Absolute

JEE ADVANCED

2500+ Questions

MATHEMATICS

This Book Consists of Advanced Level Questions with 3 Mock Test

- Single Correct Choice Questions
- One or Multiple Correct Choice Question
- Matrix match/List type Questions
- Integer / Numerical type Questions
- Comprehension type Questions

CONTENTS

100 Important Concepts/Formulas

RELATION AND FUNCTIONS

CONCEPT-1

A relation R from a set A to a set B is a subset of the cartesian product $A \times B$ obtained by describing a relationship between the first element x and the second element y of the ordered pairs in $A \times B$.

CONCEPT-2

Function: A function from a set A to a set B is a specific type of relation for which every element x of set A has one and only one image y in set B. We write $f : A \rightarrow B$, where $f(x) = y$.

CONCEPT-3

A function $f : X \rightarrow Y$ is one-one (or injective) if

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \forall x_1, x_2 \in X$ only

CONCEPT-4

A function $f : X \rightarrow Y$ is onto (or surjective) if given any $y \in Y$, $\forall\ x \in X$ such that $f(x) = y$.

CONCEPT-5

Classification of function

(*a*) Many-One Function :

A function f: $A \rightarrow B$ is called many- one, if two or more different elements of A have the same f- image in B.

(*b*) Into function :

A function f: $A \rightarrow B$ is into if there exist at least one element in B which is not the f-image of any element in A.

(*c*) Many One-Onto function :

A function f: $A \rightarrow R$ is said to be many one- onto if f is onto but not one-one.

(*d*) Many One-Into function:

A function is said to be many one-into if it is neither one-one nor onto.

CONCEPT-6

A function f: $X \rightarrow Y$ is invertible if and only if f is one-one and onto.

TRIGONOMETRIC FUNCTIONS AND EQUATIONS

CONCEPT-7

General Solution of the equation

$\sin\theta = 0$:

when $\sin\theta = 0$

$\theta = n\pi : n \in I$ *i.e.*, $n = 0, \pm 1, \pm 2$…......

General solution of the equation

$\cos\theta = 0$:

when $\cos\theta = 0$

$\theta = (2n + 1)\ \pi/2,\ n \in I.$ *i.e.*, $n = 0, \pm 1, + 2$…….

General solution of $\tan\theta = 0$ is $\theta = n\pi;\ n \in I$

CONCEPT-8

General solution of the equation

(*a*) $\sin\theta = \sin\alpha : \theta = n\pi + (-1)^n\alpha\ ;\ n \in I$

(*b*) $\sin\theta = k$, where $-1 < k < 1.\ \alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\theta = n\pi + (-1)^n\alpha$ where $n \in I$ and $\alpha = \sin^{-1}k$

(*c*) $\cos\theta = \cos\alpha : \theta = 2n\pi \pm \alpha,\ n \in I,\ \alpha \in (0, \pi)$

(*d*) $\cos\theta = k$, where $-1 < k < 1$.

$\theta = 2n\pi \pm \alpha$, where $n \in I$ and $\alpha = \cos^{-1}k$

(*e*) $\tan\theta = \tan\alpha : \theta = n\pi + \alpha\ ;\ n \in I$

(*f*) $\tan\theta = k,\ \theta = n\pi + \alpha$, where $n \in I$ and $\alpha = \tan^{-1}$ k

(*g*) $\sin^2\theta = \sin^2\alpha : \theta = n\pi \pm \alpha\ ;\ n \in I$

(*h*) $\cos^2\theta = \cos^2\alpha : \theta = n\pi \pm \alpha\ ;\ n \in I$

(*i*) $\tan^2\theta = \tan^2\alpha : \theta = n\pi \pm \alpha\ ;\ n \in I$

CONCEPT-9

$\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \ldots\ldots$ to n terms

$$= \frac{\sin\left[\alpha + \left(\frac{n-1}{2}\right)\beta\right]\left[\sin\left(\frac{n\beta}{2}\right)\right]}{\sin(\beta/2)}; \beta \neq 2n\pi$$

CONCEPT-10

$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \ldots\ldots$ to n terms

$$= \frac{\cos\left[\alpha + \left(\frac{n-1}{2}\right)\beta\right]\left[\sin\left(\frac{n\beta}{2}\right)\right]}{\sin\left(\frac{\beta}{2}\right)}; \beta \neq 2n\pi$$

CONCEPT-11

In a triangle ABC, $a = BC$, $b = CA$, $c = AB$ side lengths

$$\tan\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{b+c}\right)\cot\left(\frac{A}{2}\right)$$

CONCEPT-12

$$\sin\left(\frac{A}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

CONCEPT-13

$$\tan\left(\frac{A}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

CONCEPT-14

$$R = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C} = \frac{abc}{4\Delta}$$

CONCEPT-15

$$r = 4R\sin\left(\frac{A}{2}\right)\cdot\sin\left(\frac{B}{2}\right)\cdot\sin\left(\frac{C}{2}\right)$$

CONCEPT-16

Projection formula

$a = b\cos C + c\cos B$

CONCEPT-17

Maximum value of $a\sin\theta + b\cos\theta = \sqrt{a^2+b^2}$

and minimum value of $a\sin\theta\, b\cos\theta = -\sqrt{a^2+b^2}$

INVERSE TRIGONOMETRIC FUNCTIONS

CONCEPT-18

Properties of inverse trigonometric function

$\tan^{-1}x + \tan^{-1}y$

$$= \begin{cases} \tan^{-1}\left(\frac{x+y}{1-xy}\right), & \text{if } xy < 1 \\ \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right), & \text{if } x > 0, y > 0 \\ \text{and } xy > 1 & \\ -\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right), & \text{if } x < 0, y < 0 \\ \text{and } xy > 1 & \end{cases}$$

$\tan^{-1}x - \tan^{-1}y$

$$= \begin{cases} \tan^{-1}\left(\frac{x-y}{1+xy}\right), & \text{if } xy > -1 \\ \pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right), & \text{if } x > 0, y < 0 \text{ and } xy < -1 \\ -\pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right), & \text{if } x < 0, y > 0 \text{ and } xy < -1 \end{cases}$$

$\sin^{-1}x + \sin^{-1}y$

$$= \begin{cases} \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\}, & \text{if } -1 \le x, y \le 1 \text{ and } x^2+y^2 \le 1 \text{ or if } xy < 0 \text{ and } x^2+y^2 > 1 \\ \pi - \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\}, & \text{if } 0 < x, y \le 1 \text{ and } x^2+y^2 > 1 \\ -\pi - \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\}, & \text{if } -1 \le x, y < 0 \text{ and } x^2+y^2 > 1 \end{cases}$$

$\cos^{-1}x + \cos^{-1}y$

$$2\sin^{-1}x = \begin{cases} \sin^{-1}\left(2x\sqrt{1-x^2}\right), & \text{if } -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}} \\ \pi - \sin^{-1}\left(2x\sqrt{1-x^2}\right), & \text{if } \frac{1}{\sqrt{2}} \le x \le 1 \\ -\pi - \sin^{-1}\left(2x\sqrt{1-x^2}\right), & \text{if } -1 \le x \le -\frac{1}{\sqrt{2}} \end{cases}$$

$$2\tan^{-1}x = \begin{cases} \tan^{-1}\left(\frac{2x}{1-x^2}\right), & \text{if } -1 < x < 1 \\ \pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right), & \text{if } x > 1 \\ -\pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right), & \text{if } x < -1 \end{cases}$$

QUADRATIC EQUATIONS AND INEQUALITIES

CONCEPT-19

Roots of a Quadratic Equation : The roots of the quadratic equation are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Nature of roots: In Quadratic equation $ax^2 + bx + c = 0$. The term $b^2 - 4ac$ is called discriminant of the equation. It is denoted by Δ or D.

(A) Suppose $a, b, c \in R$ and $a \neq 0$

(i) If $D > 0 \Rightarrow$ Roots are Real and unequal

(ii) If $D = 0 \Rightarrow$ Roots are Real and equal and each equal to $-b/2a$

(iii) If $D < 0 \Rightarrow$ Roots are imaginary and unequal or complex conjugate.

(B) Suppose $a, b, c \in Q$ and $a \neq 0$

(i) If $D > 0$ and D is perfect square $\Rightarrow$ Roots are unequal and Rational

(ii) If $D > 0$ and D is not perfect square $\Rightarrow$ Roots are irrational and unequal.

CONCEPT-20

If $p + iq$ (p and q being real) is a root of the quadratic equation, where $i = \sqrt{-1}$, then $p - iq$ is also a root of the quadratic equation.

CONCEPT-21

Every equation of nth degree ($n \geq 1$) has exactly n roots and if the equation has more than n roots, it is an identity.

