ABHYAS Practice BOOSTER
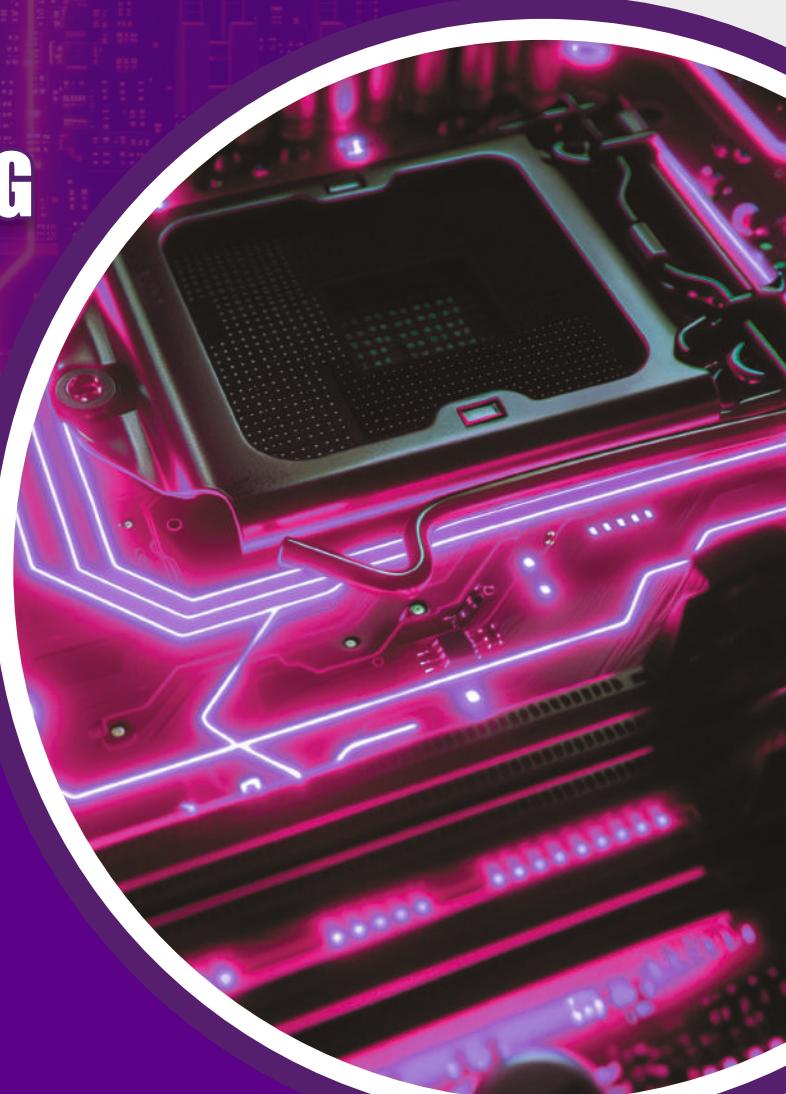
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ELECTRICAL ENGINEERING AND ELECTRONICS & COMMUNICATION ENGINEERING

VOLUME -1

Subjects Covered

- Network Theory
- Control System
- Signal and System
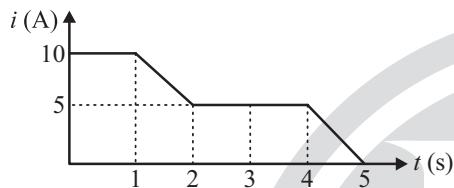- Digital Electronics
- Analog Electronics

PW

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❑❑❑

01 CHAPTER

Basic Concept of Network

1. [NAT] **[Level-I]**

The current through an element is shown in the figure. The total charge of _____ coulomb is passed through the element at $t = 3$s.

2. [NAT] **[Level-II]**

A constant current of 3A for 4 hours is required to charge an automotive battery. If the terminal voltage is $10 + t/2$V, where t is in hours, the energy dissipated is _____ joules.

3. [NAT] **[Level-II]**

Find the power delivered by the source in watt.
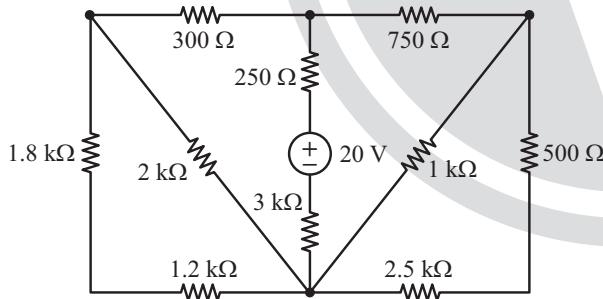
4. [NAT] **[Level-I]**

A battery is capable of providing 30mA current for 4 hours. If the energy stored in battery is 1300 joules, then the terminal voltage of battery is _____ volts:

5. [MSQ] **[Level-II]**
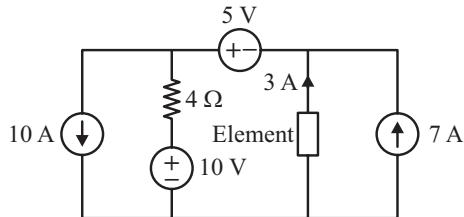
In the given network, consider two elements are connected in such a way that either they are delivering the power or they are absorbing.

Analyze the circuit and mark the correct statement (s)

(a) Power delivered by 5V battery is 50W

(b) Power delivered by 10V battery is 100W

(c) Power absorbed by 10A current source is 100W

(d) Power delivered by 7A current source is 35W

6. [MSQ] **[Level-I]**

The charge flowing in a conductor varies with time as $Q = at - bt^2$. Then, the current

(a) Decreases linearly with time

(b) Reaches a maximum and then decreases

(c) Falls to zero at time $t = a/2b$

(d) Changes at a rate $-2b$

7. [MCQ] **[Level-I]**

Consider the given circuit.
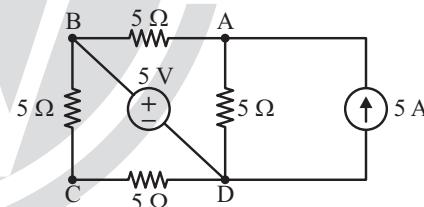
The voltage difference between A and C is

(a) 15 V   (b) 2.5 V

(c) 10 V   (d) 12.5 V

8. [MCQ] **[Level-II]**

Current I_0 flowing in the given circuit will be _____ if V-I characteristic of load is $V = 3I^2 + 2$.
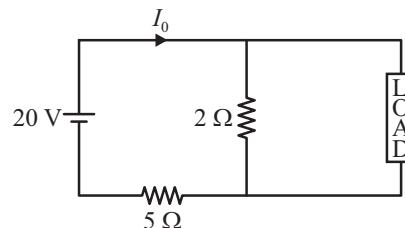
(a) 3.11 A   (b) 2.464 A

(c) 3.464 A   (d) 2.11 A

9. **[MCQ]** **[Level-I]**

In the given circuit, the voltage drop across 5 Ω resistance is:
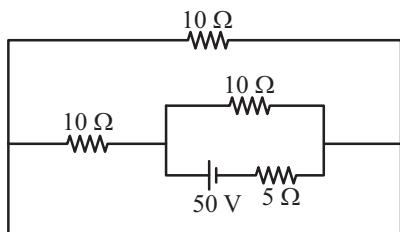
(a) 25 V   (b) 50 V

(c) 12.5 V   (d) 10 V

10. **[NAT]** **[Level-I]**

The value of current flowing through 4 Ω resistance in circuit shown in figure is_____ A.
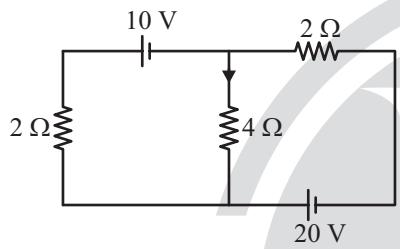
11. **[NAT]** **[Level-I]**

Consider the circuit shown below.

The value of current I_1 is _____ A.
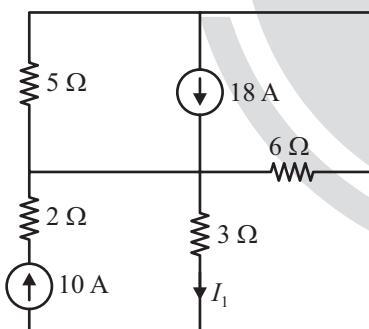
12. **[MCQ]** **[Level-II]**

For the circuit shown below, the power loss in 5 Ω resistor is 500W. The integral value of n is
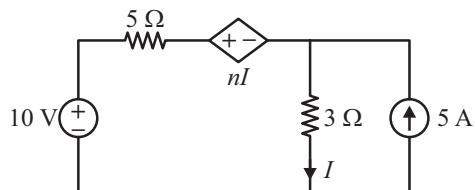
(a) 15   (b) 6

(c) −6   (d) −15

13. **[NAT]** **[Level-II]**

The value of voltage V_1 is _____ volts.
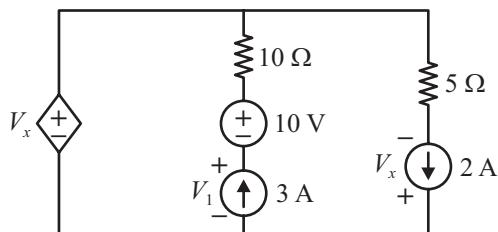
14. **[MCQ]** **[Level-II]**

In the circuit, the value of I_x is smallest natural number when m is

(Sky Level)
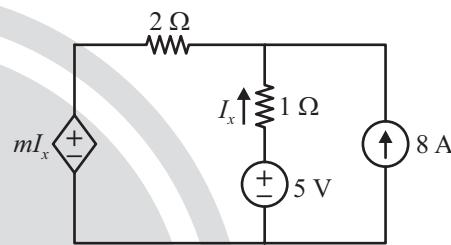
(a) 8   (b) −3

(c) −14   (d) 14

15. **[NAT]** **[Level-II]**

In the given network, the value of V_0 is _____ volts.
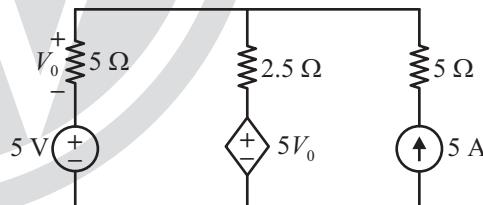
16. **[MCQ]** **[Level-II]**

In the circuit shown below, the value of V_x is

(a) − 1.56V   (b) − 2.33 V

(c) − 5.96 V   (d) − 4.28 V

17. **[MSQ]** **[Level-II]**

Which of the following circuits will make super node ?

