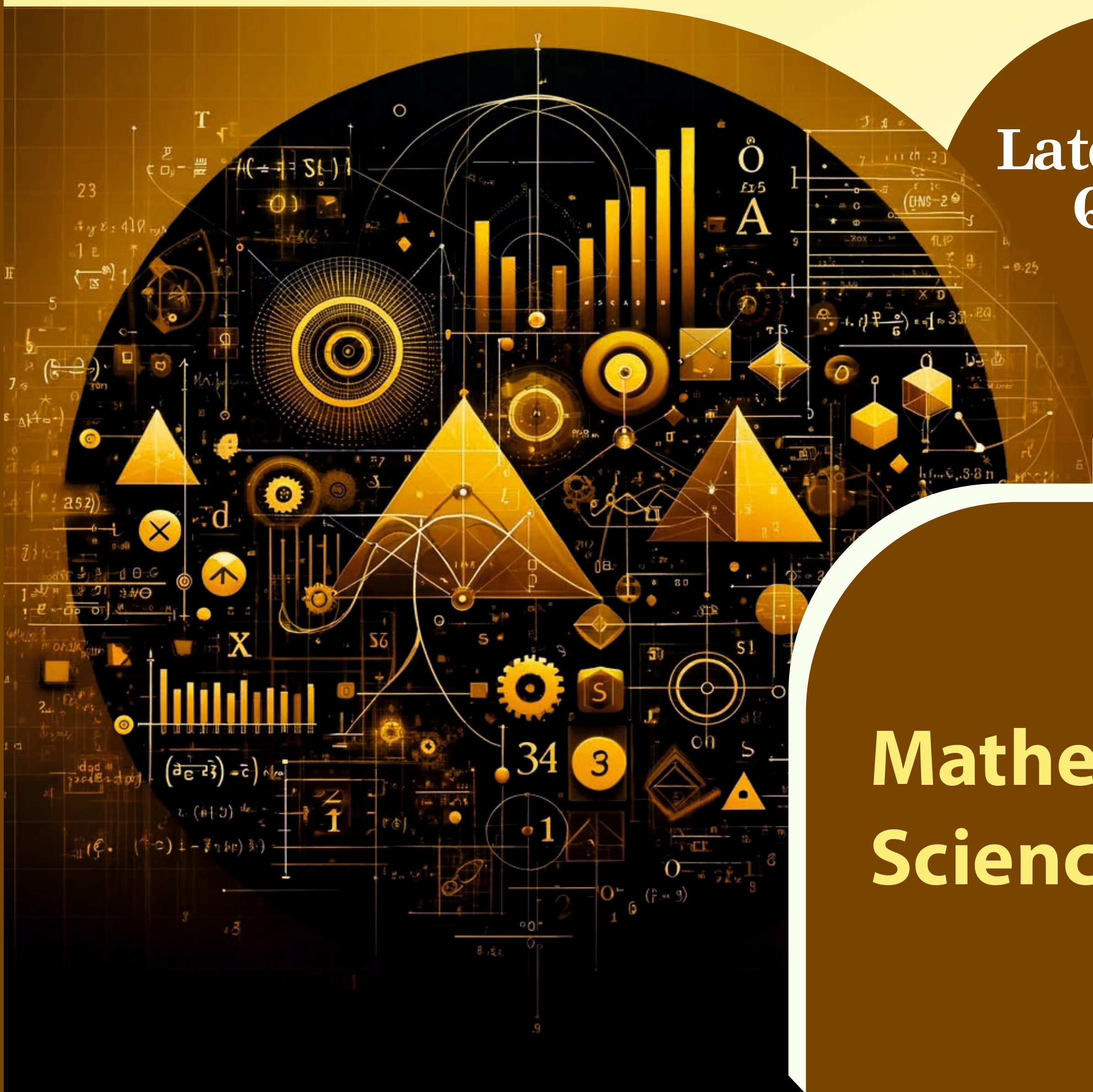
Latest Edition

CSIR-NET RECAP

Chapter-wise and Year-wise Previous Years Solved Papers

(2015-JUNE 2024)



Latest Exam Questions Covered

Mathematical Sciences

Also useful for IIT-JAM, GATE, TIFR, NBHM Exams

Trend Analysis

Topic	No. of Questions Year Wise											
	2017 June	2017 Dec.	2018 June	2018 Dec.	2019 June	2019 Dec.	2020 Nov.	2022 Feb.	2022 Sep.	2023 June	2023 Dec.	2024 June
Real Analysis	13	16	12	12	16	13	17	8	14	12	14	17
Modern Algebra	18	9	10	10	9	11	8	10	10	6	7	8
Complex Analysis	8	8	8	8	8	5	8	8	7	5	7	8
Linear Algebra	7	10	12	15	14	14	14	13	11	6	14	14
Numerical Analysis	3	3	2	2	3	1	3	3	2	1	1	1
Topology	3	5	5	5	3	7	3	0	1	0	0	2
Ordinary Differential Equations (ODEs)	2	5	5	5	4	6	5	6	3	5	3	7
Partial Differential Equations (PDEs)	4	3	5	3	4	4	5	4	3	5	3	2
Calculus of Variation	2	3	1	2	2	3	3	3	2	1	2	1
Linear Integral Equations	5	7	5	5	3	6	2	5	4	3	2	3
Total	**65**	**69**	**65**	**67**	**66**	**70**	**68**	**60**	**57**	**44**	**53**	**63**

Contents

1 June 2024

1. Let $u = u(x,t)$ be the solution of the following initial value problem $\begin{cases} u_t + 2024u_x = 0, & x \in R, t > 0 \\ u(x,0) = u_0(x) & x \in R \end{cases}$ where $u_0 : R \to R$ is an arbitrary C^1 function. Consider the following statements:S_1 : If $A_t := \{x \in R : u(x, t) < 1\}$ and $|A_t|$ denotes the Lebesgue measure of A_t for every $t \geq 0$, then $|A_t| = |A_0|$ $\forall t > 0$.S_2 : If v_0 is Lebesgue integrable, then for every $t > 0$, the function $x \mapsto v(x, t)$ is Lebesgue integrable.