COMPLEX NUMBERS

CONCEPT-22

Exponential Form: If $z = x + iy$ is a complex number then its exponential form is $z = re^{i\theta}$ where r is modulus and $\theta$is amplitude of complex number.

CONCEPT-23

(i) $|z_1| + |z_2| \geq |z_1 + z_2|$; here equality holds when arg $(z_1/z_2) = 0$ i.e. z_1 and z_2 are parallel.

(ii) $|z_1| - |z_2| \geq |z_1 - z_2|$; here equality holds when arg $(z_1/z_2) = 0$ *i.e.*, z_1 and z_2 are parallel.

(iii) $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\left(|z_1|^2 + |z_2|^2\right)$

CONCEPT-24

$\arg(z_1 z_2) = \theta_1 + \theta_2 = \arg(z_1) + \arg(z_2)$

CONCEPT-25

$$\arg\left(\frac{z_1}{z_2}\right) = \theta_1 - \theta_2 = \arg(z_1) - \arg(z_2)$$

CONCEPT-26

For any integer$k$,

$$i^{4k} = 1, i^{4k+1} = i, i^{4k+2} = -1, i^{4k+3} = -i$$

CONCEPT-27

$|z - z_1| + |z - z_2| = \lambda$, represents an ellipse if $|z_1 - z_2| < \lambda$, having the points z_1 and z_2 as its foci. And if $|z_1 - z_2| = \lambda$, then z lies on a line segment connecting z_1 and z_2.

CONCEPT-28

Properties of Cube Roots of Unity

(i) $1 + \omega + \omega^2 = 0$

(ii) $\omega^3 = 1$

(iii) $1 + \omega^n + \omega^{2n} = 3$ (if n is multiple of 3)

(iv) $1 + \omega^n + \omega^{2n} = 0$ (if n is not a multiple of 3).

BINOMIAL THEOREM

CONCEPT-29

Greatest binomial coefficients: In a binomial expansion binomial coefficients of the middle terms are called as greatest binomial coefficients.

(*a*) If n is even : When $r = \frac{n}{2}$ *i.e.*, ${}^nC_{n/2}$ takes maximum value.

(*b*) If n is odd : $r = \frac{n-1}{2}$ or $\frac{n+1}{2}$ i.e.,

${}^nC_{\frac{n-1}{2}} = {}^nC_{\frac{n+1}{2}}$ and take maximum value.

CONCEPT-30

Important Expansions:

If $|x| < 1$ and n ÎQ but $n \notin N$, then

(*a*) $$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots\ldots + \frac{n(n-1)\ldots..(n-r+1)}{r!}x^r + \ldots\ldots.$$

(*b*) $$(1-x)^n = 1 - nx + \frac{n(n-1)}{2!}x^2 - \frac{n(n-1)(n-2)}{3!}x^3 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}(-x)^r + \ldots$$

PERMUTATIONS AND COMBINATIONS

CONCEPT-31

The number of permutations of n different things, taken r at a time, where repetition is allowed, is n^r.

CONCEPT-32

Selection of Objects with Repetition :

The total number of selections of r things from n different things when each thing may be repeated any number of times is ${}^{n+r+1}C_r$

CONCEPT-33

Selection from distinct objects :

The number of ways (or combinations) of n different things selecting at least one of them is ${}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_n = 2^n - 1$. This can also be stated as the total number of combination of n different things.

CONCEPT-34

Selection fromidentical objects :

The number of ways to select some or all out of $(p + q + r)$ things where $p$are alike of first kind, q are alike of second kind and r are alike of third kind is

$(p + 1)(q + 1)(r + 1) -1$

CONCEPT-35

Selection when both identical and distinct objects are present:

If out of $(p + q + r + t)$ things, $p$are alike one kind, q are alike of second kind, r are alike of third kind and t are different, then the total number of combinations is

$(p + 1)(q + 1)(r + 1)2^t - 1$

CONCEPT-36

Circular permutations:

(a) Arrangements round a circular table:

The number of circular permutations of n different things taken all at a time is $\frac{{}^nP_n}{n} = (n-1)\,!$, if clockwise and anticlockwise orders are taken as different.

(b) Arrangements of beads or flowers (all different) around a circular necklace or garland: The number of circular permutations of 'n' different things taken all at a time is $\frac{1}{2}(n-1)!$, if clockwise and anticlockwise orders are taken to be some.

CONCEPT-37

Sum of numbers :

(a) For given n different digits $a_1, a_2, a_3 \ldots a_n$ the sum of the digits in the unit place of all numbers formed (if numbers are not repeated) is $(a_1 + a_2 + a_3 + ... + a_n)\,(n - 1)!$

(b) Sum of the total numbers which can be formed with given n different digits $a_1, a_2, \ldots a_n$ is $(a_1+a_2+a_3+\ldots+a_n)(n - 1)!.(111\ldots n\text{times})$

SEQUENCE AND SERIES

CONCEPT-38

Properties related to$A.P.$:

(i) Common difference of AP is given by$d = S_2 - 2S_1$ where S_2 is sum of first two terms and S_1 is sum of first term.

(ii) If for an AP sum of p terms is q, sum of q terms is p, then sum of $(p + q)$ term is $(p + q)$.

(iii) In an $A.P.$ the sum of terms equidistant from the beginning and end is constant and equal to sum of first and last terms.

(iv) If terms $a_1, a_2, \ldots, a_n, a_{n+1}, \ldots, a_{2n+1}$ are in $A.P.$, then sum of these terms will be equal to $(2n + 1)\, a_{n+1}$.

(v) If for an $A.P.$ sum of p terms is equal to sum of q terms then sum of $(p + q)$ terms is zero

(vi) Sum of n AM's inserted between a and b is equal to n times the single AM between a and b i.e. $\sum_{r=1}^{n} A_r = nA$ where $A = \frac{a+b}{2}$

CONCEPT-39

The geometric mean $(G.M.)$ of any two positive numbers a and b is given by $\sqrt{ab}$ *i.e.*, the sequence a, G, b is $G.P.$

nGM's between two given numbers: If in between two numbers 'a' and 'b', we have to insert $nGM G_1, G_2, \ldots G_n$ then $a_1, G_1, G_2, \ldots G_n, b$ will be in GP.

CHAPTER 3

Trigonometry

Solved Examples

1. Let $f: R \to R$ be such that $f(\tan x + \tan y) + f(\tan x - \tan y) = 2f(\tan x) \cdot f(\tan y)$ and $f\left(\frac{1}{2}\right) = -1$. Then the value of

$$\left(1-\cos^2 1\right)\cdot\sum_{k=1}^{20}\cot k\cdot\sec k\cdot\operatorname{cosec}\left(k+f(k)\right)$$

(a) $\frac{\sin 20}{\sin 19}$ (b) $\frac{\cos 20}{\cos 19}$

(c) $\frac{\sin 20}{\sin 21}$ (d) $\frac{\sin 21}{\sin 20}$

Sol. (c) Let $\tan x = x$ and $\tan y = y$

$\Rightarrow \quad f(x+y)+f(x-y)=2f(x)\cdot f(y)$

$f\left(\frac{1}{2}\right)=-1$

Put $y = 0 \Rightarrow 2f(x)\,[1 - f(0)] = 0$

$\Rightarrow \quad f(0) = 1$

Put $x=\frac{1}{2}, y=\frac{1}{2}$

$f(2) = 1$

$\Rightarrow \quad f(k) = 1$ for all integer

Now

$$\left(1-\cos^2 1\right)\cdot\sum_{k=1}^{20}\cot k\cdot\sec k\cdot\operatorname{cosec}\left(k+f(k)\right)$$

$$=\left(\sin^2 1\right)\cdot\sum_{k=1}^{20}\frac{\cos k}{\sin k}\cdot\frac{1}{\cos k}\cdot\frac{1}{\sin\left(k+f(k)\right)}$$

$$=\left(\sin^2 1\right)\left[\sum_{k=1}^{20}\frac{1}{\sin k\cdot\sin\left(k+f(k)\right)}\right]$$

$$=\left(\sin 1\right)\left[\sum_{k=1}^{20}\frac{\sin 1}{\sin k\cdot\sin\left(k+f(k)\right)}\right]$$

$$=\left(\sin 1\right)\left[\sum_{k=1}^{20}\frac{\sin\left\{(k+1)-k\right\}}{\sin k\cdot\sin\left(k+f(k)\right)}\right]$$

$$=\sin 1\left(\sum_{k=1}^{20}\left[\cot k-\cot(k+1)\right]\right)$$

$= \sin 1\ (\cot 1 - \cot 21)$

$$=\sin 1\cdot\frac{\sin 20}{\sin 1\cdot\sin 21}$$

$$=\frac{\sin 20}{\sin 21}$$

2. The value of $\tan\left(\frac{11\pi}{24}\right)$ is:

(a) $\sqrt{2}+\sqrt{3}+2-\sqrt{6}$ (b) $\sqrt{2}+\sqrt{3}+2+\sqrt{6}$

(c) $\sqrt{2}-\sqrt{3}-2+\sqrt{6}$ (d) $3\sqrt{2}-\sqrt{3}-\sqrt{6}$

Sol. (b) $\tan\left(\frac{11\pi}{24}\right)=\tan\left(\frac{\pi}{2}-\frac{\pi}{24}\right)$

$$=\cot\frac{\pi}{24}$$

Now

$$\cot\theta=\frac{1+\cos 2\theta}{\sin 2\theta}$$

Converting $\frac{\pi}{24}$ into degree i.e., $\frac{\pi}{24}\times\frac{180}{\pi}=\frac{15°}{2}$

$$\Rightarrow \quad \cot\left(\frac{15}{2}\right)^{o}=\frac{1+\cos 15°}{\sin 15°}$$