(a)

(b)

(c)

(d)

18. **[NAT]** **[Level-III]**

In the given circuit, power delivered by dependent source is _____ watts.

19. **[MSQ]** **[Level-III]**

For the given circuit, value of i_R is:

(a) $i_R = 0$ for $x = 1$

(b) $i_R = \dfrac{ri_s}{2R + r}$ for $x = 0.5$

(c) $i_R = \dfrac{ri_s(1-x)}{2R + r(1-x)}$

(d) $i_R = \dfrac{ri_s(1-x)}{R + r(1-x)}$

20. **[MSQ]** **[Level-I]**

For the given circuit, the value of i_0 & i are:

(a) $i_0 = \dfrac{1}{9}$ A   (b) $i_0 = \dfrac{2}{9}$ A

(c) $i = \dfrac{10}{9}$ A   (d) $i = \dfrac{1}{9}$ A

21. **[NAT]** **[Level-II]**

In the given circuit, the value of i_x is_____ A.

22. **[NAT]** **[Level-II]**

In the network shown below, the current through 11Ω resistor is _____ Amp.

ANSWER KEY

		GATE			
1.	(22.5)	**2.**	(475200)	**3.**	(0.374)
4.	3 (3 to 3)	**5.**	(a, c, d)	**6.**	(a, c, d)
7.	(d)	**8.**	(a)	**9.**	(a)
10.	(–3)	**11.**	(13.33)	**12.**	(c)
13.	(25)	**14.**	(c)	**15.**	(–2.14)
16.	(d)	**17.**	(c, d)	**18.**	(800)
19.	(a, b, d)	**20.**	(a, c)	**21.**	(1.6)
22.	(0.25)	**23.**	(–6, –288)	**24.**	(1)
25.	(–2.33)	**26.**	(27.2)		

		ESE & PSUs			
1.	(c)	**2.**	(a)	**3.**	(d)
4.	(b)	**5.**	(c)	**6.**	(c)
7.	(b)	**8.**	(b)	**9.**	(d)

SOLUTIONS

GATE

1. **(22.5)**

$$Q = \int i(t)\, dt$$

$Q|_{t=3\,sec}$ = Area of the curve upto 3 sec

$$= 5 \times 3 + (10-5) \times 1 + \frac{1}{2} \times (10-5) \times 1$$

$$= 22.5 \text{ Coulomb}$$

2. **(475200)**

$$v(t) = 10 + \frac{t}{2} \text{ volts}$$

$$I = 3A$$

$$E = \int_0^4 v(t) \cdot I \, dt$$

$$E = \int_0^4 (30 + 1.5t)\, dt$$

$$E = [30t + 0.75t^2]_0^4 = 0.132 \text{ kWh}$$

$$E = 0.132 \times 3600000 \text{ Joules}$$

$$E = 4755200 \text{ Joules}$$

3. **(0.374)**

The circuit will be reduced to

$$P = \frac{V^2}{R_{eq}} = \frac{20^2}{1068.18} = 0.374 \text{ watts}$$

4. **(3)**

$$E = VI \times t$$
$$1300 \text{ Joules} = V \times 30 \times 10^{-3} \times 40 \times 3600$$
$$V = 3 \text{ volts}$$

5. **(a, c, d)**
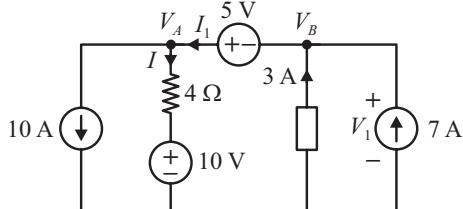
Apply KCL at node B,

$$I_1 = 3 + 7 = 10 \text{ A}$$

Apply KCL at node A,

$$I_1 = I + 10$$
$$I = I_1 - 10 = 10 - 10 = 0 \text{ A}$$
$$\therefore \quad V_A = 10 - 4 \times 0 = 10 \text{ V}$$
$$P_{5V} = 5I_1 = 5 \times 10 = 50 \text{ W}$$
$$P_{10V} = 0 \text{ W}$$

Power absorbed by 10 A source is

$$P_{10A} = 10V_A = 10 \times 10 = 100 \text{ W}$$
$$V_1 = 10 - 5 = 5V$$
$$P_{7A} = 7V_1 = 7 \times 5 = 35 \text{ W}$$
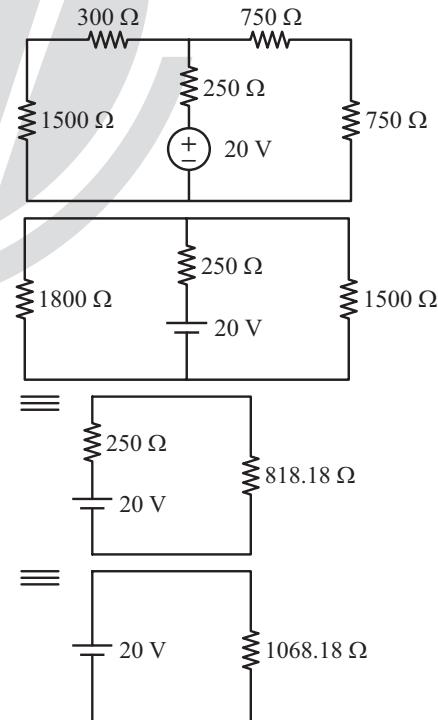
6. **(a, c, d)**

$$Q = at - bt^2$$
$$i = \frac{dQ}{dt} = \frac{d}{dt}[at - bt^2]$$
$$i = a - 2bt$$

It has a negative slope, hence it will decrease with time.
If it will be zero, then

$$A = 2bt$$
$$t = \frac{a}{2b}$$
$$\frac{di}{dt} = -2b$$

Hence, the current changes at a rate of $-2b$.

7. **(d)**
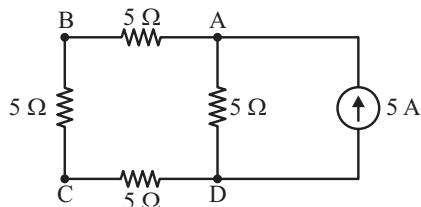
$$V_B = 5V, V_D = 0V$$

KCL at node A,

$$\frac{V_A - 5}{5} + \frac{V_A}{5} = 5$$
$$2V_A - 5 = 25$$
$$V_A = 15V$$
$$V_C = \frac{5}{2} = 2.5 \text{ V}$$
$$\therefore \quad V_A - V_C = 15 - 2.5 = 12.5 \text{ V}$$

8. **(a)**
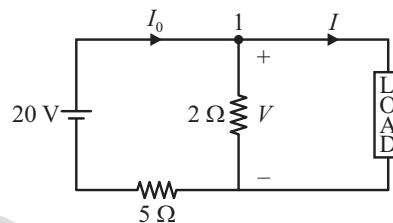
$$V = 3I^2 + 2$$

KCL at node 1,

$$I + \frac{V - 0}{2} + \frac{V - 20}{5} = 0$$
$$10I = 5V + 2V - 40 = 0$$
$$10I + 7V - 40 = 0$$
$$10I + 7(3I^2 + 2) - 40 = 0$$
$$10I + 21I^2 + 14 - 40 = 0$$
$$21I^2 + 10I - 26 = 0$$
$$I = 0.9 \text{ A}$$
$$V = 3I^2 + 2$$
$$V = 3(0.9)^2 + 2 = 4.43 \text{ V}$$
$$I_0 = \frac{V}{2} + I$$
$$I_0 = \frac{4.43}{2} + 0.9 = 3.115A$$

9. **(a)**
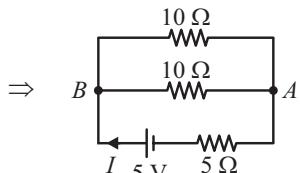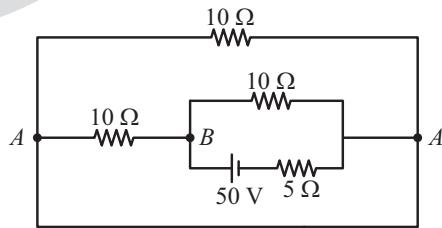
$$I = \frac{50}{5 + 5} = 5A$$
$$\therefore \quad V_{5\Omega} = 5 \times 5 = 25V$$

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- Communication System
- Electronic Devices & Circuits
- Electromagnetic Field Theory

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01 CHAPTER

Angle Modulation

1. **(NAT)** **[Level-II]**

An angle modulated signal with carrier frequency f_c is given by,

$S(t) = 5 \cos [2\pi f_c t + 4 \cos (1000\pi t) + 3 \sin (1000\pi t)]$

The maximum frequency deviation of the signal $S(t)$ will be _____ (in KHz).