Then

(*a*) both S_1 and S_2 are true

(*b*) S_1 is true but S_2 is false

(*c*) S_2 is true but S_1 is false

(*d*) both S_1 and S_2 are false

2. Let u be the solution of the Volterra integral equation $\int_0^t \left[\frac{1}{2} + \sin(t-\tau)\right] u(\tau)\, d\tau = \sin t$

Then the value of $u(1)$ is.

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) $2e^{-1}$

3. If $u = u(x, t)$ is the solution of the initial value problem

$$\begin{cases} u_t = u_{xx} & x \in R, t > 0 \\ u(x,0) = \sin(4x) + x + 1, & x \in R \end{cases}$$

satisfying $|u(x,t)| < 3e^{x^2}$ for all $x \in R$ and $t > 0$, then

(*a*) $u\left(\frac{\pi}{8},1\right) + u\left(-\frac{\pi}{8},1\right) = 2$

(*b*) $u\left(\frac{\pi}{8},1\right) = u\left(-\frac{\pi}{8},1\right)$

(*c*) $u\left(\frac{\pi}{8},1\right) + 2u\left(-\frac{\pi}{8},1\right) = 2$

(*d*) $u\left(\frac{\pi}{8},1\right) = -u\left(-\frac{\pi}{8},1\right)$

4. Let V be the real vector space of 2×2 matrices with entries in R. Let $T : V \to V$ denote the linear transformation defined by $T(B) = AB$ for all $B \in V$, where $A = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$.

What is the characteristic polynomial of T?

(*a*) $(x-2)(x-1)$ (*b*) $x^2(x-2)(x-1)$

(*c*) $(x-2)^2(x-1)^2$ (*d*) $(x^2-2)(x^2-1)$

5. Let f be an entire function. Which of the following statements is FALSE?

(*a*) If Re(f), Im (f) are bounded then f is constant

(*b*) If $e^{|\text{Re}(f)|+|\text{Im}(f)|}$ is bounded, then f is constant

(*c*) If the sum Re(f) + Im (f) and the product Re(f) Im (f) are bounded, then f is constant

(*d*) If (Re(f) + Im (f)) is bounded, then f is constant

6. Consider the initial value problem (IVP)

$$\begin{cases} y'(x) = \sqrt{|y(x) + \epsilon|}, \ x \in R \\ y(0) = y_0 \end{cases}$$

Consider the following statements:

S_1: There is an $\epsilon > 0$ such that for all $y_0 \in R$, the IVP has more than one solution.

S_2: There is a $y_0 \in R$ such that for all $\epsilon > 0$, the IVP has more than one solution.

(*a*) both S_1 and S_2 are true

(*b*) S_1 is true but S_2 is false

(*c*) S_1 is false but S_2 is true

(*d*) both S_1 and S_2 are false

7. What is the cardinality of the set of real solutions of $e^x + x = 1$?

(*a*) 0 (*b*) 1

(*c*) Countably infinite (*d*) Uncountable

8. Let φ denote the solution to the boundary value problem

$$\text{(BVP)} \begin{cases} (xy')' - 2y' + \dfrac{y}{x} = 1, & 1 < x < e^4 \\ y(1) = 0 & y(e^4) = 4e^4 \end{cases}$$

Then the value of $\varphi(e)$ is

(*a*) $-\dfrac{e}{2}$ (*b*) $-\dfrac{e}{3}$ (*c*) $\dfrac{e}{3}$ (*d*) e

9. Let $A : R^m \to R^n$ be a non-zero linear transformation. Which of the following statements is true?

(*a*) If A is one-to-one but not onto, then $m > n$

(*b*) If A is onto but not one-to-one, then $m < n$

(*c*) If A is bijective, then $m = n$

(*d*) If A is one-to-one, then $m = n$

10. Let a, b be two real numbers such that $a < 0 < b$. For a positive real number r, define $\gamma_r(t) = re^{it}$ (where $t \in [0, 2\pi]$) and $I_r = \dfrac{1}{2\pi i}\int_{\gamma r} \dfrac{z^2+1}{(z-a)(z-b)}dz$.

Which of the following statements is necessarily true?

