train Your Brain

**Example 1:** A 15.9 g sample of sodium carbonate is added to a solution of acetic acid weighing 20.0 g. The two substances react, releasing carbon dioxide gas to the atmosphere. After reaction, the contents of the reaction vessel weigh 29.3 g. What is the mass of carbon dioxide given off during the reaction?

**Sol.** The total mass of reactants taken = 15.9 + 20.0 = 35.9 gm. From the conservation of mass, the final mass of the contents of the vessel should also be 35.9 gm. But it is only 29.3 gm. The difference is due to the mass of released carbon dioxide gas. Hence, the mass of carbon dioxide gas released = 35.9 – 29.3 = 6.6 gm

**Example 2:** The following are the results of analysis of two samples of the same or two different compounds of phosphorus and chlorine. From these results, decide whether the two samples are from the same or different compounds. Also state the law, which will be obeyed by the given samples.

| | Amount of P | Amount of C*l* |
|---|---|---|
| Compound A | 1.156 gm | 3.971 gm |
| Compound B | 1.542 gm | 5.297 gm |

**Sol.** The mass ratio of phosphorus and chlorine in compound A, $m_P : m_{Cl} = 1.156 : 3.971 = 0.2911 : 1.000$

The mass ratio of phosphorus and chlorine in compound B, $m_P : m_{Cl} = 1.542 : 5.297 = 0.2911 : 1.000$

As the mass ratio is same, both the compounds are **same** and the samples obey the **law of definite proportion**.

**Example 3:** 2.5 ml of a gaseous hydrocarbon exactly requires 12.5 ml oxygen for complete combustion and produces 7.5 ml carbon dioxide and 10.0 ml water vapour. All the volumes are measured at the same pressure and temperature. Show that the data illustrates Gay Lussac's law of volume combination.

**Sol.** $V_{hydrocarbon} : V_{oxygen} : V_{carbon\ dioxide} : V_{water\ vapour}$

$= 2.5 : 12.5 : 7.5 : 10.0$

$= 1 : 5 : 3 : 4$ (simple whole no. ratio)

Hence, the data is according to the **law of volume combination**.

## Concept Application

1. A sample of pure carbon dioxide, irrespective of its source contains 27.27% carbon and 72.73% oxygen. The data support:
   (*a*) Law of constant composition.
   (*b*) Law of conservation of mass.
   (*c*) Law of reciprocal proportions.
   (*d*) Law of multiple proportions.
2. The percentage of hydrogen in water and hydrogen peroxide is 11.1 and 5.9 respectively. These figures illustrate:
   (*a*) Law of multiple proportions.
   (*b*) Law of conservation of mass.
   (*c*) Law of constant proportions.
   (*d*) Law of combining volumes.
3. 1.0 g of an oxide of A contained 0.5 g of A. 4.0 g of another oxide of A contained 1.6 g of A. The data indicate the law of:
   (*a*) Reciprocal proportions.
   (*b*) Constant proportions.
   (*c*) Conservation of energy.
   (*d*) Multiple proportions.
4. Carbon is found to form two oxides which contain 42.9% & 27.3% of carbon respectively show that these figures shows the
   (*a*) Law of multiple proportion
   (*b*) Law of definite proportion
   (*c*) Law of mass conservation
   (*d*) All of these

## ATOMIC MASS & MOLECULAR MASS

**Relative Atomic Mass:** One of the most important concept from Dalton's atomic theory was that of relative atomic mass or relative atomic weight. This is done by expressing mass of one atom with respect to a fixed standard. Dalton used hydrogen as the standard (H = 1). Later on oxygen (O = 16) replaced hydrogen as the reference.

- The present standard unit which was adopted internationally in 1961, is based on the mass of one carbon-12 isotopic atom, taken as exactly 12.000 u (amu).