2. **[MSQ]** **[Level-III]**

Consider a signal $x(t) = e^{-\frac{t^2}{2 \times 10^{-8}}}$ modulating a carrier signal of $f_c = 0.1$ MHz. Assume the modulating signal bandwidth as 3dB bandwidth of signal $x(t)$ and frequency sensitivity is 3000 Hz/V, the FM signal will have?

(a) $\Delta f_{max} = 4.5$ KHz

(b) BW = 8.65 KHz

(c) $\Delta f_{max} = 3$ KHz

(d) BW = 12.32 KHz

3. **[NAT]** **[Level-III]**

A carrier signal is amplitude and frequency modulated by a single tone message signal having bandwidth fm. The maximum frequency deviation in FM is equal to two times of transmission bandwidth of AM. The amplitude of side band spaced at $f_c + f_m$ of FM and AM are same. The modulation index of AM will be _____.
(Upto two decimal places)

Hint:

$J_1(2) = 0.57, J_1(4) = 0.06, J_1(8) = 0.235, J_1(6) = 0.28.$

4. **[MSQ]** **[Level-III]**

An exponential modulated signal with carrier frequency $f_c = 10^5$ Hz is given as,

$X(t) = 10 \cos (\omega_c t + 5 \sin 3000\pi t + 10 \sin 2000 \pi t)$

Which of the following statement is /are correct.

(a) The power of the modulated signal is 50 W.

(b) The peak frequency deviation is 10 kHz.

(c) The peak phase deviation is 15 rad.

(d) The peak frequency deviation is 20 kHz.

5. **[NAT]** **[Level-III]**

A periodic signal $m(t)$ is given below:
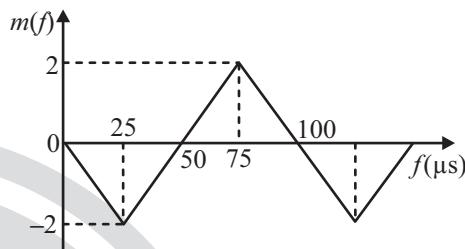
Assume the essential bandwidth of $m(t)$ as the frequency of its third harmonics.

The signal is expanded by a factor of 2 and applied as an input to phase modulator that has phase sensitivity $k_p = 5\pi$ radian/Volt.

Similarly, the expanded signal is applied as an input to frequency modulator that has frequency sensitivity $k_f = 2 \times 10^5$ Hz/Volt. then ratio of bandwidth of PM signal to bandwidth of FM is _____ KHz.

6. **[MSQ]** **[Level-II]**

Which of the following statement is/are correct?

(a) RF amplifier improves the overall noise performance of the receiver

(b) RF amplifier provides impedance matching between receiver and antenna

(c) transformer coupling is used at RF stage

(d) transformer coupling is used at IF stage

7. **[MCQ]** **[Level-II]**

An FM signal with a frequency deviation of 10 kHz at a modulation frequency of 5 kHz is applied to two frequency multiplier connected on cascade. The first multiplier doubles the frequency and second multiplier triples the frequency. Determine the frequency deviation and modulation index of the FM signal obtained at the second multipliers output.

(a) 50 kHz, 10

(b) 60 kHz, 6

(c) 60 kHz, 12

(d) 50 kHz, 5

30. **[MCQ]** [Level-III]

Let $m(t) = \cos[(4\pi \times 10^3)t]$ be the message signal and $c(t) = 5\cos[(2\pi \times 10^6)t]$ be the carrier.

$c(t)$ and $m(t)$ are used to generate an FM signal.

If the peak frequency deviation of the generated FM signal is three times, the transmission bandwidth of the AM signal,

Then the coefficient of the term $\cos[2\pi(1008 \times 10^3 t)]$ in the FM signal (in terms of the Bessel coefficients, $j_n(\beta)$) is

(a) $5J_4(3)$

(b) $\dfrac{5}{2}J_8(3)$

(c) $\dfrac{5}{2}J_8(4)$

(d) $5J_4(6)$

31. **[MCQ]** [Level-III]

A superheterodyne FM receiver operates in the frequency range of 88-108 MHz. The IF and local oscillator frequency are chosen such that $f_{cx} < f_{LO}$. The image frequency must fall outside of the 88-108 MHz region. The range of variation in f_{LO} is

(a) [98,128] MHz

(b) [78,128] MHz

(c) [98,118] MHz

(d) [78, 98] MHz

32. **[MCQ]** [Level-II]

What will be the carrier frequencies of the signal at position (3)?
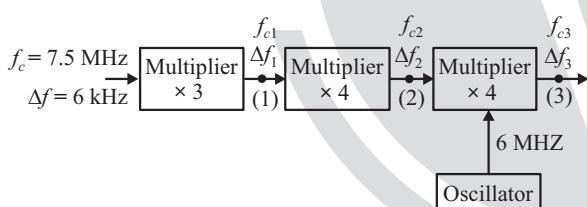
(a) 96 MHz

(b) 84 MHz

(c) Any of (a) and (b)

(d) None of these

Common Data for 33 and 34:

An FM modulator with frequency sensitivity $k_f = 10$ Hz/V has the message signal shown in figure below.
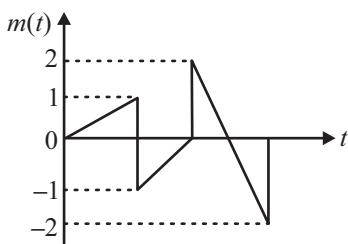
33. **[MCQ]** [Level-III]

The plot of frequency deviation for the signal is
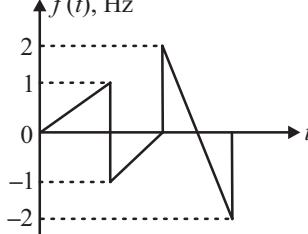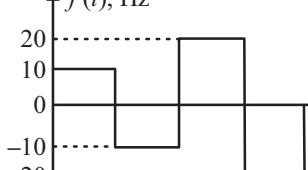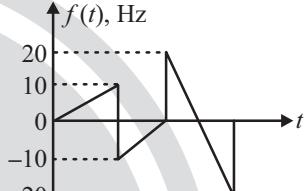
(d) None of these

34. **[MCQ]** [Level-III]

What will be the phase deviation for $0 \le t < 1$?

(a) $10\pi t^2$

(b) $20\pi t^2$

(c) $20\pi t$

(d) None of these

ESE & PSUs

1. An angle modulated signal is given as
$$x(t) = 100\cos[2\pi f_c t + 4\sin 2000\pi t]$$
where, $f_c = 10$ MHz
The peak-frequency deviation is

(a) 8000

(b) 4000

(c) 8000π

(d) 4000π

2. An angle modulated signal is given as
$$x(t) = 100\cos[2\pi f_c t + 4\sin 2000\pi t]$$
The correct statement is

(a) Signal x(t) is PM

(b) Signal x(t) is FM

(c) Signal x(t) is Both PM and FM

(d) Signal x(t) is AM

3. The primary purpose of narrowband FM is

(a) To be able to use frequencies above 108 MHz

(b) Reduce channel noise

(c) To conserve spectrum space

(d) To be able to use frequencies below 88 MHz

ANSWER KEY

GATE

1.	(2.5)	**2.**	(b, c)	**3.**	(0.12)
4.	(a, c)	**5.**	(0.277)	**6.**	(a, b, d)
7.	(c)	**8.**	(c)	**9.**	(1040)
10.	(3.83)	**11.**	(0.886)	**12.**	(137.372)
13.	(b)	**14.**	(1.001)	**15.**	(500)
16.	(c)	**17.**	(0.2)	**18.**	(6.25)
19.	(14)	**20.**	(1.67)	**21.**	(b)
22.	(b)	**23.**	(d)	**24.**	(250)
25.	(b, d)	**26.**	(5140)	**27.**	(1.220)
28.	(4.52)	**29.**	(5)	**30.**	(d)
31.	(c)	**32.**	(c)	**33.**	(c)
34.	(a)				

ESE & PSUs

1.	(b)	**2.**	(c)	**3.**	(c)
4.	(a)	**5.**	(b)	**6.**	(a)
7.	(c)	**8.**	(d)	**9.**	(d)

SOLUTIONS

GATE

1. (2.5)

Given:

Angle $\theta(t) = 2\pi fct + 4 \cos(1000 \pi t) + 3 \sin(1000 \pi t)$

Instantaneous frequency

$$f_i = \frac{1}{2\pi} \frac{d\theta(t)}{dt}$$

$$\frac{d\theta(t)}{dt} = 2\pi f_c + 4(1000\pi)(-\sin(1000 \pi t))$$
$$+ 3(1000 \pi) \cos(1000 \pi t)$$

$$\frac{d\theta(t)}{dt} = 2\pi f_c + 1000\pi (3 \cos(1000 \pi t) - 4 \sin(1000 \pi t))$$

$$f_i = \frac{1}{2\pi} [2\pi f_c + 1000\pi (3 \cos(1000 \pi t) - 4 \sin(1000 \pi t))]$$

$$f_i = f_c + 500 (3 \cos(1000 \pi t) - 4 \sin(1000 \pi t))$$

$$|f_i - f_c| = 500 \times 5 \cos(1000 \pi t + \alpha)$$

$$|f_i - f_c|_{max} = 2500$$

Maximum frequency deviation

$$\Delta f = |f_i - f_c|_{max} = 2500$$

In KHz, $\Delta f = 2.5$

2. (b, c)

$$X(t) = e^{-t^2/\tau}$$

$$e^{-at^2} \longrightarrow \sqrt{\frac{\pi}{a}} e^{-\omega^2/4a}$$

$$e^{-t^2/\tau} \longrightarrow \sqrt{\frac{\pi}{1/\tau}} e^{-\omega^2/4/\tau} = \sqrt{\pi\tau} \, e^{-\pi^2 f^2 \tau}$$

$$X(f) = \sqrt{\pi\tau} \, e^{-\pi^2 f^2 \tau}$$

For 3 dB frequency f_0

$$X(f_0) = \frac{X(f)|_{max}}{\sqrt{2}}$$

$$\Rightarrow \quad \sqrt{\pi\tau} \, e^{-\pi^2 f_0^2 \tau} = \frac{\sqrt{\pi\tau}}{\sqrt{2}}$$

$$\Rightarrow \quad e^{\pi^2 f_0^2 \tau} = \sqrt{2} \quad \Rightarrow f_0 = \sqrt{\frac{\ln(\sqrt{2})}{\pi^2 \tau}} = 1.325 \text{ kHz}$$

$$\Delta f_{max} = k_f \, x(t)_{max}$$
$$\Delta f_{max} = 3000 = 3 \text{ KHz}$$
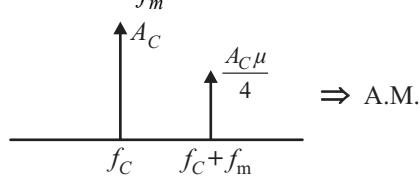$$\text{B.W} = 2(\Delta f_{max} + f_0) = 2(3 + 1.325)$$
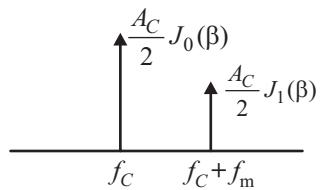
$$\text{B.W} = 8.65 \text{ kHz}$$

3. **(0.12)**

$$\Delta f_{max} = 2(\text{B.W.})_{AM}$$

$$\Rightarrow \quad \Delta f_{max} = 2(2f_m) = 4f_m$$

$$\beta = \frac{\Delta f_{max}}{f_m} = 4$$

$$\Rightarrow \text{A.M.}$$

$$\frac{A_C \mu}{4} = \frac{A_C J_1(\beta)}{2}$$

$$\Rightarrow \quad \mu = 2J_1(\beta)\big|_{\beta=4} = 2J_1(4)$$

$$\mu = 2(0.06) = 0.12$$

4. **(a, c)**

$$x(t) = 10\cos\left(\omega_c t + \underbrace{5\sin 3000\pi t + 10\sin 2000\pi t}_{\Delta\phi}\right)$$

$$\Delta\phi\big|_{max} = |5\sin\omega_1 t + 10\sin\omega_2 t|_{max} = 15$$

$$\text{Power} = 10^2 \times \frac{1}{2} = 50 \text{ W}$$

$$\theta_i(t) = \omega_c t + 5\sin 3000\pi t + 10\sin 2000\pi t$$

$$f_i = \frac{1}{2\pi}\frac{d\theta_i}{dt} = f_c + \frac{\begin{array}{c}5\times3000\pi\cos(3000\pi t)\\ +10\times2000\pi\cos(2000\pi t)\end{array}}{2\pi}$$

$$f_i = f_c + \frac{\overbrace{7500\cos w_1 t + 10000\cos w_2 t}^{\Delta f}}{}$$

$$\Delta f_{max} = 10000 + 7500 = 17.5 \text{ kHz.}$$

5. **(0.277)**
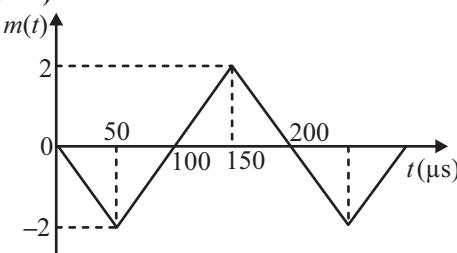
$$T = 200 \text{ μs} = 2\times10^{-4}$$

$$f_0 = \frac{1}{T} = \frac{1}{2\times10^{-4}} = 5 \text{ kHz}$$

$$f_m = 3f_0 = 3\times5 \text{ kHz} = 0.15\times10^5 \text{ Hz}$$

For PM signal

$$\theta_i(t) = \omega_C t + K_p m_1(t)$$

$$f_i = \frac{1}{2\pi}\frac{d\theta_i}{dt} = f_C + \frac{K_p}{2\pi}\frac{dm_1(t)}{dt}$$

$$\Delta f_{max} = \frac{K_p}{2\pi}\left|\frac{dm_1(t)}{dt}\right|_{max} = \frac{5\pi}{2\pi}\times\frac{2}{50\times10^{-6}}$$

$$\Delta f_{max} = 10^5 \text{ Hz}$$

$$(BW)_{PM} = 2(\Delta f_{max} + f_m) = 2(10^5 + 0.15 + 10^5)$$

$$(BW)_{PM} = 2.3\times10^5 \text{ Hz} = 230 \text{ kHz}$$

For FM signal

$$f_i = f_C + K_f m_1(t)$$

$$\Delta f_{max} = K_f |m_1(t)|_{max} = 2\times10^5 \times 2 = 4\times10^5 \text{ Hz}$$

$$(BW)_{FM} = 2(4\times10^5 + 0.15\times10^5)$$

$$(BW)_{FM} = 8.3\times10^5 \text{ Hz} = 830 \text{ kHz}$$

$$\frac{(BW)_{PM}}{(BW)_{FM}} = \frac{230}{830} = 0.277$$

6. **(a, b, d)**

(a) RF amplifier improves the overall noise performance of the receiver

(b) RF amplifier provides impedance matching between receiver and antenna

(d) Transformer coupling is used at IF stage

Hence, Option (a), (b), (d) are correct.

7. **(c)**

The overall frequency multiplication ratio is

$$n = 2\times3 = 6$$

FM wave at the output of the second frequency multipliers is

$$f(t) = nf_c + n\Delta f \cos(2\pi f_m t)$$

∴ Frequency deviation of FM is

$$n\Delta f = 6\times10\times10^3 = 60 \text{ kHz}$$

And modulation index is

$$\frac{n\Delta f}{f_m} = \frac{60 \text{ kHz}}{5 \text{ kHz}} = 12$$

8. **(c)**

$$\begin{array}{cc} m_1(t) & m_2(t) \\ \Downarrow & \Downarrow \\ fm_1 = 4 \text{ KHz} & 2 \text{ KHz} \\ \Downarrow & \Downarrow \\ AM & FM \end{array}$$

$$B_{FM} = 4$$

$$\mu = ?$$

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CONTENTS

ENGINEERING MATHEMATICS

GENERAL APTITUDE

❑❑❑

01 CHAPTER

Linear Algebra

Topic 1: Basics of Determinants

1. [MCQ]
If the determinant of a 4 × 4 matrix 'A' is 2, then the value of determinant of adj(adj(A)) is
(a) 1024
(b) 256
(c) 32
(d) 512

2. [MSQ]
If the adjoint of 3 × 3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$ then the determinant of P is (are)
(a) 2
(b) –2
(c) –1
(d) 1

3. [MCQ]
A matrix $A = \begin{bmatrix} p & q & r \\ q & r & p \\ r & p & q \end{bmatrix}$, where p, q and r are real positive numbers. If $pqr = 2$ and $A^T A = I$, then the value of $p^3 + q^3 + r^3$ should be equal to _____.
(a) 3
(b) 4
(c) 7
(d) 10

4. [MCQ]
If $ax^4 + bx^3 + cx^2 + dx + e = \begin{vmatrix} 2x & x-1 & x+1 \\ x+1 & x^2-x & x-1 \\ x-1 & x+1 & 3x \end{vmatrix}$
Then the value of e, is:
(a) 2
(b) 1.5
(c) 1
(d) 0

5. [MCQ]
The value of the determinant is:
$\begin{vmatrix} \underset{x \to 0}{Lt} \dfrac{\sin x}{x} & 2 & 4 \\ \underset{x \to 0}{Lt} x^2 \cdot \dfrac{\sin x}{x} & \displaystyle\int_0^{\pi/2} \sin x\, dx & -8 \\ \underset{x \to \infty}{Lt} \dfrac{\sin x}{x} & 0 & \left|\left(\dfrac{1}{2}\right)\right| \end{vmatrix}$