Now, $\cos 15° = \cos(45°-30°)=\frac{\sqrt{3}+1}{2\sqrt{2}}$

Similarly, $\sin 15°=\frac{\sqrt{3}-1}{2\sqrt{2}}$

Sol. (c) $5\cos^2\theta - 3\sin^2\theta + 6\sin\theta\cos\theta = 7$

$\Rightarrow 5\left(\frac{1+\cos 2\theta}{2}\right) - 3\left(\frac{1-\cos 2\theta}{2}\right) + 3\sin 2\theta = 7$

$\Rightarrow 4\cos 2\theta + 3\sin 2\theta = 6$,

but $4\cos 2\theta + 3\sin 2\theta \le \sqrt{4^2+3^2} = 5$

$\therefore$ Solution does not exist.

8. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, then general solution for θ is ($n \in z$)

(a) $2n\pi \pm \frac{\pi}{8}$ (b) $n\pi + \frac{\pi}{8}$

(c) $n\pi + (-1)^n\frac{\pi}{8}$ (d) None of these.

Sol. (b) $\sin\theta + \cos\theta = \sqrt{2}\cos\theta \Rightarrow \tan\theta = \sqrt{2}-1$

$\Rightarrow \theta = \frac{\pi}{8} \Rightarrow n\pi + \frac{\pi}{8}, n \in z.$

9. Number of solutions of 11 $\sin x = x$ is

(a) 4 (b) 6

(c) 8 (d) None of these.

Sol. (d) 11 $\sin x = x$. . . (1)

On replacing x by $-x$, we have 11 $\sin(-x) = -x$

$\Rightarrow 11\sin x = x$

So, for every positive solution, we have negative solution also and $x = 0$ is satisfying (1), so number of solution will always be odd. Therefore, (d) is appropriate choice.

10. If $\sqrt{3}\sin\pi x + \cos\pi x = x^2 - \frac{2}{3}x + \frac{19}{9}$, then x is equal to

(a) $-\frac{1}{3}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) None of these.

Sol. (b)

L.H.S. $= \sqrt{3}\sin\pi x + \cos\pi x = 2\sin\left(\pi x + \frac{\pi}{6}\right) \le 2$

and equality holds for $x = \frac{1}{3}$

and R.H.S. $= x^2 - \frac{2}{3}x + \frac{19}{9} = \left(x - \frac{1}{3}\right)^2 + 2 \ge 2$

equality holds if $x = \frac{1}{3}$.

Thus L.H.S. = R.H.S. for $x = \frac{1}{3}$ only.

11. General solution for θ if ($n \in z$)

$\sin\left(2\theta + \frac{\pi}{6}\right) + \cos\left(\theta + \frac{5\pi}{6}\right) = 2$, is

(a) $2n\pi + \frac{7\pi}{6}$ (b) $2n\pi + \frac{\pi}{6}$

(c) $2n\pi - \frac{7\pi}{6}$ (d) None of these.

Sol. (a) $\sin\left(2\theta + \frac{\pi}{6}\right) + \cos\left(\theta + \frac{5\pi}{6}\right) = 2$ . . . (1)

$\because \sin\left(2\theta + \frac{\pi}{6}\right) \le 1$ and $\cos\left(\theta + \frac{5\pi}{6}\right) \le 1$

$\therefore$ (1) may holds true iff $\sin\left(2\theta + \frac{\pi}{6}\right)$ and $\cos\left(\theta + \frac{5\pi}{6}\right)$ both equal to 1 simultaneously. First common value of θ is $\frac{7\pi}{6}$ for which

$\sin\left(2\theta + \frac{\pi}{6}\right) = \sin\frac{5\pi}{2} = \sin\frac{\pi}{2} = 1$

and $\cos\left(\theta + \frac{5\pi}{6}\right) = \cos\left(\frac{7\pi}{6} + \frac{5\pi}{6}\right) = \cos 2\pi = 1$

and since periodicity of $\sin\left(2\theta + \frac{\pi}{6}\right)$ is π

and periodicity of $\cos\left(\theta + \frac{5\pi}{6}\right)$ is 2π, therefore,

periodicity of $\sin\left(2\theta + \frac{\pi}{6}\right) + \cos\left(\theta + \frac{5\pi}{6}\right)$ is 2π.

Therefore, general solution is $\theta = 2n\pi + \frac{7\pi}{6}$.

12. If $\tan\alpha$ and $\tan\beta$ are the roots of $x^2 - 3x - 1 = 10$, then the value of $\tan(\alpha + \beta)$ is

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{3}{2}$ (d) None of these.

Sol. (*c*) $\because$ $\tan\alpha, \tan\beta$ are the roots of $x^2 - 3x - 1 = 0$

$\therefore$ $\tan\alpha + \tan\beta = 3$ and $\therefore$ $\tan\alpha + \tan\beta = -1$.

$\therefore$ $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \dfrac{3}{2}$.

13. Number of solutions of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$ is

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 3

Sol. (*c*) The given equation can be written as

$\sin x + 1 = 2\cos^2 x$

$\Rightarrow \quad \sin x + 1 = 2(1 - \sin^2 x)$

$\Rightarrow \quad 2\sin^2 x + \sin x - 1 = 0$

$\sin x = \dfrac{-1 \pm \sqrt{1+8}}{4} = \dfrac{-1 \pm 3}{4} = \dfrac{1}{2}$ or -1

$\Rightarrow \quad x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

Hence, the required number of solutions is 2.

14. If $\tan m\theta + \cot n\theta = 0$, then the general value of θ is $(r \in I)$

(*a*) $\dfrac{(2r+1)\pi}{2(m-n)}$ (*b*) $\dfrac{(2r+1)\pi}{2(m+n)}$

(*c*) $\dfrac{r\pi}{m+n}$ (*d*) $\dfrac{r\pi}{m-n}$

Sol. (*a*) The given equation can be written as

$\tan m\theta = -\cot n\theta = \tan(\pi/2 + n\theta)$

$\therefore \quad m\theta = r\pi + \left(\dfrac{\pi}{2} + n\theta\right), r \in I$

or $\quad (m-n)\theta = \dfrac{1}{2}(2r+1)\pi, r \in I$

$\therefore \quad \theta = \dfrac{(2r+1)\pi}{2(m-n)}, r \in I$.

15. The general solution of the equation $(\sqrt{3}-1)\sin\theta + (\sqrt{3}+1)\cos\theta = 2$ is $(n \in I)$

(*a*) $n\pi + (-1)^n \dfrac{\pi}{4} - \dfrac{\pi}{12}$ (*b*) $2n\pi \pm \dfrac{\pi}{4} - \dfrac{\pi}{12}$

(*c*) $n\pi + (-1)^n \dfrac{\pi}{4} + \dfrac{\pi}{12}$ (*d*) $2n\pi \pm \dfrac{\pi}{4} + \dfrac{\pi}{12}$

Sol. (*d*) Let $\sqrt{3}+1 = r\cos\alpha, \sqrt{3}-1 = r\sin\alpha$

$\therefore r^2 = (\sqrt{3}+1)^2 + (\sqrt{3}-1)^2 = 8$ or $r = 2\sqrt{2}$

and $\tan\alpha = \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = \dfrac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$

or $\tan\alpha = \tan(45° - 30°) = \tan 15°$

$\therefore \quad \alpha = 15° = \dfrac{\pi}{12}$

Using these in the given equation, we get

$r\cos(\theta - \alpha) = 2$

or $\cos\left(\theta - \dfrac{\pi}{12}\right) = \dfrac{2}{r} = \dfrac{2}{2\sqrt{2}} = \dfrac{1}{\sqrt{2}} = \cos\left(\dfrac{\pi}{4}\right)$

$\therefore \quad \theta - \dfrac{\pi}{12} = 2n\pi \pm \dfrac{\pi}{4}$

or $\theta = 2n\pi \pm \dfrac{\pi}{4} + \dfrac{\pi}{12}, n \in I$.