  Relative atomic mass (R.A.M)

$$= \frac{\text{Mass of one atom of an element}}{\frac{1}{12} \times \text{Mass of one } C^{12} \text{ atom}}$$

**Atomic Mass Unit (or amu):** The atomic mass unit (amu) is equal to $\left(\frac{1}{12}\right)^{th}$ of mass of one atom of carbon-12 isotope.

$$\therefore \quad 1 \text{ amu} = \frac{1}{12} \times \text{mass of one } C^{12} \text{ isotopic atom}$$

$$\simeq \text{mass of one nucleon in } C^{12} \text{ atom.}$$

**1 amu = $1.66 \times 10^{-24}$ gm or $1.66 \times 10^{-27}$ kg**

- Today, amu has been replaced by 'u' which is known as unified atomic mass
- **One amu is also called One Dalton (Da).**
- Atomic mass = R.A.M × 1 amu

- Relative atomic mass indicates the number of nucleons present in the atom.

### Average / Mean Atomic Mass

The weighted average of the isotopic masses of the element's naturally occurring isotopes.

Mathematically, average atomic mass of X ($A_x$)

$$= \frac{a_1x_1 + a_2x_2 + ..... + a_nx_n}{100}$$

Where:

$a_1, a_2, a_3$ ..... atomic mass of isotopes.

and $x_1, x_2, x_3$ ..... mole% or % of natural abundance of isotopes.

**Key Note**

- Atomic weights of many elements are not whole numbers due to the presence of stable isotopes.
- The number of atoms of a particular isotope present in 100 atoms of a natural sample of that element is called its relative abundance which always remains constant for a given element.
- Shortcut for % determination if average atomic weight is given for X having isotopes XA & XB.

$$\% \text{ of XA} = \left| \frac{\text{Average atomic weight} - \text{wt of } X^B}{\text{difference in weight of } X^A \text{ \& } X^B} \right| \times 100$$

**Illustration:** Naturally occurring chlorine is 75% $Cl^{35}$ which has an atomic mass of 35 amu and 25% $Cl^{37}$ which has a mass of 37 amu. Calculate the average atomic mass of chlorine:

(*a*) 35.5amu (*b*) 36.5amu (*c*) 71amu (*d*) 72amu

**Sol.** (*a*) Average atomic mass

$$= \frac{(\%\text{ of I isotope}) \times (\text{its A.M}) + (\%\text{ of II isotope}) \times (\text{its A.M.})}{100}$$

$$= \frac{75 \times 35 + 25 \times 37}{100} = 35.5 \text{ amu}$$

**Relative molecular mass:**

$$= \frac{\textbf{mass of one molecule of the substance}}{\frac{1}{12} \times \textbf{mass of one } C^{12} \textbf{ – atom}}$$

- Molecular mass = Relative molecular mass × 1 amu

### Mean Molar Mass or Molecular Mass

The average molar mass of the different substance present in the container = $\frac{n_1M_1 + n_2M_2 + .....n_nM_n}{n_1 + n_2 + ....n_n}$.

Where:

$M_1, M_2, M_3$ ..... are molar masses.

$n_1, n_2, n_3$ ..... are moles of substances.

### Formula Mass

The formula mass of a substance is defined as the sum of the atomic masses of constituent atoms in an ionic compound. This is generally used for ionic compounds which do not contain discrete molecules, but ions as their constituent units.

**For example:** Formula mass of NaCl is:

Formula mass = mass of sodium atom + mass of chlorine atom

= (23 + 35.5) u = 58.5 u

**Illustration:** The molar composition of polluted air is as follows:

| Gas | At. wt. | mole percentage composition |
|---|---|---|
| Oxygen | 16 | 16% |
| Nitrogen | 14 | 80% |
| Carbon dioxide | - | 03% |
| Sulphur dioxide | - | 01% |

What is the average molecular weight of the given polluted air? (Given, atomic weights of C and S are 12 and 32 respectively.

**Sol.** $M_{avg} = \frac{\sum_{j=1}^{j=n} n_jM_j}{\sum_{j=1}^{j=n} n_j}$ Here $\sum_{j=1}^{j=n} n_j = 100$

$$\therefore M_{avg} = \frac{16 \times 32 + 80 \times 28 + 44 \times 3 + 64 \times 1}{100}$$

$$= \frac{512 + 2240 + 132 + 64}{100} = \frac{2948}{100} = 29.48 \text{ Ans.}$$

**1.** 1.80 g of a certain metal burnt in oxygen gave 3.0 g of its oxide. 1.50 g of the same metal heated in steam gave 2.50 g of its oxide. The law shown by above data is:

(*a*) Law of constant proportion

(*b*) Law of multiple proportion

(*c*) Law of reciprocal proportion

(*d*) All of the above

**Sol.** In the first sample of oxide,

Weight of metal = 1.80 g;

Weight of oxygen = (3.0 – 1.80) g = 1.2 g

$$\therefore \frac{\text{wt of metal}}{\text{wt of oxygen}} = \frac{1.80\text{g}}{1.2\text{g}} = 1.5$$

In the second sample of the oxide,

Weight of metal = 1.50 g;

Weight of oxygen = (2.50 – 1.50) g = 1 g

$$\frac{\text{wt of metal}}{\text{wt of oxygen}} = 1.5$$

Thus, in both samples of the oxide, the proportions of the weights of the metal and oxygen are fixed. Hence the results follow the law of constant proportion.

Therefore, option (*a*) is the correct answer.

**2.** Calculate the total charge present on 4.2 gm of $N^{3-}$.

(*a*) $8.67 \times 10^4$ C (*b*) $9.05 \times 10^4$ C

(*c*) $8.67 \times 10^3$ C (*d*) $7.67 \times 10^4$ C

**Sol.** $\text{Mole} = \frac{\text{wt.in gm}}{\text{Ionic wt.}} = \frac{4.2}{14} = 0.3$

Total no. of ions = $0.3 \times N_A$ ions.

Total charge = $0.3\ N_A \times 3 \times 1.6 \times 10^{-19}$ C

$= 0.3 \times 6.022 \times 10^{23} \times 3 \times 1.6 \times 10^{-19} = 8.67 \times 10^4$ C

Therefore, option (*a*) is the correct answer.

**3.** How many carbon atoms are present in 0.35 mol of $C_6H_{12}O_6$?

(*a*) $6.022 \times 10^{23}$ carbon atoms.

(*b*) $1.26 \times 10^{23}$ carbon atoms.

(*c*) $1.26 \times 10^{24}$ carbon atoms.

(*d*) $6.022 \times 10^{24}$ carbon atoms.

**Sol.** $\because$ 1 mol of $C_6H_{12}O_6$ has 6 $N_A$ atoms of C

$\therefore$ 0.35 mol of $C_6H_{12}O_6$ has $6 \times 0.35\ N_A$ atoms of C

$= 2.1\ N_A$ atoms $= 2.1 \times 6.022 \times 10^{23} = 1.26 \times 10^{24}$ carbon atoms.

Therefore, option (*c*) is the correct answer.

**4.** How many molecules are present in 5.23 gm of glucose ($C_6H_{12}O_6$)?

(*a*) $1.65 \times 10^{22}$ (*b*) $1.75 \times 10^{22}$

(*c*) $1.75 \times 10^{21}$ (*d*) None of these

**Sol.** $\because$ 180 gm glucose has = $N_A$ molecules

$$\therefore \text{ 5.23 gm glucose has} = \frac{5.23 \times 6.022 \times 10^{23}}{180}$$

$= 1.75 \times 10^{22}$ molecules

Therefore, option (*b*) is the correct answer.

**5.** A sample of ($C_2H_6$) ethane has the same mass as $10^7$ molecules of methane. How many $C_2H_6$ molecules does the sample contain?

(*a*) $5.34 \times 10^6$ (*b*) $1.26 \times 10^8$

(*c*) $4.26 \times 10^6$ (*d*) $6.022 \times 10^6$

**Sol.** Moles of $CH_4 = \frac{10^7}{N_A}$

Mass of $CH_4 = \frac{10^7}{N_A} \times 16 =$ mass of $C_2H_6$

So Moles of $C_2H_6 = \frac{10^7 \times 16}{N_A \times 30}$

So no. of molecules of $C_2H_6 = \frac{10^7 \times 16}{N_A \times 30} \times N_A = 5.34 \times 10^6$.

Therefore, option (*a*) is the correct answer.

**6.** From 160 g of $SO_2$ (g) sample, $1.2046 \times 10^{24}$ molecules of $SO_2$ are removed then find out the volume of left over $SO_2$ (g) at STP.

(*a*) 11.2 L (*b*) 12.5 L

(*c*) 9.5 L (*d*) 10.8 L

**Sol.** Given moles = $\frac{160}{64} = 2.5$.

Removed moles = $\frac{1.2046 \times 10^{24}}{6.022 \times 10^{23}} = 2$.