(a) $\dfrac{\sqrt{\pi}}{2}$
(b) $\sqrt{\pi}$
(c) $2\sqrt{\pi}$
(d) π

6. [NAT]
Let $A = \begin{bmatrix} 9 & 2 & 7 & 1 \\ 0 & 7 & 2 & 1 \\ 0 & 0 & 11 & 6 \\ 0 & 0 & -5 & 0 \end{bmatrix}$
Then the value of $|\det\ (8I - A)^3|$ is _____. (Enter in integer)

7. [NAT]
If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x+1) & x(x+1) \\ 3x(x-1) & x(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix}$
Then f (100) is equal to_____. (Enter in integer)

8. [MCQ]
If the matrix, $M = \left[M_{ij} \right] \forall \begin{array}{l} 1 \le i \le 2025, \\ 1 \le j \le 2025 \end{array}$
Such that $M_{ij} = \displaystyle\int_i^j \dfrac{x^2 \ln x}{2x} dx + \lim_{x \to \infty} \dfrac{\sin(i+j)x}{x}$
Then the determinant of the matrix 'M' is _____.
(a) 1
(b) –1
(c) 2025
(d) 0

9. [MCQ]
Let A be a 4 × 4 matrix with real entries. you are given the following information.
- det (A) = 6
- B is a matrix obtained by interchanging two rows of A
- C is obtained by multiplying one row of A by 3
- D is a matrix obtained by adding a multiple of one row of A to another row.

What are determinants of B, C & D?

(a) det $(B) = -6$, det $(C) = 18$, det $(D) = 6$
(b) det $(B) = 6$, det $(C) = 18$, det $(D) = 6$
(c) det $(B) = -6$, det $(C) = 2$, det $(D) = 6$
(d) det $(B) = 6$, det $(C) = 2$, det $(D) = 6$

10. [MCQ]

Let $A \in R^{4 \times 4}$ be partitioned as: $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

Where, $A_{11} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, $A_{12} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$, $A_{21} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$,

$A_{22} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

What will be det (A)?

(a) 12 (b) 13
(c) 8 (d) 6

11. [NAT]

Given the matrix $B = \begin{bmatrix} a & 1 & 2 & 3 \\ 0 & b & 4 & 5 \\ 0 & 0 & c & 6 \\ 0 & 0 & 0 & d \end{bmatrix}$, the determinant

is 120 and the trace is 14. The value of $|a - d|$ if $(a < b < c < d)$ are positive integers with , will be _____. (Enter in integer)

12. [NAT]

If matrix 'A' $= [a_{ij}]$ is defined by the element $a_{ij} = \int_i^j \sin^{(j+i)} x\, dx$ where $1 \le i \le 3; 1 \le j \le 3$.

The determinant of the matrix A is_____. (Round off to one decimal places)

Topic 2: Basics of Matrices

---

13. [MCQ]

Let $B = \begin{bmatrix} 1+2i & 3 \\ 4 & 2-i \end{bmatrix}$. If the inverse of B exists then

which of the following is correct form of B^{-1}?

(a) $\frac{1}{15} \begin{bmatrix} 2-i & -3 \\ -4 & 1+2i \end{bmatrix}$ (b) $\frac{1}{-8+3i} \begin{bmatrix} 2-i & -3 \\ -4 & 1+2i \end{bmatrix}$

(c) $\frac{1}{10} \begin{bmatrix} 2-i & -3 \\ -4 & 1+2i \end{bmatrix}$ (d) $\frac{1}{10} \begin{bmatrix} 2+i & 3 \\ 4 & 1-2i \end{bmatrix}$

14. [MCQ]

If a matrix 'A' shifts the vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ to $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ to

$\begin{bmatrix} 0 \\ 2 \end{bmatrix}$, then $A^2 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is:

(a) $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} -2 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 \\ -1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 \\ -2 \end{bmatrix}$

15. [NAT]

The minimum number of multiplications required to calculate the product ABC where $A_{3 \times 4}$, $B_{4 \times 5}$, $C_{5 \times 3}$ are the matrices, is _____ (Enter in integer).

16. [NAT]

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 0 & a & 1 \\ b & 3 & -2 \\ -1 & -1 & c \end{bmatrix}$ then

$\frac{a+b}{c} = $ _____. (Enter in integer)

17. [MSQ]

Let $[M]$ be a square matrix of order 3×3 such that

$a_{ij} = \begin{cases} 0, & i \neq j \\ \int_0^{\pi/2} \sin^i x\, dx & i = j \end{cases}$ then which of the following

is/are true?

(a) Trace of the matrix M is $\frac{\pi}{4} + \frac{5}{3}$

(b) Trace of the matrix M is $\frac{7\pi}{12} + 1$

(c) Determinant of matrix M is $\frac{\pi^2}{12}$

(d) Determinant of the matrix M is $\frac{\pi}{6}$

18. [MCQ]

If matrix A such that $A^2 = 2A - I$, where I is the identity matrix then for $n \ge 2$, An is equal to

(a) $nA - (n-1)I$ (b) $nA - I$
(c) $2^{n-1}A - (n-1)I$ (d) $2^{n-1}A - I$

19. [MCQ]

For $\alpha, \beta, \gamma \in R$ let $A = \begin{bmatrix} \alpha^2 & 6 & 8 \\ 3 & \beta^2 & 9 \\ 4 & 5 & \gamma^2 \end{bmatrix}$

and $B = \begin{bmatrix} 2\alpha & 3 & 5 \\ 2 & 2\beta & 6 \\ 1 & 4 & 2\gamma - 3 \end{bmatrix}$

If tr(A) = tr(B), then the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$ is:

(a) 1 (b) 2
(c) 4 (d) 3

20. [MCQ]

Let A and B be two matrices such that AB exists. Consider the following statements regarding the existence of BA:

1. BA must exist if A and B are both invertible.
2. If AB exists, then BA always exists.
3. If A and B are square matrices of the same order then BA exists.
4. If A has dimensions $m \times n$ and B has deminsion $n \times p$, then BA exists if and only if $p = m$

Which of the above statements is/are correct?

(a) Only 1 and 3   (b) Only 1 and 4
(c) Only 2, 3 and 4   (d) Only 1, 3 and 4

21. [MCQ]

A matrix 'A' is defined by $a_{ij} = i^2 - j^2 \; \forall \; \begin{matrix} 1 \le i \le 2013 \\ 1 \le j \le 2013 \end{matrix}$

The value of sum of all the elements in the matrix is

(a) 1   (b) –1
(c) 0   (d) 5

22. [MCQ]

If a matrix $A = [a_{ij}]$ where $a_{ij} = i^j - j^i$ where $1 \le i \le 3$, $1 \le j \le 3$, then the value of $\displaystyle\sum_{i=1}^{3}\sum_{j=1}^{3} a_{ij}$ is

(a) 207   (b) 0
(c) 217   (d) 343

23. [MCQ]

If $A = A^T$ and $B = -B^T$ then the matrix '$AB + BA$' always is:

(a) Symmetric   (b) Skew-Symmetric
(c) Orthogonal   (d) Singular

24. [MCQ]

If $A = (a_{ij})_{n \times n}$, where $a_{ij} = i^2 - j^2$ is a square matrix of even order then

(a) A is symmetric and $|A|$ is a perfect square
(b) A is symmetric and $|A| = 0$
(c) A is a skew-symmetric matrix and $|A| = 0$
(d) None of these

Topic 3: Rank of Matrix

25. [MCQ]

Let $v_1 = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. Find the value of the coefficient in the expression $v_1 = v_2 + e$, which minimizes the length of the error.

(a) 13/9   (b) 11/9
(c) 17/9   (d) 2/9

26. [NAT]

Let $v_1 = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \& v_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. The error vector $\vec{e} = v_1 - \alpha v_2$

has the modulus $\|e\|$. Then, at what value of α $\|e\|$ is minimized?

27. [NAT]

If $'A' = \begin{bmatrix} 1/9 & 8/9 & -4/9 \\ 4/9 & -4/9 & -7/9 \\ 8/9 & 1/9 & 4/9 \end{bmatrix}$ is an orthogonal matrix and

$\vec{x} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ is a vector, then the length of the vector $A\vec{x}$ is_____. (Round off to two decimals)

28. [MCQ]

What will be the value of 'a' such that the rank of 4×4 matrix $A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ a & 2 & 2 & 2 \\ 9 & 9 & a & 3 \end{bmatrix}$ is 3?

(a) $a = \{2, -6\}$ (b) $a = \{1, -6\}$
(c) $a = \{2, 6\}$ (d) $a = \{2, 1\}$

29. [MCQ]

The rank of the matrix $[A]_{3 \times 3} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix}$ will be :

(a) 1   (b) 2
(c) 3
(d) The rank cannot be determined

30. [MCQ]

Let A be an $n \times m$ matrix. Consider the following statements:

1. The rank of A is equal to the maximum number of linearly independent rows.
2. The rank of A is equal to the maxinaun number of linearly independent columns.
3. If A is an $n \times m$ matrix with $n > m$ then the rank of A can never exceed m.
4. If A is a Singular Square matrix, then its rank is equal to its size.

Which of the above statement is/are correct?