(*a*) $I_r \neq 0$ if $r > \max\{|a|, b\}$

(*b*) $I_r \neq 0$ if $r < \max\{|a|, b\}$

(*c*) $I_r = 0$ if $r > \max\{|a|, b\}$ and $|a| = b$

(*d*) $I_r = 0$ if $|a| < r < b$

11. Let $(a_n)_{n \geq 1}$ be a bounded sequence in R. Which of the following statements is FALSE?

(*a*) if $\lim_{n\to\infty}\inf a_n = \lim_{n\to\infty}\sup a_n$, then (a_n) is convergent

(*b*) if $\inf\{a_n \mid n \geq 1\} = \limsup_{n\to\infty} a_n$, then (a_n) is convergent

(*c*) if $\sup\{a_n \mid n \geq 1\} = \liminf_{n\to\infty} a_n$, then (a_n) is constant

(*d*) if $\sup\{a_n \mid n \geq 1\} = \inf\{a_n \mid n \geq 1\}$, then (a_n) is constant

12. How many arrangements of the digits of the number 1234567 are there, such that exactly three of them occur in their original position. (E.g., in the arrangement 5214763, exactly the digits 2,4 and 6 are in their original positions. In the arrangement 1243576, exactly the digits 1,2 and 5 are in their original positions.)

(*a*) 525 (*b*) 35 (*c*) 840 (*d*) 315

13. For a complex number a such that $0 < |a| < 1$, which of the following statements is true?

(*a*) If $|z| < 1$, then $|1 - z| < |z - a|$

(*b*) If $|z - a| = |1 - \bar{a}z|$, then $|z| = 1$

(*c*) If $|z| = 1$, then $|z - a| < |1 - \bar{a}z|$

(*d*) If $|1 - \bar{a}z| < |z - a|$, then $|z| < 1$

14. Consider the set $A = \{x \in Q : 0 < (\sqrt{2}-1)x < \sqrt{2}+1\}$ as a subset of R. Which of the following statements is true?

(*a*) $\sup A = 2 + 2\sqrt{3}$ (*b*) $\sup A = 3 + 2\sqrt{2}$

(*c*) $\inf A = 2 + 2\sqrt{3}$ (*d*) $\inf A = 3 + 2\sqrt{2}$

15. Consider the contour γ given by

$$\gamma(\theta) = \begin{cases} e^{2i\theta} & \text{for } \theta \in [0, \pi/2] \\ 1 + 2e^{2i\theta} & \text{for } \theta \in [\pi/2, 3\pi/2] \\ e^{2i\theta} & \theta \in [3\pi/2, 2\pi] \end{cases}$$

Then what is the value of $\int_\gamma \dfrac{dz}{z(z-2)}$?

(*a*) 0 (*b*) πi (*c*) $-\pi i$ (*d*) $2\pi i$

16. Let $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$, and consider the symmetric bilinear form on R^4 given by $\langle v,w \rangle = v^tAw$ for $v, w \in R^4$. Which of the following statements is true?

(*a*) A is invertible

(*b*) There exist non-zero vectors v, w such that $\langle v,w \rangle = 0$

(*c*) $\langle u, v \rangle \neq \langle u, w \rangle$ for all non-zero vectors u, v, w with $v \neq w$

(*d*) Every eigenvalue of A^2 is positive

17. For each $n \geq 1$ define $f_n : R \to R$ by

$f_n(x) = \dfrac{x^2}{\sqrt{x^2 + \dfrac{1}{n}}}$, $x \in R$ where $\sqrt{}$ denotes the non-negative square root. Wherever $\lim_{n\to\infty} f_n(x)$ exists, denote it by $f(x)$.

Which of the following statements is true?

(*a*) There exists $x \in R$ such that $f(x)$ is not defined

(*b*) $f(x) = 0$ for all $x \in R$

(*c*) $f(x) = x$ for all $x \in R$

(*d*) $f(x) = |x|$ for all $x \in R$

18. For a quadratic form $f(x, y, z) \in R[x, y, z]$, we say that $(a, b, c) \in R^3$ is a zero of f if $f(a, b, c) = 0$. Which of the following quadratic forms has at least one zero different from (0, 0, 0)?

(*a*) $x^2 + 2y^2 + 3z^2$

(*b*) $x^2 + 2y^2 + 3z^2 - 2xy$

(*c*) $x^2 + 2y^2 + 3z^2 - 2xy - 2yz$

(*d*) $x^2 + 2y^2 - 3z^2$

19. Let S be a dense subset of R and $f : R \to R$ a given function. Define $g : S \to R$ by $g(x) = f(x)$. Which of the following statements is necessarily true?

(*a*) If f is continuous on the set S, then f is continuous on the set $R \setminus S$

(*b*) If g is continuous, then f is continuous on the set S

(*c*) If g is identically 0 and f is continuous on the set $R \setminus S$, then f is identically 0

(*d*) If g is identically 0 and f is continuous on the set S, then f is identically 0

20. Let $S = \left\{ x \in R : x > 1 \text{ and } \dfrac{1-x^4}{1-x^3} > 22 \right\}$.

Which of the following is true about S?

(*a*) S is empty.

(*b*) There is a bijection between S and N

(*c*) There is a bijection between S and R

(*d*) There is a bijection between S and a nonempty finite set

21. Let C be the collection of all sets S such that the power set of S is countably infinite. Which of the following statements is true?

(*a*) There exists a non-empty finite set in C

(*b*) There exists a countably infinite set in C

(*c*) There exists an uncountable set in C

(*d*) C is empty

22. Consider the ring $R = \{\sum_{n \in Z} a_n X^n \mid a_n \in Z$; and $a_n \neq 0$ only for finitely many $n \in Z\}$ where addition and multiplication are given by

$$\sum_{n \in Z} a_n X^n + \sum_{n \in Z} b_n X^n = \sum_{n \in Z} (a_n + b_n) X^n$$

$$\left(\sum_{n \in Z} a_n X^n\right)\left(\sum_{n \in Z} b_m X^m\right) = \sum_{k \in Z} \left(\sum_{n+m=k} a_n b_m\right) X^k$$

Which of the following statements is true?