So left moles = 0.5.

Volume left at STP = $0.5 \times 22.4 = 11.2$ L.

Therefore, option (*a*) is the correct answer.

**7.** 14 g of Nitrogen gas and 22 g of $CO_2$ gas are mixed together. Find the volume of gaseous mixture at STP.

(*a*) 10.2 L (*b*) 12.2 L

(*c*) 15.5 L (*d*) 22.4 L

**Sol.** Moles of $N_2 = \frac{14}{28} = 0.5$.

Moles of $CO_2 = \frac{22}{44} = 0.5$.

So total moles = 0.5 + 0.5 = 1.

So vol. at STP = $1 \times 22.4 = 22.4$ L.

Therefore, option (*d*) is the correct answer.

**8.** How many years it would take to spend Avogadro's number of rupees at the rate of 1 million rupees per second?

(*a*) $19.098 \times 10^{19}$ years (*b*) 19.098 years

(*c*) $19.098 \times 10^9$ years (*d*) None of these

## FUNDAMENTAL QUANTITIES, LAWS OF CHEMICAL COMBINATION

**1.** Express the result of (0.582 + 324.65) to the appropriate number of significant figures:
(*a*) 325.24 (*b*) 325.23
(*c*) 325.2 (*d*) 325.232

**2.** The correctly reported answer of the area of rectangle which is 12.34 cm long and 1.23 cm wide is :
(*a*) 15.2 $m^2$ (*b*) 15.2 $cm^2$
(*c*) 15.1 $cm^2$ (*d*) 15.17 $cm^2$

**3.** Two elements X and Y combine in gaseous state to form XY in the ratio 1:35.5 by mass. The mass of Y that will be required to react with 2 g of X is:
(*a*) 7.1 g (*b*) 3.55 g (*c*) 71 g (*d*) 35.5 g

**4.** 4.4 g of an oxide of nitrogen gives 2.24 L of nitrogen and 60 g of another oxide of nitrogen gives 22.4 L of nitrogen at S.T.P. The data illustrates:
(*a*) Law of conservation of mass
(*b*) Law of constant proportions
(*c*) Law of multiple proportions
(*d*) Law of reciprocal proportions

**5.** Two elements X and Y combine to form compounds A, B and C. The ratio of different masses of Y which combine with a fixed mass of X in A, B and C is 1 : 3 : 5. If 32 parts by mass of X combines with 84 parts by mass of Y in B, then in C, 16 parts by mass of X will combine with;
(*a*) 14 parts by mass of Y (*b*) 42 parts by mass of Y
(*c*) 70 parts by mass of Y (*d*) 84 parts by mass of Y

## ATOMIC MASS & MOLECULAR MASS, MOLE CONCEPT AND APPLICATIONS

**6.** 1 mol of $CH_4$ contains:
(*a*) $6.02 \times 10^{23}$ atoms of H
(*b*) 4 *g*-atom of Hydrogen
(*c*) $1.81 \times 10^{23}$ molecules of $CH_4$
(*d*) 3.0 *g* of carbon

**7.** 7.5 grams of a gas occupy 5.6 litres of volume at STP, the gas is:
(*a*) NO (*b*) $N_2O$ (*c*) CO (*d*) $CO_2$

**8.** The number of atoms in 4.25 *g* of $NH_3$ is approximately:
(*a*) $1 \times 10^{23}$ (*b*) $2 \times 10^{23}$
(*c*) $4 \times 10^{23}$ (*d*) $6 \times 10^{23}$

**9.** One litre of a gas at STP weighs 1.16 *g*. The possible gas is:
(*a*) $C_2H_2$ (*b*) CO
(*c*) $O_2$ (*d*) $CH_4$

**10.** If $N_A$ is Avogadro's number, then number of valence electrons in 4.2 *g* of nitride ions ($N^{3-}$) is:
(*a*) 2.4 $N_A$ (*b*) 4.2 $N_A$
(*c*) 1.6 $N_A$ (*d*) 3.2 $N_A$

**11.** The number of molecules at STP in 1 ml of an ideal gas will be:
(*a*) $6 \times 10^{23}$ (*b*) $2.69 \times 10^{19}$
(*c*) $2.69 \times 10^{23}$ (*d*) None of these