(a) only 1 and 2   (b) Only 1, 2 and 3
(c) Only 1, 2 and 4   (d) 1, 2, 3 and 4

Topic 4: Non-Homogeneous System

31. **[MCQ]**

Consider the following system of equations:

$x + y + z = 3$

$2x + 3y + z = 7$

$3x + 4y + 2z = 10$

Which of the following statement is correct?

(a) The system has a unique solution

(b) The system has no solution

(c) The system has infinitely many solutions

(d) The system has exactly two soultion

32. **[MCQ]**

Which of the following ordered pair (m, n) of the lienar equations:

$x + 2y + 3z = 4$

$3x + 4y + 5z = m$

$4x + 9y + 9z = n$

is consistent.

(a) (4, 1) (b) (5, 2)

(c) (6, 3) (d) For any values of m, n

33. **[MCQ]**

Consider the following system of linear equations?

$x + 2y + z = 4$

$2x + 4y + 2z = 8$

$3x + 6y + kz = 12$

Which of the following statement is correct?

(a) The system has infinitely many solution for $k = 2$.

(b) The system has no. solution for $k = 3$.

(c) The system has unique solution for $k = 1$

(d) The rank of confined matrix is always 2 regardless of k.

34. **[MCQ]**

A system of linear equation given below.

$x + 2y + \mu z = \lambda$

$x + y + z = 6$

$x + 2y + 3z = 10$

The system has no solution if

(a) $\lambda = 10, \mu = 3$

(b) $\mu \neq 3, \lambda = 10$

(c) $\mu = 3, \lambda \neq 10$

(d) $\mu \neq 3$

35. **[NAT]**

A d.c circuit involves 3 closed loops. Applying Kirchoff's laws to the closed loops give the following equations for current flow in milliamperes.

$2I_1 + 3I_2 - 4I_3 = 26$

$I_1 - 5I_2 - 3I_3 = -87$

$-7I_1 + 2I_2 + 6I_3 = 12$

The value of 'I_3' in milli amperes is _____. (Enter an integer)

36. **[MCQ]**

Consider the system of equations:

$4x - 2y + 6z = 5$

$2x + 2y + 4z = 3$

$10x - 2y + \lambda z = b$

If the system has infinitely many solutions, then the values of λ & b will be:

(a) $\lambda = 14, b = 13$ (b) $\lambda = 13, b = 12$

(c) $\lambda = 14, b = 10$ (d) $\lambda = 16, b = 13$

37. **[MCQ]**

Consider the following linear system of equations:

$2x + 4y - 6z = p$

$4x + 6y + 6z = q$

$10x + 18y - 12z = r$

Which of the following option is correct?

(a) The system is consistent for all values of p, q, r

(b) The system is consistent if $p + q + r = 0$

(c) The system is consistent if $p - q + 2r = 0$

(d) The system is consistent if $3p + q - r = 0$

38. **[MCQ]**

A company needs to allocate n resources to m tasks. The allocation is modeled as a system of linear equations with m equations and n variables. Consider the following scenarios.

1. If the number of task m is less than the number of resources n, then there may exist infinitely many ways to allocate recourses.

2. If $m > n$, the allocation becomes impossible for any configuration.

3. If $m = n$, the allocation can be determined uniquely if the allocation matrix is invertible.

Which of the following is correct?

(a) Only 1 is correct

(b) Only 3 is correct

(c) Both 1 and 3 are correct

(d) None of these are correct

39. **[MCQ]**

Consider the system of equations:

$x + y - z = 4$

$x + 3y + z = 10$

$x - y + 2z = 3$

Which of the following statement is correct?

(a)   The system has no solutions

(b)   The system has infinitely many solutions

(c)   The system has a unique solutions

(d)   The system has inconsistent equations

40.   [MCQ]

For what value of λ, the given system have a solution:

$2x + y + z = 1$

$2x + 2y + 4z = \lambda$

$6x + 6y + 12z = \lambda^2$

(a) $\dfrac{7 \pm \sqrt{17}}{2}$ (b) $\dfrac{5 \pm \sqrt{17}}{2}$

(c) $\dfrac{1 \pm \sqrt{17}}{2}$ (d)   0, 3

41.   [MCQ]

Consider the following equations:

$x + 2y - z = 4$

$2x + 5y + kz = 10$

$3x + 7y + 3z = 14$

For what value of 'k' the above system of equation has infinitely many solutions?

(a) $k \neq 4$ (b) $k = 4$

(c) $k = 2$ (d) $k \neq 2$

42.   [MCQ]

Given the matrix A as follows:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{bmatrix}$$

What can be concluded about the system $Ax = b$ for any vector b?

(a)   The system has a unique solution for any b.

(b)   The system has no solution for any b.

(c)   The system has infinitely many solutions for any b.

(d)   The system may have no solutions or infinitely many solutions depending on b.

43.   [MCQ]

Consider the system of equations:

$x + 3z = 5$

$-2x + 5y - z = 0$

$-x + 4y + z = 4$

Which of the following statements is correct about the consistency of the system?

(a)   The system is consistent with a unique solution.

(b)   The system is inconsistent and has no solution.

(c)   The system is consistent and has infinitely many solution.

(d)   The system has a solution only if $x = 0$.

44.   [MSQ]

Consider the following system of equation:

$x + y + z = 1$

$x + 2y + 4z = n$

$x + 4y + 10z = n^2$

For what values of 'n' does the system have infinite solutions and what is the corresponding general solution for x, y and z?

(a) $n = 1$, with the solution $x = 1 + 2z, y = -3z, z = z$

(b) $n = 1$, with the solution $x = 2z, y = 1 - 3z, z = z$

(c) $n = 2$, with the solution $x = 1 + 2z, y = -3z, z = z$

(d) $n = 2$, with the solution $x = 2z, y = 1 - 3z, z = z$

45.   [MCQ]

If A is 4×5 matrix and the system of equations $AX = B$ is inconsistent then the highest possible rank of A will be

(a) $\rho(A) \leq 2$ (b) $\rho(A) \leq 3$

(c) $\rho(A) \leq 4$ (d) $\rho(A) \leq 5$

Topic 5: Homogeneous System

46.   [MCQ]

Consider a 3×5 matrix A defined as follows

$$A = \begin{bmatrix} 1 & 4 & 5 & a & 18 \\ 0 & 1 & 7 & 19 & b \\ 0 & 0 & 1 & 11 & 15 \end{bmatrix}$$

Where a and b are real numbers. Choose the correct statement regarding the matrix A based on values of a and b.

(a)   There are specific values of a and b for which the columns of A become linearly independent.

(b)   There are values of a and b for which the equation $Ax = 0$ has only the trivial solution $x = 0$.

(c)   For any values of a and b, the rows of A span a 3-d subspace in R^5.

(d)   There exist values of a and b such that the rank A is equal to 2.

47.   [MCQ]

Consider the following system of linear equations:

$2x + 2py + pz = 0$

$2x + 2qy + qz = 0$

$2x + 2ry + rz = 0$

where $p, q, r \in R$ are non-zero and distinct: has a non-zero solution, then choose correct option.

(a) $p + q + r = 0$

(b) $p + q + r = 1$

(c) p, q, r can be any combination except 0

(d) $\dfrac{1}{p}, \dfrac{1}{q}, \dfrac{1}{r}$ are in $A.P.$

ANSWER KEY

1.	(d)	**2.**	(a, b)	**3.**	(c)
4.	(d)	**5.**	(b)	**6.**	(216)
7.	**(0)**	**8.**	(d)	**9.**	(a)
10.	(b)	**11.**	(3)	**12.**	(–0.001 to 0.001)
13.	(b)	**14.**	(a)	**15.**	(96)
16.	(–0.001 to 0.001)	**17.**	(a, d)	**18.**	(a)
19.	(d)	**20.**	(d)	**21.**	(c)
22.	(b)	**23.**	(b)	**24.**	(d)
25.	(a)	**26.**	**(2)**	**27.**	(3.7 to 3.78)
28.	(a)	**29.**	(b)	**30.**	(b)
31.	(c)	**32.**	(d)	**33.**	(a)
34.	(c)	**35.**	(8 to 10)	**36.**	(d)
37.	(d)	**38.**	(c)	**39.**	(c)
40.	(d)	**41.**	(b)	**42.**	(d)
43.	(b)	**44.**	(a, d)	**45.**	(b)
46.	(c)	**47.**	(c)	**48.**	(a, d)
49.	(a)	**50.**	(c)	**51.**	(1)
52.	(a)	**53.**	(d)	**54.**	(0)
55.	(a, b)	**56.**	(c)	**57.**	(12)
58.	(b)	**59.**	(a)	**60.**	(a)
61.	(d)	**62.**	(a)	**63.**	(d)
64.	(a)	**65.**	(b)	**66.**	(a)
67.	(a)	**68.**	(c)	**69.**	(a)
70.	(a, c)	**71.**	(c)	**72.**	(d)
73.	(–0.3 to –0.4)	**74.**	(b)	**75.**	(c)
76.	(56)	**77.**	(b)	**78.**	(a)
79.	(d)	**80.**	(a)	**81.**	(c)
82.	(16)	**83.**	(a)	**84.**	(b)
85.	(c)	**86.**	(a)	**87.**	(c)
88.	(a)	**89.**	(d)	**90.**	(a)
91.	(b)	**92.**	(a)	**93.**	(a, b, c)
94.	(a, b, c, d)	**95.**	(c)	**96.**	(a)
97.	(d)	**98.**	(c)	**99.**	(c)
100.	(d)	**101.**	(37)	**102.**	(a, b, c)
103.	(b)	**104.**	(c)	**105.**	(d)
106.	(a)	**107.**	(b)	**108.**	(a)
109.	(c, d)	**110.**	(c)	**111.**	(b)
112.	(a)	**113.**	(d)	**114.**	(b)
115.	(d)	**116.**	(4)	**117.**	(b, d)
118.	(b)	**119.**	(b)	**120.**	(a)
121.	(a)	**122.**	(b)	**123.**	(b)
124.	(a, b, d)	**125.**	(4)	**126.**	(d)
127.	(c)	**128.**	(b)	**129.**	(c)