16. One solution of the equation $4\cos^2\theta\sin\theta - 2\sin^2\theta = 3\sin\theta$ is $(n \in z)$

(*a*) $x = n\pi + (-1)^n \left(\dfrac{-3\pi}{10}\right)$

(*b*) $x = n\pi + (-1)^n \left(\dfrac{3\pi}{10}\right)$

(*c*) $x = 2n\pi \pm \dfrac{\pi}{6}$

(*d*) None of these.

Sol. (*a*) The given equation can be written as

$\sin\theta[4(1 - \sin^2\theta) - 2\sin\theta - 3] = 0$

or $\sin\theta[1 - 2\sin\theta - 4\sin^2\theta] = 0$

or $\sin\theta[4\sin^2\theta + 2\sin\theta - 1] = 0$

$\therefore$ Either $\sin\theta = 0$ which gives $\theta = n\pi$

or $4\sin^2\theta + 2\sin\theta - 1 = 0$ which gives

$\sin\theta = \dfrac{-2 \pm \sqrt{4+16}}{2 \times 4} = \dfrac{-2 \pm 2\sqrt{5}}{8} = \dfrac{-1 \pm \sqrt{5}}{4}$

$= \dfrac{-1+\sqrt{5}}{4}, \dfrac{-1-\sqrt{5}}{4}$

$\Rightarrow$ $\sin 2x\,(2\cos x - 3) = \cos 2x\,(2\cos x - 3) \Rightarrow$ $\sin 2x = \cos 2x$ ($\because \cos x \neq 3/2$)

$\Rightarrow$ $\tan 2x = 1 \Rightarrow 2x = n\pi + \dfrac{\pi}{4} \Rightarrow x = \dfrac{n\pi}{2} + \dfrac{\pi}{8}$, $n \in I$.

20. Solve for x, the equation $\sin^3 x + \sin x \cos x + \cos^3 x = 1$ $(m, n \in z)$

(*a*) $2m\pi$ (*b*) $(4n+1)\dfrac{\pi}{2}$

(*c*) Both (*d*) None of these

Sol. (*c*) The given equation is $\sin^3 x + \cos^3 x + \sin x \cos x = 1$

$\Rightarrow (\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x) + \sin x \cos x - 1 = 0$

$\Rightarrow (1 - \sin x \cos x)\,[\sin x + \cos x - 1] = 0$

Either $1 - \sin x \cos x = 0 \Rightarrow \sin 2x = 2$ which is not possible

Or, $\sin x + \cos x - 1 = 0 \Rightarrow \cos(x - \pi/4) = \dfrac{1}{\sqrt{2}}$

$\Rightarrow x - \dfrac{\pi}{4} = 2m\pi \pm \dfrac{\pi}{4}$

$\Rightarrow$ $x = 2m\pi$ and $x = (4n+1)\dfrac{\pi}{2}$

21. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has

(*a*) no real solution (*b*) one real solution

(*c*) two real solutions (*d*) can't be determined

Sol. (*a*) The given equation can be written as

$e^{2\sin x} - 4e^{\sin x} - 1 = 0 \Rightarrow e^{\sin x} = \dfrac{4 \pm \sqrt{16+4}}{2} = 2 + \sqrt{5}$

$\Rightarrow$ $\sin x = \ln(2 + \sqrt{5})$ or ($\ln(2 - \sqrt{5})$ not defined as $(2 - \sqrt{5})$ is negative)

Now, $2 + \sqrt{5} > e \Rightarrow \ln(2 + \sqrt{5}) > 1 \Rightarrow \sin x > 1$

Which is not possible. Hence no real solution.

22. If $\tan(\pi \cos x) = \cot(\pi \sin x)$, then $\cos\left(x - \dfrac{\pi}{4}\right)$, is

(*a*) $\dfrac{1}{\sqrt{2}}$ (*b*) $\dfrac{1}{2\sqrt{2}}$

(*c*) 0 (*d*) None of these.

Sol. (*b*) Given that $\tan(\pi \cos x) = \cos(\pi \sin x)$

or $\tan(\pi \cos x) = \tan\left(\dfrac{\pi}{2} - \pi \sin x\right)$

$\Rightarrow$ $\pi \cos x = \dfrac{\pi}{2} - \pi \sin x$

$\Rightarrow$ $\cos x + \sin x = \dfrac{1}{2}$

$\Rightarrow$ $\dfrac{1}{\sqrt{2}}\cos x + \dfrac{1}{\sqrt{2}}\sin x = \dfrac{1}{2\sqrt{2}}$

$\Rightarrow$ $\cos\left(x - \dfrac{\pi}{4}\right) = \dfrac{1}{2\sqrt{2}}$.

EXERCISE

SINGLE CORRECT TYPE QUESTIONS (01 TO 45)

1. Value of $4\sin(27°)$ is

(*a*) $(5+\sqrt{5})^{\frac{1}{2}} - (5-\sqrt{5})^{\frac{1}{2}}$

(*b*) $(5+\sqrt{5})^{\frac{1}{2}} - (3-\sqrt{5})^{\frac{1}{2}}$

(*c*) $(5+\sqrt{5})^{\frac{1}{2}} + (5-\sqrt{5})^{\frac{1}{2}}$

(*d*) $(5+\sqrt{5})^{\frac{1}{2}} + (3-\sqrt{5})^{\frac{1}{2}}$

2. If $\tan\theta$, $2\tan\theta + 2$, $3\tan\theta + 3$ are in G.P., then the value of $\dfrac{7 - 5\cot\theta}{9 + 4\sqrt{\sec^2\theta - 1}}$ is

(*a*) $\dfrac{12}{5}$ (*b*) $\dfrac{33}{100}$ (*c*) $\dfrac{-33}{28}$ (*d*) $\dfrac{12}{13}$

3. If x is. A.M. of $\tan\dfrac{\pi}{9}$ and $\tan\dfrac{5\pi}{18}$ and y is A.M. of $\tan\dfrac{\pi}{9}$ and $\tan\dfrac{7\pi}{18}$, then

(*a*) $x > y$ (*b*) $x = y$ (*c*) $2x = y$ (*d*) $x = 2y$

4. $\dfrac{2\sec\theta+3\tan\theta+5\sin\theta-7\cos\theta+5}{2\tan\theta+3\sec\theta+5\cos\theta+7\sin\theta+8}$ equals to

(*a*) $\tan\dfrac{\theta}{2}$ (*b*) $\cot\dfrac{\theta}{2}$

(*c*) $\sec\dfrac{\theta}{2}$ (*d*) $\operatorname{cosec}\left(\dfrac{\theta}{2}\right)$

5. General solution of inequality

$\dfrac{5}{4}\sin^2 x+\sin^2 x\cos^2 x>\cos 2x, (n\in z)$ is .

(*a*) $n\pi+\dfrac{\pi}{6}<x\le n\pi+\dfrac{5\pi}{6}$

(*b*) $n\pi+\dfrac{\pi}{3}<x<n\pi+\dfrac{5\pi}{3}$

(*c*) $n\pi+\dfrac{\pi}{4}\le x<n\pi+\dfrac{\pi}{4}$

(*d*) none of these

6. If $\dfrac{\tan 3A}{\tan A}=k$, then value of k can be.

(*a*) less then $\dfrac{1}{3}$ (*b*) 2

(*c*) 3 (*d*) 1

7. If ABC is triangle and $\tan\dfrac{A}{2},\tan\dfrac{B}{2},\tan\dfrac{C}{2}$ are in H.P. then minimum value of $\cot\dfrac{B}{2}$ is.

(*a*) $\sqrt{3}$ (*b*) 2 (*c*) 3 (*d*) 4

8. Value of $\sum_{r=1}^{n-1}\cos^2\left(\dfrac{r\pi}{n}\right)$ is

(*a*) $\dfrac{n}{2}$ (*b*) $\dfrac{n}{2}+1$ (*c*) $\dfrac{n}{2}-1$ (*d*) $n+1$

9. If $2\sin\left(\dfrac{A}{2}\right)=\sqrt{1+\sin A}+\sqrt{1-\sin A}$, then $\dfrac{A}{2}$ lies in the interval (n is integer)

(*a*) $\left(2n\pi+\dfrac{\pi}{4},2n\pi+\dfrac{3\pi}{4}\right)$

(*b*) $\left(2n\pi,2n\pi+\dfrac{\pi}{2}\right)$

(*c*) $\left(2n\pi-\dfrac{\pi}{4},2n\pi+\dfrac{\pi}{4}\right)$

(*d*) $\left(2n\pi-\dfrac{\pi}{4}, 2n\pi\right)$

10. If $a\le\cos^2\theta-6\sin\theta\cos\theta+3\sin^2\theta+2\le b$, then value of $a+b$ is

(*a*) $2\sqrt{10}$ (*b*) 8

(*c*) 6 (*d*) none of these

11. Number of real solutions of the equation $\cos^7 x+\sin^4 x=1$ in the interval $[-\pi, \pi]$ is

(*a*) Two (*b*) Three

(*c*) Four (*d*) None of these

12. Smallest positive value of x (in degrees) lying in $x\in[0, 90°]$, for which

$\tan(x+100°)=\tan(x+50°)\cdot\tan x\cdot\tan(x-50°)$.