**12.** Volume of a gas at STP is $1.12 \times 10^{-7}$ cc. The number of molecules in it are:
(*a*) $3.01 \times 10^{20}$ (*b*) $3.01 \times 10^{12}$
(*c*) $3.01 \times 10^{23}$ (*d*) $3.01 \times 10^{24}$

**13.** 4.4 g of an unknown gas occupies 2.24 L of volume at standard temperature and pressure. The gas may be:
(*a*) Carbon dioxide (*b*) Carbon monoxide
(*c*) Oxygen (*d*) Sulphur dioxide

**14.** The number of oxygen atoms in 4.4 *g* of $CO_2$ is approx.:
(*a*) $1.2 \times 10^{23}$ (*b*) $6 \times 10^{22}$
(*c*) $6 \times 10^{23}$ (*d*) $12 \times 10^{23}$

**15.** The total number of protons in 10 *g* of calcium carbonate is: ($N_A = 6.022 \times 10^{23}$)
(*a*) $1.5057 \times 10^{24}$ (*b*) $2.0478 \times 10^{24}$
(*c*) $3.0115 \times 10^{24}$ (*d*) $4.0956 \times 10^{24}$

**16.** Number of molecules in 100 ml each of $O_2$,$NH_3$ and $CO_2$ at STP are:
(*a*) In the order: $CO_2 < O_2 < NH_3$
(*b*) In the order: $NH_3 < O_2 < CO_2$
(*c*) The same in all
(*d*) In the order: $NH_3 < CO_2 < O_2$

**17.** The number of water molecules in 1 litre of water is:
(*a*) 18 (*b*) $18 \times 1000$
(*c*) $N_A$ (*d*) 55.55 $N_A$

**18.** 2 g of oxygen contains number of atoms equal to that in:
(*a*) 0.5 g of hydrogen (*b*) 4 g of sulphur
(*c*) 7 g of nitrogen (*d*) 2.3 g of sodium

## PERCENTAGE COMPOSITION, EMPIRICAL FORMULA & MOLECULAR FORMULA

**19.** Caffeine has a molecular weight of 194. If it contains 28.9% by mass of nitrogen, number of atoms of nitrogen in one molecule of caffeine is:
(*a*) 4 (*b*) 6
(*c*) 2 (*d*) 3

**20.** A compound (60 *g*) on analysis gave C = 24 g, H = 4 *g*, O = 32 *g*. Its empirical formula is:
(*a*) $C_2H_2O_2$ (*b*) $C_2H_2O$
(*c*) $CH_2O_2$ (*d*) $CH_2O$

**1.** A sample of calcium phosphate $Ca_3(PO_4)_2$ contains 8 mol of O atoms. The number of mole of Ca atoms in the sample is:
(*a*) 4 (*b*) 1.5 (*c*) 3 (*d*) 8

**2.** Ratio of masses of $H_2SO_4$ and $Al_2(SO_4)_3$ each containing 32 grams of S is _____.
(*a*) 0.86 (*b*) 1.72 (*c*) 0.43 (*d*) 2.15

**3.** Which has maximum number of atoms of oxygen?
(*a*) 10 ml $H_2O$(l)
(*b*) 0.1 mole of $V_2O_5$
(*c*) 12 gm $O_3$(g)
(*d*) $12.044 \times 10^{22}$ molecules of $CO_2$

**4.** Mass of one atom of the element A is $3.9854 \times 10^{-23}$g. How many atoms are contained in 1g of the element A?
(*a*) $2.509 \times 10^{22}$ (*b*) $6.022 \times 10^{23}$
(*c*) $12.044 \times 10^{23}$ (*d*) None of these

**5.** The number of atoms present in 0.5 g-atoms of nitrogen is same as the atoms in:
(*a*) 12 g of C (*b*) 32 g of S
(*c*) 8 g of oxygen (*d*) 24 g of Mg

**6.** How many moles of magnesium phosphate $Mg_3(PO_4)_2$ will contain 0.25 mole of oxygen atoms?
(*a*) 0.02 (*b*) $3.125 \times 10^{-2}$
(*c*) $1.25 \times 10^{-2}$ (*d*) $2.5 \times 10^{-2}$