(*a*) R is not commutative

(*b*) The ideal $(X - 1)$ is a maximal ideal in R

(*c*) The ideal $(X - 1, 2)$ is a prime ideal in R

(*d*) The ideal $(X, 5)$ is a maximal ideal in R

23. Let $\begin{pmatrix} 2 & a \\ b & c \end{pmatrix}$ be a 2×2 real matrix for which 6 is an eigenvalue. Which of the following statements is necessarily true?

(*a*) $24 - ab = 4c$ (*b*) $a + b = 8$

(*c*) $c = 6$ (*d*) $ab = 0$

24. The number of group homomorphisms from $Z/150Z$ to $Z/90Z$ is

(*a*) 30 (*b*) 60 (*c*) 45 (*d*) 10

25. Let A be a 10×10 real matrix. Assume that the rank of A is 7. Which of the following statements is necessarily true?

(*a*) There exists a vector $v \in R^{10}$ such that $Av \neq 0$ and $A^2 v = 0$

(*b*) There exists a vector $v \in R^{10}$ such that $A^2 v \neq 0$

(*c*) A must have a non-zero eigenvalue

(*d*) $A^7 = 0$

26. Consider R and $Q[x]$ as vector spaces over Q. Which of the following statements are true?

(*a*) There exists an injective Q-linear transformation $T : R \to Q[x]$

(*b*) There exists an injective Q-linear transformation $T : Q[x] \to R$

(*c*) The Q-vector spaces $Q[x]$ and R are isomorphic

(*d*) There do not exist non-zero Q-linear transformations $T : R \to Q[x]$

27. For which of the following values of q, does a finite field of order q have exactly 6 subfields?

(*a*) $q = 2^{18}$ (*b*) $q = 2^{32}$ (*c*) $q = 2^{12}$ (*d*) $q = 2^{243}$

28. Let $(a_n)_{n \geq 1}$ be a sequence of positive real numbers. Let

$$b_n = \frac{a_n}{\max\{a_1, \ldots, a_n\}}, n \geq 1$$

Which of the following statements are necessarily true?

(*a*) If $\lim_{n\to\infty} b_n$ exists in R, then $\{a_n : n \geq 1\}$ is bounded

(*b*) If $\lim_{n\to\infty} b_n = 1$, then $\lim_{n\to\infty} a_n$ exists in R

(*c*) If $\lim_{n\to\infty} b_n = \frac{1}{2}$, then $\lim_{n\to\infty} a_n$ exists in R

(*d*) If $\lim_{n\to\infty} b_n = 0$, then $\lim_{n\to\infty} a_n = 0$

29. For $z \in C \setminus \{0\}$, let $f(z) = \dfrac{1}{z}\sin\left(\dfrac{1}{z}\right)$ and $g(z) = f(z)\sin(z)$.

Which of the following statements are true?

(*a*) f has an essential singularity at 0

(*b*) g has an essential singularity at 0

(*c*) f has a removable singularity at 0

(*d*) g has a removable singularity at 0

30. Let $\sum_{n=1}^{\infty} a_n$ be a convergent series of real numbers.

For $n \geq 1$ define

$$A_n = \begin{cases} a_n, & \text{if } a_n > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$B_n = \begin{cases} a_n, & \text{if } a_n < 0 \\ 0, & \text{otherwise} \end{cases}$$

Which of the following statements are necessarily true?

(a) $A_n \to 0$ and $B_n \to 0$ as $n \to \infty$

(b) If $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, then both $\sum_{n=1}^{\infty} A_n$ and $\sum_{n=1}^{\infty} B_n$ are absolutely convergent

(c) Both $\sum_{n=1}^{\infty} A_n$ and $\sum_{n=1}^{\infty} B_n$ are convergent

(d) If $\sum_{n=1}^{\infty} a_n$ is not absolutely convergent, then both $\sum_{n=1}^{\infty} A_n$ and $\sum_{n=1}^{\infty} B_n$ are divergent

31. Let A be a 4 × 4 real matrix whose minimal polynomial is $x^2 + x + 1$ and let $B = A + I_4$. Which of the following statements are necessarily true?

(a) The minimal polynomial of B is $x^2 + x + 1$

(b) The minimal polynomial of B is $x^2 - x + 1$

(c) $B^3 = I_4$

(d) $B^3 + I_4 = 0$

32. Consider the real vector space $V = R[x]$ equipped with an inner product. Let W be the subspace of V consisting of polynomials of degree at most 2. Let $W^{\perp}$ denote the orthogonal complement of W in V. Which of the following statements are true?

(a) There exists a polynomial $p(x) \in W$ such that $x^4 - p(x) \in W^{\perp}$

(b) $W^{\perp} = \{0\}$

(c) W and $W^{\perp}$ have the same dimension over R

(d) $W^{\perp}$ is an infinite dimensional vector space over R

33. Let f be an entire function such that for every integer $k \geq 1$ there is an infinite set X_k such that $f(z) = \frac{1}{k}$ for all $z \in X_k$. Which of the following statements are necessarily true?