(*a*) 30° (*b*) 45° (*c*) 65° (*d*) 20°

13. If A, B and C are angles of a triangle, then value of $\tan^2\dfrac{A}{2}+\tan^2\dfrac{B}{2}+\tan^2\dfrac{C}{2}$ is

(*a*) ≥ 1 (*b*) ≤ 1

(*c*) 1 (*d*) none of these

14. Value of

$\cos^4\left(\dfrac{\pi}{8}\right)+\cos^4\left(\dfrac{3\pi}{8}\right)+\cos^4\left(\dfrac{5\pi}{8}\right)+\cos^4\left(\dfrac{7\pi}{8}\right)$

is

(*a*) $\dfrac{5}{2}$ (*b*) $\dfrac{3}{2}$ (*c*) $\dfrac{1}{2}$ (*d*) 1

15. If $\tan\beta=2\sin\alpha\cdot\sin\gamma\cdot\operatorname{cosec}(\alpha+\gamma)$, then $\cot\alpha$, $\cot\beta$, $\cot\gamma$ are in

(*a*) A.P. (*b*) G.P. (*c*) H.P. (*d*) A.G.P.

16. The value of $\cos^2 10°-\cos 10°\cos 50°+\cos^2 50°$ is

(*a*) $\dfrac{4}{3}$ (*b*) $\dfrac{1}{3}$ (*c*) $\dfrac{3}{4}$ (*d*) 3

17. If $\cos^6\alpha+\sin^6\alpha+k\sin^2 2\alpha=1\ \forall\ \alpha\in(0,\pi/2)$, then the value of k is

(*a*) $\dfrac{3}{4}$ (*b*) $\dfrac{1}{4}$ (*c*) $\dfrac{1}{3}$ (*d*) $\dfrac{1}{8}$

18. $\dfrac{\sin^3\theta-\cos^3\theta}{\sin\theta-\cos\theta}-\dfrac{\cos\theta}{\sqrt{1+\cot^2\theta}}-2\tan\theta\cot\theta=-1$, if

(*a*) $\theta\in\left(0,\dfrac{\pi}{2}\right)$ (*b*) $\theta\in\left(\dfrac{\pi}{2},\pi\right)$

(*c*) $\theta\in\left(\pi,\dfrac{3\pi}{2}\right)$ (*d*) $\theta\in\left(\dfrac{3\pi}{2},2\pi\right)$

32. If $\cos(\sin x) = \frac{1}{\sqrt{2}}$, then x must lie in the interval

(*a*) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (*b*) $\left(-\frac{\pi}{4}, 0\right)$

(*c*) $\left(\frac{3\pi}{4}, \pi\right)$ (*d*) None of these

33. The most general solution for the equation

$$\left(4\sqrt{\cos\frac{x}{2}} - 5 - \frac{\sqrt{2}}{2}\right)^2 + \sqrt{2}\left(4\sqrt{\cos\frac{x}{2}} - 5 - \frac{\sqrt{2}}{2}\right)$$

$-\frac{\cos x}{2} = 0$ *is* $x = (n \in z)$

(*a*) $4n\pi$ (*b*) $\frac{n\pi}{6}$

(*c*) $\frac{n\pi}{3}$ (*d*) None of these

34. The equation $a \sin x + \cos 2x = 2a - 7$ possesses a solution if

(*a*) $a > 6$ (*b*) $2 \le a \le 6$

(*c*) $a > 2$ (*d*) None of these

35. If $\tan^2 x + \sec x - a = 0$ has at least one solution, then complete set of values of a is

(*a*) $(-\infty, 1]$ (*b*) $[-1, \infty)$

(*c*) $\left[\frac{9}{4}, \infty\right)$ (*d*) $[1, \infty)$

36. If $\sin(\theta - x) = a$, $\cos(\theta - y) = b$, then $\cos(x - y) =$

(*a*) $a\sqrt{(1-b^2)} + b\sqrt{(1-a^2)}$

(*b*) ab

(*c*) $a\sqrt{(1-b^2)} - b\sqrt{(1-a^2)}$

(*d*) $2\,ab$

37. The product cot 123°·cot 133°·cot137°·cot147°, when simplified is equal to

(*a*) –1 (*b*) tan 37° (*c*) cot 33° (*d*) 1

38. If $(1 + \tan 1°)\cdot(1 + \tan 2°)\cdot(1 + \tan 3°)....(1 + \tan 45°) = 2^n$, then '$n$' is equal to

(*a*) 16 (*b*) 23

(*c*) 30 (*d*) None of these

39. If $4^{\sin 2x + 2\cos^2 x} + 4^{1 - \sin 2x + 2\sin^2 x} = 65$, then ($\sin 2x + \cos 2x$) has the value equal to

(*a*) –1 (*b*) 2 (*c*) $\sqrt{2}$ (*d*) 1

40. If $P = \cos\frac{\pi}{20}\cdot\cos\frac{3\pi}{20}\cdot\cos\frac{7\pi}{20}\cdot\cos\frac{9\pi}{20}$ and

$Q = \cos\frac{\pi}{11}\cdot\cos\frac{2\pi}{11}\cdot\cos\frac{4\pi}{11}\cdot\cos\frac{8\pi}{11}\cdot\cos\frac{16\pi}{11}$,

then $\frac{P}{Q}$ is

(*a*) Not defined (*b*) 1

(*c*) 2 (*d*) 5

41. The solution of the equation $4\sin^4 x + \cos^4 x = 1$ is:

(*a*) $x = 2n\pi$ (*b*) $x = n\pi + \frac{\pi}{2}$

(*c*) $x = n\pi$ (*d*) None of these

42. If A, B, C, D are the angles of a cyclic quadrilateral, then the value of $\cos A + \cos B + \cos C + \cos D$ is

(*a*) 4 (*b*) 1 (*c*) 0 (*d*) –1

43. In a ΔPQR, if $3 \sin P + 4 \cos Q = 6$ and $4 \sin Q + 3 \cos P = 1$, then the angle R is equal to

(*a*) $\frac{\pi}{4}$ (*b*) $\frac{3\pi}{4}$ (*c*) $\frac{5\pi}{6}$ (*d*) $\frac{\pi}{6}$

44. If $\tan\theta_1 = k\cot\theta_2$ then $\frac{\cos(\theta_1 - \theta_2)}{\cos(\theta_1 + \theta_2)} =$

(*a*) $\frac{1+k}{1-k}$ (*b*) $\frac{1-k}{1+k}$ (*c*) $\frac{k+1}{k-1}$ (*d*) $\frac{k-1}{k+1}$

45. $\frac{1}{\cos\alpha + \cos 3\alpha} + \frac{1}{\cos\alpha + \cos 5\alpha} + \ldots + \frac{1}{\cos\alpha + \cos(2n+1)\alpha}$

is equal to

(*a*) $\operatorname{cosec}\alpha\left[\tan(n+1)\alpha - \tan\alpha\right]$

(*b*) $\sec\alpha\left[\tan(n+1)\alpha - \tan\alpha\right]$

(*c*) $\frac{1}{2}\sec\alpha\left[\tan(n+1)\alpha - \tan\alpha\right]$

(*d*) $\frac{1}{2}\operatorname{cosec}\alpha\left[\tan(n+1)\alpha - \tan\alpha\right]$

MULTIPLE CORRECT TYPE QUESTIONS (46 TO 71)

46. If A lies between 270° and 360° and $\sin A = \frac{-7}{25}$, then

(a) $\sin 2A = \frac{-336}{625}$ (b) $\cos\frac{A}{2} = \frac{\sqrt{2}}{5}$

(c) $\tan\frac{A}{2} = -\frac{1}{7}$ (d) $\sin\frac{A}{2} = \frac{-\sqrt{2}}{10}$

47. Solutions of the equation $9\cos^{12}(x) + \cos^2(2x) + 1 = 6\cos^6 x \cos 2x + 6\cos^6 x - 2\cos 2x$ are given by $x = (n \in z)$

(a) $n\pi + \frac{\pi}{2}$ (b) $n\pi + \cos^{-1}\left(\sqrt[4]{\frac{2}{3}}\right)$

(c) $n\pi - \cos^{-1}\left(\sqrt[4]{\frac{2}{3}}\right)$ (d) None of these

48. If $\cos(x - y)$, $\cos x$ and $\cos(x - y)$ are in Harmonic progression then value of $\cos x \sec\frac{y}{2}$.

(a) $\sqrt{2}$ (b) $-\sqrt{2}$

(c) 2 (d) None of these

49. If $P_{n+1} = \sqrt{\frac{1}{2}(1+P_n)}$,

then $\cos\left(\frac{\sqrt{1-P_n}}{P_1 P_2 P_3 \ldots\text{ to } \infty}\right)$ is equal to

(a) 1 (b) -1 (c) P_0 (d) $\frac{1}{P_0}$

50. If $N = \sin\left(\frac{\pi}{6} - \alpha\right)\sin\left(\frac{\pi}{6} + \alpha\right) - \cos^2\alpha$, then $\log_{1.3}(-N) =$ is equal to.