**7.** 64 g of an organic compound has 24 g carbon and 8 g hydrogen and the rest is oxygen. The empirical formula of the compound is:
(*a*) $CH_4O$ (*b*) $CH_2O$
(*c*) $C_2H_4O$ (*d*) None of these

**8.** Two elements X (atomic mass = 75) and Y (atomic mass = 16) combine to give a compound having 75.8% of X. The formula of the compound is:
(*a*) $X_2Y_3$ (*b*) $X_2Y$ (*c*) $X_2Y_2$ (*d*) XY

**9.** A definite amount of gaseous hydrocarbon was burnt with just sufficient amount of $O_2$. The volume of all reactants was 600 ml, after the explosion the volume of the products [$CO_2$(g) and $H_2O$(g)] was found to be 700 ml under the similar conditions. The possible molecular formula of the compound is:
(*a*) $C_3H_8$ (*b*) $C_3H_6$ (*c*) $C_3H_4$ (*d*) $C_4H_{10}$

**10.** Mole fraction of ethyl alcohol in aqueous ethyl alcohol ($C_2H_5OH$) solution is 0.25. Hence, percentage of ethyl alcohol by weight is:
(*a*) 54% (*b*) 25%
(*c*) 75% (*d*) 46%

**11.** Weight of oxygen in $Fe_2O_3$ and FeO in the simple ratio for the same amount of iron, is:
(*a*) 3 : 2 (*b*) 1 : 2
(*c*) 2 : 1 (*d*) 3 : 1

**12.** A person needs on average of 2.0 mg of riboflavin (vitamin $B_2$) per day. How many gm of butter should be taken by the person per day if it is the only source of riboflavin? (Butter contains 5.5 microgram riboflavin per gm.)
(*a*) 363.6 gm (*b*) 2.75 mg
(*c*) 11 gm (*d*) 19.8 gm

**13.** The oxide of a metal contains 30% oxygen by weight. If the atomic ratio of metal and oxygen is 2 : 3, determine the atomic weight of metal.
(*a*) 12 u (*b*) 56 u (*c*) 27 u (*d*) 52 u

**14.** When a mixture of 10 mole of $SO_2$ and 15 mole of $O_2$ was passed over catalyst, 8 mole of $SO_3$ was formed. How many mole of $SO_2$and $O_2$ did not enter into combination?
(*a*) 2 moles of $SO_2$, 11 moles of $O_2$
(*b*) 3 moles of $SO_2$, 11.5 moles of $O_2$
(*c*) 2 moles of $SO_2$, 4 moles of $O_2$
(*d*) 8 moles of $SO_2$, 4 moles of $O_2$

**15.** $C_6H_5OH(g) + O_2(g) \rightarrow CO_2(g) + H_2O(l)$
Magnitude of volume change if 30 ml of $C_6H_5OH$ (g) is burnt with excess amount of oxygen, is:
(*a*) 30 ml (*b*) 60 ml (*c*) 20 ml (*d*) 10 ml

**16.** Mass of sucrose $C_{12}H_{22}O_{11}$ produced by mixing 84 gm of carbon, 12 gm of dihydrogen and 56 lit. $O_2$ at 1 atm & 273 K according to given reaction, is:
$C(s) + H_2(g) + O_2(g) \rightarrow C_{12}H_{22}O_{11}(s)$
(*a*) 138.5 (*b*) 155.5
(*c*) 172.5 (*d*) 199.5

**17.** What volume (in ml) of 0.2 M $H_2SO_4$ solution should be mixed with the 40 ml of 0.1 M NaOH solution such that the resulting solution has the concentration of $H_2SO_4$ as $\frac{6}{55}$ M.
(*a*) 70 (*b*) 45 (*c*) 30 (*d*) 58

**18.** For the reaction; $2x + 3y + 4z \rightarrow 5w$, initially 1 mol of x, 3 mol of y and 4 mol of z is taken. If 1.25 mol of w is obtained then % yield of this reaction is:
(*a*) 50% (*b*) 60% (*c*) 70% (*d*) 40%

**19.** If 10 g of Ag reacts with 1 g of sulphur , the amount of $Ag_2S$ formed will be:
(*a*) 7.75 g (*b*) 0.775 g
(*c*) 11 g (*d*) 10 g