(a) There exists an infinite set X such that $f(z) = 0$ for all $z \in X$

(b) There exists a non-empty closed set X such that $f(z) = 0$ for all $z \in X$

(c) The set X_k is unbounded for each $k \geq 1$

(d) If there exists a bounded sequence $(z_k)_{k\geq 1}$ such that $z_k \in X_k$ for each $k \geq 1$, then f has a zero

34. Let $M_5(C)$ be the complex vector space of 5 × 5 matrices with entries in C. Let V be a non-zero subspace of $M_5(C)$ such that every non-zero $A \in V$ is invertible. Which among the following are possible values for the dimension of V?

(a) 1 (b) 2

(c) 3 (d) 5

35. Let I be an ideal of the ring $F_2[t]/(t^2(1-t)^2)$. Which of the following are the possible values for the cardinality of I?

(a) 1 (b) 8 (c) 16 (d) 24

36. Consider the initial boundary value problem (IBVP)

$$\begin{cases} u_t + u_x = 2u, & x > 0, t > 0 \\ u(0,t) = 1 + \sin t, & t > 0 \\ u(x,0) = e^x \cos x, & x > 0 \end{cases}$$

If u is the solution of the IBVP, then the value of $\frac{u(2\pi,\pi)}{u(\pi,2\pi)}$ is

(a) $e^{2\pi}$ (b) $e^{-\pi}$ (c) $-e^{\pi}$ (d) $-e^{-\pi}$

37. The extremizer of the problem

$$\min\left[\frac{1}{2}\int_{-1}^{1}\left[(y'(x))^2 + (y(x))^2\right]dx\right] \text{ subject to}$$

$y \in C^1[-1,1], \int_{-1}^{1} xy(x)\,dx = 0$ and $y(-1) = y(1) = 1$ is

(a) $\frac{e}{1+e^2}\left(e^x + e^{-x}\right) + x^2 - 1$

(b) $\frac{e}{1+e^2}\left(e^x + e^{-x}\right) + 1 - x^2$

(c) $\frac{e}{1+e^2}\left(e^x + e^{-x}\right)$

(d) $\frac{e}{1+e^2}\left(e^x + e^{-x}\right) + \sin(2\pi x)$

38. Let $K \subseteq R$ be non-empty and $f: K \to K$ be continuous such that $|x - y| \leq |f(x) - f(y)|\ \forall x, y \in K$.

Which of the following statements are true?

(a) f need not be surjective

(b) f must be surjective if $K = [0, 1]$

(c) f is injective and $f^{-1}: f(K) \to K$ is continuous

(d) f is injective, but $f^{-1}: f(K) \to K$ need not be continuous

39. Let $T: R^4 \to R^4$ be a linear map with four distinct eigenvalues and satisfying $T^4 - 15T^2 + 10T + 24I = 0$.

Which of thefollowing statements are necessarily true?

(a) There exists a non-zero vector $v_1 \in R^4$ such that $Tv_1 = 2v_1$

(b) There exists a non-zero vector $v_2 \in R^4$ such that $Tv_2 = v_2$

(c) For every non-zero vector $v \in R^4$, the set $\{2v, 3Tv\}$ is linearly independent

(d) T is a one-one function

40. Define $f : R^2 \to R$ by $f(x,y) = \begin{cases} \frac{y\sqrt{x^2+y^2}}{x} & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

Which of the following statements are true?

(*a*) $\frac{\partial f}{\partial x}(0,0)$ exists

(*b*) $\frac{\partial f}{\partial y}(0,0)$ exists

(*c*) f is not continuous at (0, 0)

(*d*) f is not differentiable at (0, 0)

41. Let R and S be non-zero commutative rings with multiplicative identities 1_R, 1_S, respectively. Let $f : R \to S$ be a ring homomorphism with $f(1_R) = 1_S$. Which of the following statements are true?

(*a*) If $f(a)$ is a unit in S for every non-zero element $a \in R$, then S is a field

(*b*) If $f(a)$ is a unit in S for every non-zero element $a \in R$, then $f(R)$ is a field

(*c*) If R is a field, then $f(a)$ is a unit in S for every non-zero element $a \in R$

(*d*) If a is a unit in R, then $f(a)$ is a unit in S

42. Let $f : [0, 1) \to [1, \infty)$ be defined by $f(x) = \frac{1}{1-x}$. For $n \geq 1$, let $p_n(x) = 1 + x + \cdots + x^n$. Then which of the following statements are true?

(*a*) $f(x)$ is not uniformly continuous on [0, 1)

(*b*) The sequence $(p_n(x))$ converges to $f(x)$ pointwise on [0, 1)

(*c*) The sequence $(p_n(x))$ converges to $f(x)$ uniformly on [0, 1)

(*d*) The sequence $(p_n(x))$ converges to $f(x)$ uniformly on $[0, c]$ for every $0 < c < 1$

43. For $c \in R$, consider the following Fredholm integral equation $y(x) = 1 + x + cx^2 + 2\int_0^1 (1-3xt)\, y(t)\, dt$

Then the values of c for which the integral equation admits a solution are

(*a*) –8 (*b*) –6 (*c*) 2 (*d*) 6

44. Suppose that f is an entire function such that $|f(z)| \geq 2024$ for all $z \in C$. Which of the following statements are necessarily true?