(a) 1 (b) -1

(c) 0 (d) independent of α

51. If $T_n = \sin^n\theta + \cos^n\theta$, then $\frac{T_6 - T_4}{T_6} = m$ holds for values of m satisfying ($\theta \in R$):

(a) $m \in \left[-1, \frac{1}{3}\right]$ (b) $m \in \left[0, \frac{1}{3}\right]$

(c) $m \in [-1, 0]$ (d) $m \in \left[-1, -\frac{1}{2}\right]$

52. The equation $\sin^4 x + \cos^4 x + \sin 2x + K = 0$ must have real solution if:

(a) $K = 0$ (b) $|K| \le \frac{1}{2}$

(c) $-\frac{3}{2} \le K \le \frac{1}{2}$ (d) $-\frac{1}{2} \le K \le \frac{3}{2}$

53. If $x = \cos 39° + \sin 57°$, $y = \cos 40° + \sin 58°$ and $z = \cos 41° + \sin 59°$, then:

(a) $x > y$ (b) $y > z$

(c) $x = y = z$ (d) $x + z > y$

54. The least difference between the roots of the equation $4\cos x\,(2 - 3\sin^2 x) + \cos 2x + 1 = 0\ \forall\ x \in R$ is:

(a) equal to $\frac{\pi}{2}$ (b) greater than $\frac{\pi}{10}$

(c) less than $\frac{\pi}{2}$ (d) less than $\frac{\pi}{3}$

55. Which of the following function have the maximum value unity?

(a) $\sin^2 x - \cos^2 x$

(b) $\sqrt{\frac{6}{5}}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{3}}\cos x\right)$

(c) $\cos^6 x + \sin^6 x$

(d) $\cos^4 x + \sin^4 x$

56. If $\cos\beta$ is geometric mean between $\sin\alpha$ and $\cos\alpha$, where $0 < \alpha, \beta < \frac{\pi}{2}$, then $\cos 2\beta =$

(a) $-2\sin^2\left(\frac{\pi}{4} - \alpha\right)$ (b) $-2\cos^2\left(\frac{\pi}{4} + \alpha\right)$

(c) $2\sin^2\left(\frac{\pi}{4} + \alpha\right)$ (d) $2\cos^2\left(\frac{\pi}{4} - \alpha\right)$

57. Consider the statements about $y = \frac{\sin 3x}{\sin x}(\sin x \ne 0)$. Which of the following is(are) correct?

(a) The minimum value of y is (-1)

(b) The maximum value of y is 3

(c) The minimum value of y is NOT defined

(d) The maximum value of y is NOT defined

68. If $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$, for $k = 1, 2, 3, ...$, then

(a) $f_4(x) - f_6(x) = \frac{1}{12}$ (b) $f_4\left(\frac{\pi}{8}\right) = \frac{1}{2}$

(c) $f_4\left(\frac{\pi}{12}\right) = \frac{7}{32}$ (d) $f_1(2) < 0$

69. If $\sec\theta + \tan\theta = 1$, then root of the equation $(a - 2b + c)x^2 + (b - 2c + a)x + (c - 2a + b) = 0$ is

(a) $\sec\theta$ (b) $\tan\theta$ (c) $\sin\theta$ (d) $\cos\theta$

70. If $\sin\theta + \sin\phi = a$ and $\cos\theta + \cos\phi = b$, then:

(a) $\cos\left(\frac{\theta-\phi}{2}\right) = \pm\frac{1}{2}\sqrt{a^2+b^2}$

(b) $\cos\left(\frac{\theta-\phi}{2}\right) = \pm\sqrt{a^2-b^2}$

(c) $\cos\left(\frac{\theta-\phi}{2}\right) = \pm\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$

(d) $\cos(\theta-\phi) = \frac{a^2+b^2-2}{2}$

71. If $\sin(x - y) = \cos(x + y) = \frac{1}{2}$, then the values of x and y lying between 0 and π are given by

(a) $x = \frac{\pi}{4}, y = \frac{3\pi}{4}$ (b) $x = \frac{\pi}{4}, y = \frac{\pi}{12}$

(c) $x = \frac{5\pi}{4}, y = \frac{5\pi}{12}$ (d) $x = \frac{11\pi}{12}, y = \frac{3\pi}{4}$

NUMERICAL TYPE QUESTIONS (72 TO 77)

PART-I: DECIMAL TYPE QUESTIONS (72 to 77)

72. The number of distinct solutions of the equation $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \sin^6 x + \cos^6 x = 2$ in the interval $[0, \pi]$ is

73. If the set of all values of x in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfying $|4\sin x + \sqrt{2}| < \sqrt{6}$ is $\left(\frac{a\pi}{24}, \frac{b\pi}{24}\right)$, then value of $\left|\frac{a+b}{3}\right|$ is

74. If maximum value of the expression

$$\frac{1}{\left(11\sin^2\theta + 24\sin\theta\cdot\cos\theta + 29\cos^2\theta\right)}$$ is 'a'

then the value of a is ____.

75. Let $a = \sum_{r=1}^{11} \tan^2\left(\frac{r\pi}{24}\right)$ and $b = \sum_{r=1}^{11} (-1)^{r-1}\tan^2\left(\frac{r\pi}{24}\right)$, then the value of $\log_{(2b-a)}(2a - b)$ is

76. Number of values of θ lying in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ the equation $(1 - \tan\theta)(1 + \tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$ is

77. If $\frac{9a}{\cos\theta} + \frac{5b}{\sin\theta} = 56$, $\frac{9a\sin\theta}{\cos^2\theta} - \frac{5b\cos\theta}{\sin^2\theta} = 0$ and if the value of $\left[(9a)^{\frac{2}{3}} + (5b)^{\frac{2}{3}}\right]^3 = (8k)^2$, then k is

PART-II: INTEGER TYPE QUESTIONS (78 to 89)

78. The angle A of the ΔABC is obtuse $x = 2635 - \tan B \tan C$. If $[x]$ denotes the greatest integer function, the value of $[x] - 2630$ is

79. If $\frac{1}{\sin 20^\circ} - \frac{1}{\sqrt{3}\cos 20^\circ} = k$, then $18k^4 + 162k^2 - 1371$ is

80. If $\alpha + \beta = \gamma$ and $\tan\gamma = 22$, a is the arithmetic and b is the geometric mean respectively between $\tan\alpha$ and $\tan\beta$, the value of $\frac{a^3}{(1-b^2)^3} - 1330$ is equal to

81. Value of $\sin^2 12^\circ + \sin^2(21^\circ) + \sin^2(39^\circ) + \sin^2(48^\circ) - \sin^2(9^\circ) - \sin^2(18^\circ)$ is

82. In a triangle ABC, $a < \cos A + \cos B + \cos C < b$, then value of $a + 2b$ is

83. Given that the angles α, β, γ are connected by the relation $2\tan^2\alpha\tan^2\beta\tan^2\gamma + \tan^2\alpha\tan^2\beta + \tan^2\beta\tan^2\gamma + \tan^2\gamma\tan^2\alpha = 1$. Find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

84. If $\sec A\tan B + \tan A\sec B = 91$, then the value of $(\sec A\sec B + \tan A\tan B)^2 - (91)^2$ is equal to

85. If the sum of all solution of the equation $\tan^2 33x = \cos 2x - 1$ lying in the interval $[0, 314]$ is $k\pi$, then the value of $k/10$, is

86. Let M denotes the maximum value of expression $f(\theta)=\dfrac{7+6\cos 2\theta}{4-3\cos 2\theta}, \theta \in R$ and m denotes the minimum value of expression $g(\theta) = 2\sin^2\theta + \cos^4\theta + 3, \theta \in R$. Then the value of $(M + m)$ is