**20.** A solution of A (MM = 20) and B (MM = 10), [Mole fraction $X_B = 0.6$] has density 0.7 gm/ml then molarity and molality of B in this solution will be _____ and _____ respectively.
(*a*) 30 M, 75 m (*b*) 40 M, 75 m
(*c*) 30 M, 65 m (*d*) 50 M, 55 m

# VIDYAPEETH
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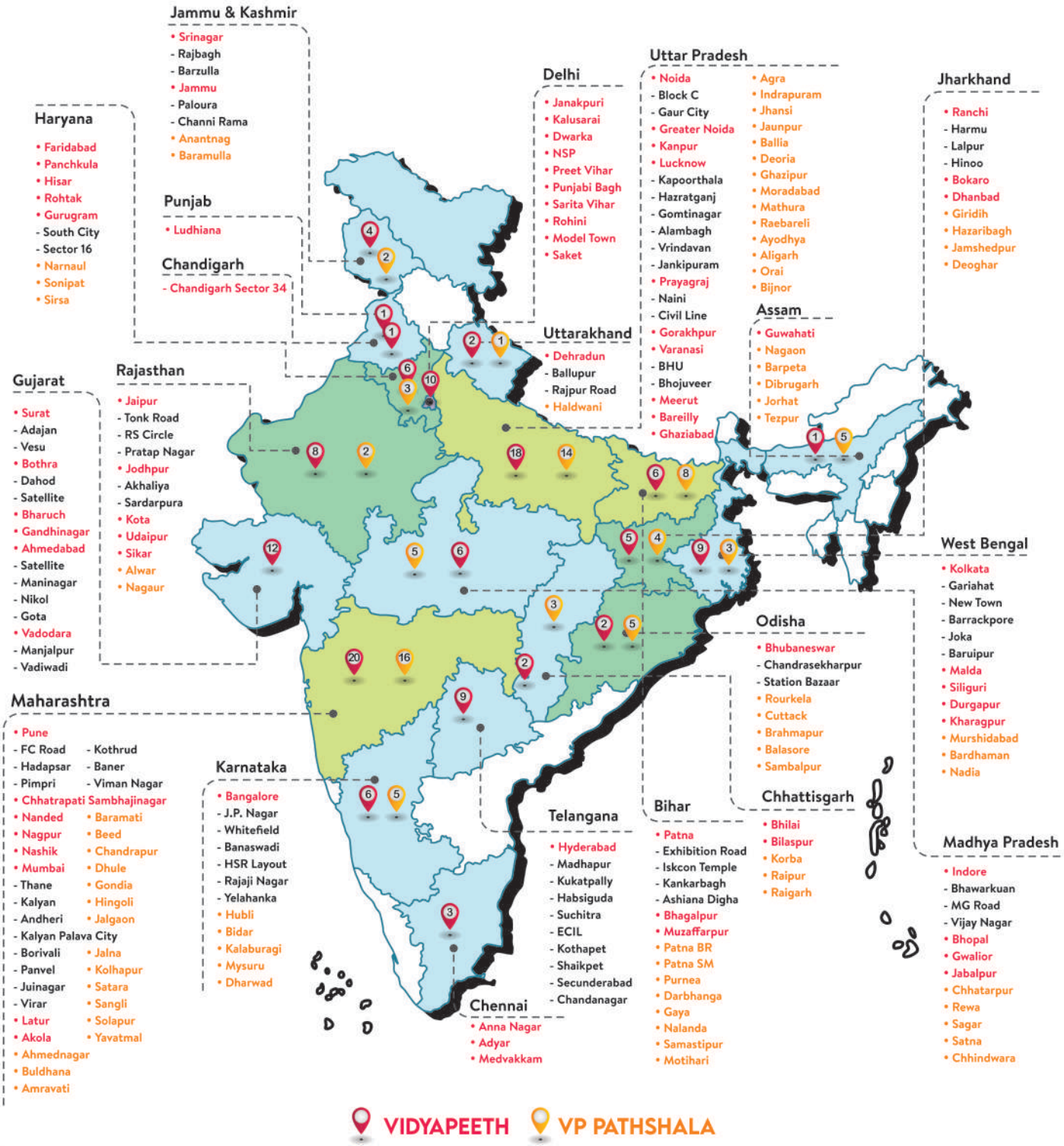


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