(*a*) $f(z) = 2024$ for all $z \in C$

(*b*) f is a constant function

(*c*) f is an injective function

(*d*) f is a bijective function

45. The infimum of the set

$$\left\{\int_a^b \sqrt{1+\left(y'(t)\right)^2}\, dt : y \in C^1[a,b], y(a) = a^2,\ y(b) = b-5\right\} \text{ is}$$

(*a*) $\frac{19\sqrt{2}}{8}$ (*b*) $19\sqrt{2}$ (*c*) $\frac{19}{8}$ (*d*) $\frac{19}{2\sqrt{2}}$

46. Let $f : R^2 \to R^3$ be a differentiable function such that $(Df)(0, 0)$ has rank. Write $f = (f_1, f_2, f_3)$. Which of the following statements are necessarily true?

(*a*) f is injective in a neighbourhood of (0,0)

(*b*) There exists an open neighbourhood U of (0,0) in R^2 such that f_3 is a function of f_1 and f_2

(*c*) f maps an open neighbourhood of (0,0) in R^2 onto an open subset of R^3

(*d*) (0,0) is an isolated point of $f^{-1}(\{ f(0,0) \})$

47. Let V be the subspace spanned by the vectors $v_1 = (1,0,2,3,1)$, $v_2 = (0,0,1,3,5)$, $v_3 = (0,0,0,0,1)$ in the real vector space R^5.

Which of thefollowing vectors are in V?

(*a*) $(1,1,1,1,1)$ (*b*) $(0,0,1,2,4)$

(*c*) $(1,0,1,0,1)$ (*d*) $(1,0,1,0,2)$

48. Let $g(x)$ be the polynomial of degree at most 4 that interpolates the data

x	–1	0	2	3	6
y	–30	1	c	10	19

If $g(4) = 5$, then which of the following statements are true?

(*a*) $c = 13$ (*b*) $g(5) = 6$

(*c*) $g(1) = 14$ (*d*) $c = 15$

49. Let $(a_n)_{n\geq 1}$ be a bounded sequence of real numbers such that $\lim_{n\to\infty} a_n$ does not exist. Let $S = \{l \in R$: there exists a subsequence of (a_n) converges to$l\}$. Which of the following statements arenecessarily true?

(*a*) S is the empty set

(*b*) S has exactly one element

(*c*) S has at least two elements

(*d*) S has to be a finite set

50. Which of the following conditions ensure that the power series $\sum_{n\geq 0} a_n z^n$ defines an entire function?

(*a*) The power series converges for every $z \in C$

(*b*) The power series converges for every $z \in R$

(*c*) The power series converges for every $z \in \{2^n : n \in N\}$

(*d*) The power series converges for every $z \in \left\{\frac{1}{5^n} : n \in N\right\}$

51. Which of the following numbers are order of some element of the symmetric group S_5?

(*a*) 3 (*b*) 4 (*c*) 5 (*d*) 6

52. Define $f : R \to R$ by $f(x) = x|x|$. Which of the following statements are true?

(*a*) f is continuous on R

(*b*) f is differentiable on R

(*c*) f is differentiable only at 0

(*d*) f is not differentiable at 0

53. Let X denote the topological space R with the cofinite topology (i.e., the finite complement topology) and let Y denote the topological space R with the Euclidean topology. Which of the following statements are true?

(*a*) $X \times [0,1]$ is closed in $X \times Y$ with respect to the product topology

(*b*) $X \times [0,1]$ is compact with respect to the product topology

(*c*) X is compact

(*d*) $X \times Y$ is compact with respect to the product topology

54. Let $f : R \to R$ be a continuous and one-to-one function. Which of the following statements are necessarily true?

(*a*) f is strictly increasing

(*b*) f is strictly decreasing

(*c*) f is either strictly increasing or strictly decreasing

(*d*) f is onto

55. Consider the initial value problem (IVP)

$$y'(x) = \frac{\sin(y(x))}{1 + y^4(x)}, x \in R, y(0) = y_0.$$

Then which of the following statements are true?

(*a*) There is a positive y_0 such that the solution of the IVP is unbounded

(*b*) There is a negative y_0 such that the solution of the IVP is bounded

(*c*) For every $y_0 \in R$, every solution of the IVP is bounded

(*d*) For every $y_0 \in R$, there is a solution to the IVP for all $x \in R$

56. Consider the improper integrals $I = \int_{\pi/2}^{\pi} \frac{1}{\sqrt{\sin x}} dx$ and,

for $a \geq 0$ $I_a = \int_a^{\infty} \frac{1}{x\sqrt{1+x^2}} dx$

(*a*) The integral I is convergent

(*b*) The integral I is not convergent

(*c*) The integral I_a converges for $a = \frac{1}{2}$ but not for $a = 0$

(*d*) The integral I_a converges for all $a \geq 0$

57. Let $q_1(x_1, x_2)$ and $q_2(y_1, y_2)$ be real quadratic forms such that there exist $(u_1, u_2), (v_1, v_2) \in R^2$ such that $q_1(u_1, u_2) = 1 = q_2(v_1, v_2)$. Define $q(x_1, x_2, y_1, y_2) = q_1(x_1, x_2) - q_2(y_1, y_2)$. Which of the following statements are necessarily true?

(*a*) q is a quadratic form in x_1, x_2, y_1, y_2

(*b*) There exists $(t_1, t_2) \in R^2$ such that $q_1(t_1, t_2) = 5$

(*c*) There does not exist $(s_1, s_2) \in R^2$ such that $q_2(s_1, s_2) = -5$

(*d*) Given $\alpha \in R$, there exists a vector $\omega \in R^4$ such that $q(\omega) = \alpha$

58. For $\lambda \in R$ such that $|\lambda| < \frac{5}{32}$, let $R(x, t, \lambda)$ and u denote the resolvent kernel and the solution, respective the Fredholm integral equation

$u(x) = x + \frac{\lambda}{2}\int_{-2}^{2}(xt + x^2t^2)u(t)dt$ .Then which of the following statements are true?