87. The number of solution of the equation

$$\sin(\pi - 6x) + \sqrt{3}\sin\left(\frac{\pi}{2} + 6x\right) = \sqrt{3} \text{ in } [0, 2\pi], \text{ is}$$

88. Number of solution of the equation

$$|\sin x \cos x| + \sqrt{2 + \tan^2 x + \cot^2 x} = \sqrt{3},$$

$$x \in (0, 6\pi) - \left\{n\pi, (2n+1)\frac{\pi}{2}\right\},$$

where $n \in I$ is

89. Let $f(x) = ax^2 + bx + c$ where a, b, c are integers.

If $\sin\frac{\pi}{7}\cdot\sin\frac{3\pi}{7} + \sin\frac{3\pi}{7}\cdot\sin\frac{5\pi}{7} + \sin\frac{5\pi}{7}\cdot\sin\frac{\pi}{7}$

$= f\left(\cos\frac{\pi}{7}\right)$

then the value of $f(2)$ is

MATRIX MATCH/LIST TYPE QUESTIONS

PART-I: MATRIX TYPE QUESTIONS (90 to 92)

90. Match the columns.

	Column-I		Column-II
I.	Value of $\cot 7\frac{1}{2}^\circ$ is	P.	$\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$
II.	Value of $\tan\left(7\frac{1}{2}^\circ\right)$ is	Q.	$\frac{1}{4}\left(\sqrt{4+2\sqrt{2}}+\sqrt{4-2\sqrt{2}}\right)$
III.	Value of $\sin\left(67\frac{1}{2}^\circ\right)$ is	R.	$\frac{1}{4}\left(\sqrt{4+2\sqrt{2}}-\sqrt{4-2\sqrt{2}}\right)$
IV.	Value of $\cos\left(67\frac{1}{2}^\circ\right)$ is	S.	$\sqrt{6}-\sqrt{4}-\sqrt{3}+\sqrt{2}$
		T.	$\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{5}$

91. Match the column.

	Column-I		Column-II
I.	The number of real roots of the equation is $\cos^9 x + \sin^4 x = 1$ in $(-\pi, \pi)$	P.	1
II.	The value of $\sqrt{3}$ cosec 20° – sec 20° is	Q.	4
III.	4cos36° – 4cos72° + 4sin18°cos36° equals	R.	0
IV.	The number of values of $x \in [-2\pi, 2\pi]$, which satisfy cosec$x$ = 1 + cot$x$	S.	3
		T.	2

92. Match the columns.

	Column-I		Column-II
I.	The number of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ is $(0 < x < \pi)$	P.	No solution
II.	If $\sin\theta + \sin\phi = \frac{1}{2}$ and $\cos\theta + \cos\phi = 2$, then value of $\cot\left(\frac{\theta+\phi}{2}\right)$ is	Q.	$\frac{1}{3}$
III.	The value of $\sin^2\alpha + \sin\left(\frac{\pi}{3}-\alpha\right)\sin\left(\frac{\pi}{3}-\alpha\right)$ is	R.	1
IV.	If tan θ = 3 tan ϕ, then maximum value of $\tan^2(\theta - \phi)$ is	S.	2
		T.	3/4

PART-II: LIST TYPE QUESTIONS (93 to 95)

93. Match the columns.

	Column-I		Column-II
I.	The least difference between the roots, in the first quadrant $\left(0 \le x \le \frac{\pi}{2}\right)$, of the equation $4\cos x(2 - 3\sin^2 x) + (\cos 2x + 1) = 0$ is	P.	2
II.	Number of positive values of x for which $2\cos x$, $\lvert\cos x\rvert$ and $1 - 3\cos^2 x$ are in G.P., lying in $[0, \pi]$ is	Q.	$\frac{\pi}{6}$
III.	The values of x between 0 and 2π which satisfy the equation $\sin x\sqrt{8\cos^2 x} = 1$ are in A.P. with common difference d, then $\frac{4d}{\pi}$ is	R.	$\frac{\pi}{3}$
IV.	The number of solution of the equation $\lvert \sin\theta + \cos\theta \rvert^{(1+\sin 2\theta)} = 2, -\pi \le \theta \le \pi$, is	S.	1

(*a*) (I-Q); (II-P); (III-S); (IV-P)
(*b*) (I-P); (II-P); (III-S); (IV-Q)
(*c*) (I-P); (II-P); (III-Q); (IV-S)
(*d*) (I-P); (II-Q); (III-P); (IV-S)

94. Match the columns.

	Column-I		Column-II
I.	The minimum value of $\sec^2 x + \operatorname{cosec}^2 x - 4$ is	P.	3
II.	The maximum value of $\lvert\lvert\sin x\rvert - 4\rvert - 3$ is	Q.	4
III.	$10^{\log_{10} 3} + \log_{10} \tan 1° + \log_{10} \tan 2° + \log_{10} \tan 3° + ... + \log_{10} \tan 89°$ equals	R.	1
IV.	If $\cos x + \cos^2 x = 1$, then value of $4\sin^2 x(2 - \cos^2 x)$	S.	0

(*a*) (I-S); (II-R); (III-Q); (IV-P)
(*b*) (I-S); (II-R); (III-P); (IV-Q)
(*c*) (I-S); (II-Q); (III-R); (IV-P)
(*d*) (I-P); (II-Q); (III-R); (IV-S)

95. Match the columns.

	Column-I		Column-II
I.	Let x and $y \in \left(0, \frac{\pi}{2}\right)$ such that $\cos^2(x - y) = \sin 2x \times \sin 2y$, then $x + y =$	P.	$\frac{\pi}{4}$
II.	Let $x, y, z \in \left(0, \frac{\pi}{4}\right)$ such that $\frac{1}{2}(1 - \tan x)(1 - \tan y)(1 - \tan z) = 1 - (\tan x + \tan y + \tan x)$, then $x + y + z =$	Q.	$\frac{\pi}{3}$
III.	Let $x, y \in \left(0, \frac{\pi}{2}\right)$ such that $\sin x \cos y + \sin y \cos z + \sin z \cos x =$ then $x + y + z$ is	R.	$\frac{\pi}{2}$
IV.	Let $x, y, z \in \left(0, \frac{\pi}{2}\right)$ such that $3\cos^2 x + 2\cos^2 y = 4$ and then $\frac{2}{3}(x + 2y)$ is	S.	$\frac{3\pi}{4}$

(*a*) (I-P); (II-S); (III-Q); (IV-R)
(*b*) (I-P); (II-R); (III-Q); (IV-S)
(*c*) (I-R); (II-P); (III-Q); (IV-S)
(*d*) (I-R); (II-P); (III-S); (IV-Q)

COMPREHENSION TYPE QUESTIONS (96 TO 106)

Comprehension-1

In a triangle ABC, if $\cot A + \cot B + \cot C = \cot\theta$, then

96. $\sin(A - \theta)\sin(B - \theta)\sin(C - \theta)$ equals to

(*a*) $\sin^3\theta$ (*b*) $\cos^3\theta$
(*c*) $\tan^3\theta$ (*d*) $\cot^3\theta$

97. Possible value of θ is

(*a*) 45° (*b*) 35° (*c*) 25° (*d*) 55°

Comprehension-2

If θ is an angle measured in radian $\theta \in [0, 2\pi]$, then θr is length of arc *AB*, of circle of radius *r*, subtending angle θ at the centre *O*, of the circle. Area of sector *OAB* is $\frac{1}{2}r^2\theta$.

98. The angle between minute hand and hour hand of a clock at "half past 4" equals

(*a*) 42° (*b*) 43°

(*c*) 44° (*d*) None of these

99. The wheel of a train is 1 meter in diameter and it makes 5 revolutions per second. Then the speed of the train is approximately equal to

(*a*) 57 km/hr (*b*) 66 km/hr

(*c*) 68 km/hr (*d*) 42.6 km/hr

100. Two lines drawn through a point on the circumference of a circle divide the circle into three regions of equals area. The two lines also mirror each other on the diameter passing through the point of intersection. Then the angle θ between the lines is given by

(*a*) $3\theta + 3\sin\theta = \pi$ (*b*) $6\theta + 3\sin\theta = \pi$

(*c*) $2\theta + \sin\theta = \pi$ (*d*) $\theta + \sin\theta = \frac{\pi}{2}$

Comprehension-3

We know that $\sin x \le 1$ and $\sin y \le 1$ for all *x*, *y* So, $\sin x + \sin y \le 2$ for all *x* and *y* i.e., $\sin x + \sin y = 2$ if and only if $\sin x = 1 = \sin y$

so, $x = 2n\pi + \frac{\pi}{2}$ and $y = 2m\pi + \frac{\pi}{2}$, $m, n \in z$

In general, on or more of the following extreme value conditions

I. $-1 \le \sin x \le 1 \Rightarrow |\sin x| \le 1 \Rightarrow \sin\ x \le 1$

II. $-1 \le \cos x \le 1 \Rightarrow |\cos x| \le 1$y and $\cos^2 x \le 1$

III. $-\sqrt{a^2+b^2} \le (a\sin x + b\cos x) \le \sqrt{a^2+b^2}$

$\Rightarrow |a\sin x + b\cos x| \le \sqrt{a^2+b^2}$

On the basis of above passage, answer the following questions:

101. Number of roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $[0, 2\pi]$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 4

102. The smallest positive number *p* for which the equation $\cos(p\sin x) = \sin(p\cos x)$ has a solution in $[0, \pi]$ is

(*a*) $\frac{\pi}{4}$ (*b*) $\frac{\pi}{3}$ (*c*) $\frac{\pi}{4\sqrt{2}}$ (*d*) $\frac{\pi}{2\sqrt{2}}$

Comprehension-4

Consider *f, g* and *h* be three valued functions defined on *R*. Let $f(x) = \sin 3x + \cos x$, $g(x) = \cos 3x + \sin x$ and $h(x) = f^2(x) + g^2(x)$.