130. (a)	**131.** (b)	**132.** (d)
133. (a)	**134.** (d)	**135.** (a, b)
136. (b)	**137.** (a)	**138.** (a)
139. (a, d)	**140.** (a)	**141.** (a)
142. (a, d)	**143.** (12)	**144.** (c)
145. (20)	**146.** (a)	**147.** (–6)
148. (b)	**149.** (d)	**150.** (d)
151. (a)	**152.** (b, d)	**153.** (a, c)

SOLUTIONS

1. **(d)**

Given, $|A| = 2$ and $n = 4$

For a matrix $A_{n \times n}$

$|\text{adj } A| = |A|^{n-1}$ and $|\text{adj (adj } A)| = |A|^{(n-1)^2}$

$\Rightarrow \quad |\text{adj (adj } A)| = 2^{(4-1)^2} = 2^9 = 512$

$\therefore \quad |\text{adj (adj } A)| = 512$

2. **(a, b)**

Given:

$\text{Adj } P = \begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$

$|\text{Adj } P| = \begin{vmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{vmatrix} \qquad R_2 \to R_2 - 2R_1, R_3 \to R_3 - R_1$

$|\text{Adj } P| = \begin{vmatrix} 1 & 4 & 4 \\ 0 & -7 & -1 \\ 0 & -3 & -1 \end{vmatrix} = 1(7 - 3) = 4$

$\because \quad |\text{Adj } P|_{3 \times 3} = |P|^{3-1}$ \quad (Property of Adjoint)

On comparing it, $|P|^2 = 4 \Rightarrow |P| = \pm 2$

3. **(c)**

Given, $A = \begin{bmatrix} p & q & r \\ q & r & p \\ r & p & q \end{bmatrix} \to$ Symmetric matrix $(A = A^T)$

$p, q, r \in \mathbb{R}^+; p \, q \, r = 2$

$A^T A = I \Rightarrow A^2 = I$

$\Rightarrow |A|^2 = |I| = 1$

$\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix}^2 = 1$

$\left(p\left(qr - p^2\right) - q\left(q^2 - rp\right) + r\left(pq - r^2\right) \right)^2 = 1$

$\left(3pqr - \left(p^3 + q^3 + r^3\right) \right)^2 = 1$

$6 - (p^3 + q^3 + r^3) = \pm 1$

$p^3 + q^3 + r^3 = 5 \text{ (or) } 7$

4. **(d)**

Given:

$ax^4 + bx^3 + cx^2 + dx + e = \begin{vmatrix} 2x & x-1 & x+1 \\ x+1 & x^2-x & x-1 \\ x-1 & x+1 & 3x \end{vmatrix}$

Put $x = 0$

$e = \begin{vmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{vmatrix}$

Using $C_2 \to C_2 + C_3$

$e = \begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & -1 \\ -1 & 1 & 0 \end{vmatrix}$

$\Rightarrow \quad e = 1 - 1 = 0$

5. **(b)**

$\begin{vmatrix} \underset{x \to 0}{Lt} \dfrac{\sin x}{x} & 2 & 4 \\ \underset{x \to 0}{Lt} \, x^2 \dfrac{\sin x}{x} & \displaystyle\int_0^{\pi/2} \sin x \, dx & -8 \\ \underset{x \to \infty}{Lt} \dfrac{\sin x}{x} & 0 & \sqrt{\dfrac{1}{2}} \end{vmatrix}$

$= \begin{vmatrix} 1 & 2 & 4 \\ 0 & 1 & -8 \\ 0 & 0 & \sqrt{\pi} \end{vmatrix} = 1(\sqrt{\pi} - 0) = \sqrt{\pi}$

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CONTENTS

❑❑❑

01 Basics of Energy and Environment

Ecology and Environment

1. Match the following types of ecological pyramids (List I) with their correct characteristics (List II):

List I (Ecological Pyramid)	List II (Characteristic)
A. Pyramid of Numbers	1. Always upright
B. Pyramid of Biomass	2. Can be inverted in parasitic ecosystems
C. Pyramid of Energy	3. Can be upright or inverted
D. Pyramid of Numbers (Grassland)	4. Producer > Herbivore > Carnivore

Select the correct code:
- (a) A-3, B-2, C-1, D-4
- (b) A-2, B-3, C-1, D-4
- (c) A-3, B-1, C-2, D-4
- (d) A-3, B-2, C-4, D-1

2. Match the following ecosystems (List I) with the type of biomass pyramid they usually exhibit (List II):

List I (Ecosystem)	List II (Pyramid of biomass Type)
A. Forest ecosystem	1. Inverted
B. Aquatic ecosystem	2. Upright
C. Grassland ecosystem	3. Generally upright
D. Oceanic food chain	4. Biomass of zooplankton > phytoplankton

Select the correct code:
- (a) A-2, B-1, C-3, D-4
- (b) A-2, B-4, C-1, D-3
- (c) A-3, B-4, C-2, D-1
- (d) A-3, B-1, C-2, D-4

3. Match the Trophic Level with its Description.

List I (Trophic Level)	List II (Description)
A. Producers	1. Always at the base of ecological pyramids
B. Primary Consumers	2. Herbivores
C. Secondary Consumers	3. Feed on herbivores
D. Decomposers	4. Operate at all trophic levels

Select the correct code:
- (a) A-1, B-2, C-3, D-4
- (b) A-2, B-1, C-4, D-3
- (c) A-1, B-4, C-2, D-3
- (d) A-4, B-2, C-3, D-1

4. Match the Type of Succession with its Description.

List I (Type of Succession)	List II (Description)
A. Primary Succession	1. Begins in areas with no previous life (e.g., lava)
B. Secondary Succession	2. Occurs in areas with pre-existing life but disturbed
C. Autogenic Succession	3. Driven by organisms within the ecosystem itself
D. Allogenic Succession	4. Driven by external factors like fire or flood

Select the correct code:
- (a) A-1, B-2, C-3, D-4
- (b) A-2, B-1, C-4, D-3
- (c) A-1, B-3, C-2, D-4
- (d) A-1, B-2, C-4, D-3

5. Match the Stage of Succession with its Key Features.

List I (Succession Stage)	List II (Feature)
A. Nudation	1. Exposure of a bare area
B. Invasion	2. Arrival and establishment of species
C. Competition & Coaction	3. Interaction and struggle for resources
D. Stabilization/Climax	4. Formation of a stable and mature community

Select the correct code:
- (a) A-1, B-2, C-3, D-4
- (b) A-2, B-1, C-3, D-4
- (c) A-1, B-3, C-2, D-4
- (d) A-1, B-2, C-4, D-3

6. Match the Succession Type with suitable example.

List I (Succession Type)	List II (Example)
A. Hydrarch Succession	1. Pond turning into forest over time
B. Xerarch Succession	2. Rocky surface gradually becoming forest
C. Lithosere	3. Succession on bare rock surface
D. Psammosere	4. Succession on sandy habitat like sea coasts

Select the correct code:
(a) A-1, B-2, C-3, D-4
(b) A-2, B-1, C-4, D-3
(c) A-1, B-4, C-3, D-2
(d) A-4, B-2, C-1, D-3

7. **Assertion (A):** The carbon footprint of an individual includes direct as well as indirect greenhouse gas emissions.

Reason (R): Emissions from electricity usage and transportation are direct emissions, while emissions from goods and services consumed are indirect.
(a) Both A and R are true, and R is the correct explanation of A.
(b) Both A and R are true, but R is not the correct explanation of A.
(c) A is true, but R is false.
(d) A is false, but R is true.

8. **Assertion (A):** Carbon handprint focuses on minimizing the harmful effects of human actions on the environment.

Reason (R): It quantifies the greenhouse gases added to the atmosphere by human activities.
(a) Both A and R are true, and R is the correct explanation of A.
(b) Both A and R are true, but R is not the correct explanation of A.
(c) A is true, but R is false.
(d) A is false, but R is true.

9. **Assertion (A):** Adopting lifestyle changes such as reducing meat consumption and using public transport can lower an individual's carbon footprint.

Reason (R): These lifestyle changes contribute to the net-zero emission goals of countries.
(a) Both A and R are true, and R is the correct explanation of A.
(b) Both A and R are true, but R is not the correct explanation of A.
(c) A is true, but R is false.
(d) A is false, but R is true.

10. Which one of the following is the best description of the term "ecosystem"?
(a) A community of organisms interacting with one another.
(b) That part of the Earth which is inhabited by living organisms.
(c) A community of organisms together with the environment in which they live.
(d) The flora and fauna of a geographical area.