(*a*) $R(x,t,\lambda) = \frac{3xt}{3-8\lambda} - \frac{5x^2t^2}{5-32\lambda}$

(*b*) $R(x,t,\lambda) = \frac{3xt}{3-8\lambda} + \frac{5x^2t^2}{5-32\lambda}$

(*c*) $u(1) = -\frac{5}{5-32\lambda}$

(*d*) $u(1) = \frac{3}{3-8\lambda}$

59. For two indeterminates x, y, let $R = F_3[x]$ and $S = R[y]$. Which of the following statements are true?

(*a*) S is a principal ideal domain

(*b*) $S/(y^2 + x^2)$ is a unique factorization domain

(*c*) S is a unique factorization domain

(*d*) $S/(x)$ is a principal ideal domain

60. Let R be a principal ideal domain with a unique maximal ideal. Which of the following statements are necessarily true?

(*a*) Every quotient ring of R is a principal ideal domain

(*b*) There exists a quotient ring S of R and an ideal $I \subseteq S$ which is not principal

(*c*) R has countably many ideals

(*d*) Every quotient ring $S(\neq \{0\})$ of R has a unique maximal ideal which is principal

61. Let V ($\neq \{0\}$) be a finite dimensional vector space over R and $T : V \to V$ be a linear operator. Suppose that the kernel of T equals the image of T. Which of the following statements are necessarily true?

(*a*) The dimension of V is even

(*b*) The trace of T is zero

(*c*) The minimal polynomial of T cannot have two distinct roots

(*d*) The minimal polynomial of T is equal to its characteristic polynomial

62. If $x_1 = x_1(t)$, $x_2 = x_2(t)$ is the solution of the initial value problem

$$e^{-t}\frac{dx_1}{dt} = -x_1 + x_2$$

$$e^{-t}\frac{dx_2}{dt} = -x_1 - x_2,$$

$x_1(0) = 1, x_2(0) = 0$.and $r(t) = \sqrt{x_1^2(t) + x_2^2(t)}$,

then which ofthe following statements are true?

(*a*) $r(t) \to 0$ as $t \to +\infty$ (*b*) $r(\ln 2) = e^{-1}$

(*c*) $r(\ln 2) = 2e^{-1}$ (*d*) $r(t)e^t \to 0$ as $t \to +\infty$

ANSWER KEY

1. (*d*) **2.** (*a*) **3.** (*a*) **4.** (*c*) **5.** (*d*) **6.** (*d*) **7.** (*b*) **8.** (*a*) **9.** (*c*) **10.** (*c*)
11. (*c*) **12.** (*a*) **13.** (*b*) **14.** (*b*) **15.** (*c*) **16.** (*b*) **17.** (*d*) **18.** (*d*) **19.** (*c*) **20.** (*c*)
21. (*d*) **22.** (*d*) **23.** (*a*) **24.** (*a*) **25.** (*a*) **26.** (*b*) **27.** (*a, b, c, d*) **28.** (*c, d*) **29.** (*a, b*)
30. (*a, b, d*) **31.** (*b, d*) **32.** (*a, d*) **33.** (*c, d*) **34.** (*a*) **35.** (*a, b, c*) **36.** (*a*) **37.** (*c*) **38.** (*a*)
39. (*a, d*) **40.** (*a, b, c, d*) **41.** (*c, d*) **42.** (*a, b, d*) **43.** (*a*) **44.** (*b*) **45.** (*a*) **46.** (*a*)
47. (*c, d*) **48.** (*b, c, d*) **49.** (*c*) **50.** (*a, b, c*) **51.** (*a, b, c, d*) **52.** (*a, b*)
53. (*a, b, d*) **54.** (*c*) **55.** (*b, c, d*) **56.** (*a, c*) **57.** (*a, b, d*) **58.** (*b, d*) **59.** (*c, d*) **60.** (*c, d*)
61. (*a, b, c*) **62.** (*a, b, d*)

SOLUTION

1. (*d*) $R = \{(x, y) \in D; |x - x_0| \leq a, |y - y_0| \leq b\}$

$f(x, y) = \sqrt{|y + \varepsilon|}$ which is constant.

$\Rightarrow$ Solution exists.

Also $\left|\frac{\partial f}{\partial y}\right| = \left|\frac{1}{2\sqrt{y+\varepsilon}}\right| < L$ for $y_0 \neq -\varepsilon$

Hence, S_1, S_2 both are false.
Hence, option (*d*) is correct.