103. General solution of equation $h(x) = 4$ is $(n \in I)$

(*a*) $(4n+1)\frac{\pi}{8}$ (*b*) $(8n+1)\frac{\pi}{8}$

(*c*) $(2n+1)\frac{\pi}{4}$ (*d*) $(7n+1)\frac{\pi}{4}$

104. Number of point(s) where graph of the two functions $y = f(x)$ and $y = g(x)$ interesect in $[0, \pi]$ is

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

Comprehension-5

If *n* be a natural number define polynomial $f_n(x)$ of n^{th} degree as follows:

$f_n(\cos\theta) = \cos n\theta$ i.e., $f_2(x) = 2x^2 - 1$, $f_3(x) = 4x^3 - 3x$, Then

105. $\left(x+\sqrt{x^2-1}\right)^{10} + \left(x-\sqrt{x^2-1}\right)^{10}$ is equal to

(*a*) $f_{10}(x)$ (*b*) $f_{11}(x) + f_9(x)$

(*c*) $f_{11}(x)$ (*d*) $2f_{10}(x)$

106. $f_6(x)$ is equal to

(*a*) $36x^6 - 48x^4 + 18x^2 - 5$

(*b*) $32x^6 - 48x^4 + 18x^2 - 1$

(*c*) $36x^6 - 45x^4 + 18x^2 - 8$

(*d*) $36x^6 - 48x^4 + 18x^2 - 7$

ANSWER KEY

Scan for Detailed Explanations

1. (*b*) **2.** (*b*) **3.** (*c*) **4.** (*a*) **5.** (*a*) **6.** (*a*) **7.** (*a*) **8.** (*c*) **9.** (*a*) **10.** (*b*)

11. (*b*) **12.** (*a*) **13.** (*a*) **14.** (*b*) **15.** (*a*) **16.** (*c*) **17.** (*a*) **18.** (*b*) **19.** (*a*) **20.** (*b*)

21. (*c*) **22.** (*b*) **23.** (*b*) **24.** (*b*) **25.** (*d*) **26.** (*a*) **27.** (*c*) **28.** (*b*) **29.** (*b*) **30.** (*c*)

31. (*d*) **32.** (*a*) **33.** (*a*) **34.** (*b*) **35.** (*b*) **36.** (*a*) **37.** (*d*) **38.** (*b*) **39.** (*a*) **40.** (*c*)

41. (*c*) **42.** (*c*) **43.** (*d*) **44.** (*a*) **45.** (*d*) **46.** (*a*, *d*) **47.** (*a*,*b*,*c*) **48.** (*a*, *b*) **49.** (*c*) **50.** (*b*,*d*)

51. (*c*, *d*) **52.** (*a*, *b*, *c*) **53.** (*a*, *b*, *d*) **54.** (*b*, *c*, *d*) **55.** (*a*, *b*, *c*, *d*) **56.** (*a*, *b*)

57. (*a*, *d*) **58.** (*b*, *c*) **59.** (*a*, *b*, *c*, *d*) **60.** (*a*, *b*, *d*) **61.** (*b*, *d*) **62.** (*b*, *c*) **63.** (*b*, *c*)

64. (*b*, *c*) **65.** (*a*, *b*, *d*) **66.** (*a*, *b*) **67.** (*b*, *d*) **68.** (*a*, *c*) **69.** (*a*, *d*) **70.** (*a*, *d*) **71.** (*b*, *d*)

72. (4.00) **73.** (2.67) **74.** (0.20) **75.** (2.00) **76.** (4.00) **77.** (7.00) **78.** (4) **79.** (5) **80.** (1) **81.** (1)

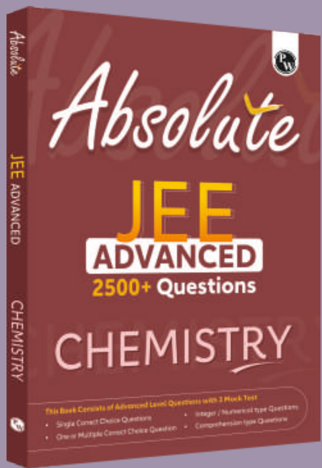
82. (4) **83.** (1) **84.** (1) **85.** (495) **86.** (17) **87.** (13) **88.** (0) **89.** (9)

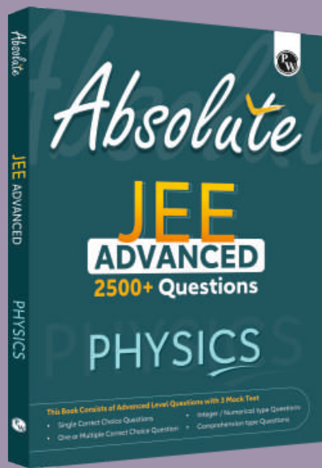
90. (I-P); (II-S), (III-Q), (IV-R) **91.** (I-S); (II-Q); (III-S); (IV-T)

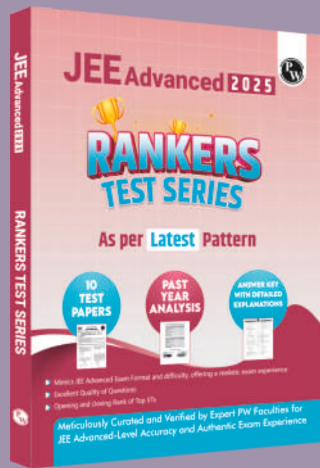
92. (I-R); (II-P); (III-T); (IV-Q) **93.** (*a*) **94.** (*b*) **95.** (*d*) **96.** (*a*) **97.** (*c*) **98.** (*d*)

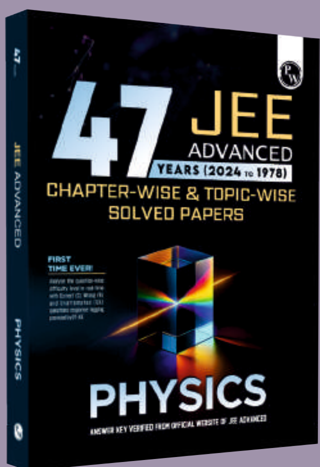
99. (*a*) **100.** (*a*) **101.** (*d*) **102.** (*d*) **103.** (*a*) **104.** (*c*) **105.** (*d*) **106.** (*b*)

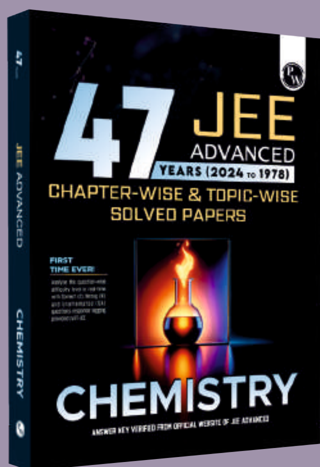
Other Helpful Books

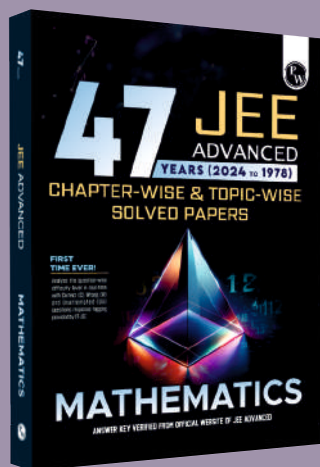












₹ 449/-

PHYSICS WALLAH PUBLICATION

To Buy PW Books
SCAN ME!

To share Feedback
SCAN ME!

ISBN 978-93-6897-345-4
9 789368 973454

39292c9b-efed-4bff-97b1-9115b390460d