2. (*a*) Convolution:
$F(t) * G(t) = \int_0^t F(t) \cdot G(t - x)dx$
Convolution Theorem:
If $L(f(t)) = f(s)$ & $L(g(t)) = g(s)$
then $L(f(t) * g(t)) = f(s) \cdot g(s)$

Given $\int_0^t \left[\frac{1}{2} + \sin(t - T)\right] u(T) dT = \sin t$.

$$\Rightarrow \sin(t) = \left(\frac{1}{2} + \sin(t)\right) u(t)$$

$$\Rightarrow \mathcal{L}(\sin(t)) = \mathcal{L}\left(\frac{1}{2} + \sin(t)\right)$$

$$\cdot \mathcal{L}(u(t))$$

$$\Rightarrow \mathcal{L}(u(t)) = \frac{\mathcal{L}(\sin(t))}{\mathcal{L}\left(\frac{1}{2} + \sin(t)\right)}$$

$$\Rightarrow u(t) = \mathcal{L}^{-1}\left\{\frac{\frac{1}{s^2+1}}{\frac{1}{2s} + \frac{1}{s^2+1}}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{2s}{(s+1)^2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{2s+2-2}{(s+1)^2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{2}{s+1}-\frac{2}{(s+1)^2}\right\}$$

$$= e^t \cdot \mathcal{L}^{-1}\left\{\frac{2}{s}-\frac{2}{s^2}\right\}$$

$$= e^t \cdot \left[\mathcal{L}^{-1}\left(\frac{2}{s}\right)-\mathcal{L}^{-1}\left(\frac{2}{s^2}\right)\right]$$

$$\Rightarrow u(t) = e^t[2-2t]$$

$$\Rightarrow u(1) = 0$$

Hence, option (*a*) is correct.

4. (*c*) Given $V = M_2(\mathbb{R})$.

Basis of $M_2(\mathbb{R})$ is given by

$$\beta = \left\{\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\1 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}\right\}$$

$$T\left(\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\right) = \begin{bmatrix}2 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} = \begin{bmatrix}2 & 0\\0 & 0\end{bmatrix}$$

$$= 2\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} + 0\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} + 0\begin{bmatrix}0 & 0\\1 & 0\end{bmatrix} + 0\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$$

Similarly we can find the other elements.

Now, $[T]_\beta^\beta$ is given by $\begin{bmatrix}2 & 0 & 0 & 0\\0 & 2 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$.

$\Rightarrow Ch_T(x) = (x-2)^2(x-1)^2$

Hence, option (*c*) is correct.

5. (*d*) Properties of Entire Function:

- If $\text{Re}(f)$, $\text{Im}(f)$ are bounded then f is constant.
- If $e^{|Ref|+|Imf|}$ is bounded, then f is constant.
- If the sum $\text{Re}(f)+\text{Im}(f)$ and the product $\text{Re}(f)\,\text{Im}(f)$ are bounded, then f is constant.

Hence, option (*d*) is false.

6. (*d*) Since $f(x,y) = \sqrt{|y+\varepsilon|}$ is constant

Hence, solution exist and

$$\left|\frac{\partial f}{\partial y}\right| = \left|\frac{1}{2\sqrt{y+\varepsilon}}\right| < L,\ y_0 \neq \varepsilon.$$

Hence, S_1, S_2 both are false.

Hence, option (*d*) is correct.

7. (*b*) Since only $x = 0$ satisfy $e^x + x = 1$

Hence, cardinality of the set is 1.

8. (*a*) Cauchy Euler Differential Equation:

$$xy'' + y' - 2y' + \frac{y}{x} = 1 \Rightarrow x^2y'' - xy' + y = x$$

Put $x = e^z \Rightarrow z = \log x$

$$[D_1(D_1-1) - D_1 + 1]y = e^z; D_1 \equiv \frac{d}{dz}$$

$$\Rightarrow [D_1^2 - 2D_1 + 1]y = e^z$$

A.E $(m-1)^2 = 0 \Rightarrow m = 1,1$

$C\cdot F = c_1e^z + c_2ze^z$

$= c_1x + c_2x\log x$

$$\text{P.I.} = \frac{1}{(D_1-1)^2}\cdot e^z$$

$$= \frac{z}{2(D_1-1)}e^z$$

$$= \frac{z^2}{2}e^z$$

$$y = c_1x + c_2x\ln x + \frac{x}{2}(\ln x)^2$$

$$y(1) = 0 \Rightarrow 0 = c_1$$

$$y(e^4) = 4e^4$$

$$\Rightarrow 4e^4 = c_2e^4\ln e^4 + \frac{e^4}{2}(\ln e^4)^2 \Rightarrow c_2 = -1$$

$$\Rightarrow y(x) = -x\ln x + \frac{x}{2}(\ln x)^2$$

$$\Rightarrow \varphi(e) = y(e) = \frac{-e}{2}$$

Hence, option (*a*) is correct.

9. (*c*) (*a*) If A is one-one but not onto then $\eta(A) = 0$

Now, By Rank-Nullity Theorem

$m = \rho(A) + \eta(A)$

$\Rightarrow m = \rho(A) \leq n$

Hence, option (*a*) is incorrect.

(*b*) If A is onto but not one-one then $n < m$.

Hence, option (*b*) is incorrect.

(*c*) If A is bijective then

$\rho(A) = n$, $\eta(A) = 0$

$\Rightarrow m = \rho(A) + \eta(A) = n$

Hence, option (*c*) is correct.

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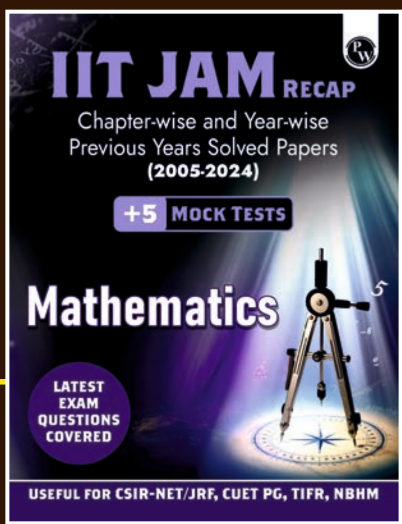

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