11. Which of the following best defines the concept of "ecotone"?
(a) A region where sunlight penetration is highest in an ecosystem
(b) A transitional zone between two distinct ecosystems with high species diversity
(c) A part of an ecosystem where only detritivores survive
(d) A nutrient-poor, stable ecosystem zone

12. Which of the following are considered "ecosystem services"?
1. Pollination
2. Climate regulation
3. Food and fiber production
4. Carbon sequestration
The correct option is:
(a) 1 and 2 only   (b) 3 and 4 only
(c) 1, 2 and 3 only (d) All of the above

13. Which one of the following terms describes not only the physical space occupied by an organism, but also its functional role in the community of organisms?
(a) Ecotone   (b) Ecological niche
(c) Habitat   (d) Home range

14. Which one of the following is the process involved in photosynthesis?
(a) Potential energy is released to form free energy
(b) Free energy is converted into potential energy and stored
(c) Food is oxidized to release carbon dioxide and water
(d) Oxygen is taken, and carbon dioxide and water vapour are given out

15. Consider the following statements regarding 'Earth Hour':
1. It is an initiative of UNEP and UNESCO.
2. It is a movement in which the participants switch off the lights for one hour on a certain day every year.
3. It is a movement to raise awareness about climate change and the need to save the planet.

Which of the statements given above is/are correct?

(a) 1 and 3 only   (b) 2 only

(c) 2 and 3 only   (d) 1, 2 and 3

16. In the context of ecosystem productivity, marine upwelling zones are important as they increase the marine productivity by bringing the:

1. Decomposer microorganisms to the surface.

2. Nutrients to the surface.

3. Bottom-dwelling organisms to- the surface.

Which of the statements given above is/are correct?

(a) 1 and 2   (b) 2 only

(c) 2 and 3   (d) 3 only

17. In the grasslands, trees do not replace the grasses as a part of an ecological succession because of :

(a) Insects and Fungi

(b) Limited sunlight and paucity of nutrients

(c) Water limits and fire

(d) None of the above

18. Which one of the following is the correct sequence of ecosystems in the order of decreasing productivity?

(a) Oceans, lakes, grasslands, mangroves

(b) Mangroves, oceans, grasslands, lakes

(c) Mangroves, grasslands, lakes, oceans

(d) Oceans, mangroves, lakes, grasslands

19. Consider the following statements about ecological succession:

1. Primary succession occurs on newly exposed surfaces like lava or sand.

2. Secondary succession takes place in areas where a previous community existed.

3. Climax community is always a forest ecosystem.

Which of the above are correct?

(a) 1 and 2 only   (b) 2 and 3 only

(c) 1 and 3 only   (d) 1, 2 and 3

20. Which one of the following ecosystems has the highest gross primary productivity (GPP) but lower net primary productivity (NPP) due to high respiration loss?

(a) Ocean   (b) Tropical rainforest

(c) Desert   (d) Grassland

21. With reference to food chains in ecosystems, consider the following statements:

1. A food chain illustrates the order in which a chain of organisms feed upon each other.

2. Food chains are found within the populations of a species.

3   A food chain illustrates the numbers of each organism which are eaten by others.

Which of the statements given above is / are correct?

(a) 1 only   (b) 1 and 2 only

(c) 1, 2 and 3   (d) None

22. Arrange the following in the correct sequence in an aquatic food chain:

1. Small fishes

2. Zooplankton

3. Large carnivorous fishes

4. Phytoplankton

(a) $4 \rightarrow 2 \rightarrow 1 \rightarrow 3$ (b) $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$

(c) $4 \rightarrow 1 \rightarrow 2 \rightarrow 3$ (d) $2 \rightarrow 4 \rightarrow 1 \rightarrow 3$

23. Which of the following is correct regarding ecological pyramids?

(a) Pyramid of biomass is always upright

(b) Pyramid of energy can be inverted in aquatic ecosystems

(c) Pyramid of numbers can be both upright and inverted

(d) Pyramid of energy may not follow 10% law

24. Which of the following statements regarding food chains is/are correct?

1. Energy transfer is efficient and continuous.

2. Longer food chains are more stable.

3. Bioaccumulation increases as we go up trophic levels.

(a) 1 only   (b) 2 and 3 only

(c) 3 only   (d) 1 and 3 only

25. Which of the following are keystone species in their ecosystems?

1. Tiger in a forest

2. Sea otter in kelp forests

3. Coral in coral reefs

4. Grass in grassland

The correct option is:

(a) 1, 2 and 3 only (b) 2, 3 and 4 only

(c) 1 and 4 only   (d) All of the above

26. In ecological niche theory, if two species occupy the same niche:

(a) Both can coexist without competition

(b) One species will be eliminated

(c) The environment adapts to accommodate both

(d) They evolve into similar species

27. Which of the following ecosystems has the lowest net primary productivity (NPP)?

(a) Desert   (b) Estuary

(c) Grassland   (d) Tropical forest

28. The concept of "trophic cascade" is associated with:
- (a) Collapse of primary producers
- (b) Progressive increase in biomass at higher levels
- (c) Impact of top predators on ecosystem structure
- (d) Herbivore-plant mutualism

29. Detritus food chain differs from grazing food chain in that:
- (a) It begins with herbivores
- (b) It includes photosynthetic producers
- (c) It starts with dead organic matter
- (d) It doesn't involve energy flow

30. Lichens, which are capable of initiating ecological succession even on a bare rock, are actually a symbiotic association of
- (a) Algae and bacteria
- (b) Algae and fungi
- (c) Bacteria and fungi
- (d) Fungi and mosses

Environmental Pollution and Degradation

1. **Assertion (A):** Coal - based thermal power stations contribute to acid-rain.

Reason (R): Oxides of carbon are emitted when coal burns.
- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

2. Which of the following statements correctly describe Black Carbon?
1. It is a greenhouse gas with a long atmospheric lifetime.
2. It is produced by incomplete combustion of fossil fuels and biomass.
3. It contributes to glacial melt in the Himalayas.
4. It is the same as soot particles.

Select the correct code:
- (a) 1 and 2 only
- (b) 2, 3 and 4 only
- (c) 1, 2, and 3 only
- (d) 1, 3, and 4 only

3. With reference to Blue Carbon, consider the following statements:
1. It refers to the carbon stored in coastal and marine ecosystems.
2. Mangroves, seagrasses, and salt marshes are important blue carbon sinks.
3. Oceans store more carbon than terrestrial forests.

Which of the above statements is/are correct?
- (a) 1 only
- (b) 1 and 2 only
- (c) 2 and 3 only
- (d) 1, 2 and 3

4. Match the Type of Carbon with its Description.

Type of Carbon	Description
A. Green Carbon	1. Carbon stored in plants and terrestrial ecosystems
B. Brown Carbon	2. Organic carbon released from biomass burning
C. Grey Carbon	3. Carbon emissions from industrial processes
D. Black Carbon	4. Strong light-absorbing component of PM2.5

Select the correct code:
- (a) A-1, B-2, C-3, D-4
- (b) A-1, B-4, C-2, D-3
- (c) A-2, B-1, C-3, D-4
- (d) A-3, B-1, C-4, D-2

5. Which of the following are some important pollutants released by the steel industry in India?
1. Oxides of sulphur 2. Oxides of nitrogen
3. Carbon monoxide 4. Carbon dioxide

Select the correct answer using the code given below.
- (a) 1, 3 and 4 only
- (b) 2 and 3 only
- (c) 1 and 4 only
- (d) 1,2, 3 and 4

6. With reference to furnace oil, consider the following statements:
1. It is a product of oil refineries.
2. Some industries use it to generate power.
3. Its use causes sulphur emissions into environment.

Which of the statements given above are correct?
- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

7. Which of the following are the reason/factors for exposure to benzene pollution?
1. Automobile exhaust
2. Tobacco smoke
3. Wood burning
4. Using varnished wooden furniture
5. Using products made of polyurethane

Select the correct answer using the code given below:
- (a) 1, 2 and 3 only
- (b) 2 and 4 only
- (c) 1, 3 and 4 only
- (d) 1, 2, 3, 4 and 5

8. Which of the following types of hydrogen is considered the cleanest and most sustainable?
- (a) Grey Hydrogen
- (b) Blue Hydrogen
- (c) Green Hydrogen
- (d) Turquoise Hydrogen

ANSWER KEY

Ecology and Environment

1.	(a)	2.	(a)	3.	(a)
4.	(a)	5.	(a)	6.	(a)
7.	(a)	8.	(c)	9.	(a)
10.	(c)	11.	(b)	12.	(d)
13.	(b)	14.	(b)	15.	(c)
16.	(b)	17.	(c)	18.	(c)
19.	(a)	20.	(b)	21.	(a)
22.	(a)	23.	(c)	24.	(c)
25.	(a)	26.	(b)	27.	(a)
28.	(c)	29.	(c)	30.	(b)

Environmental Pollution and Degradation

1.	(b)	2.	(b)	3.	(d)
4.	(a)	5.	(d)	6.	(d)
7.	(d)	8.	(c)	9.	(d)
10.	(c)	11.	(a)	12.	(c)
13.	(b)	14.	(d)	15.	(a)
16.	(b)	17.	(d)	18.	(d)
19.	(d)	20.	(a)	21.	(a)
22.	(a)	23.	(a)	24.	(a)
25.	(a)	26.	(a)	27.	(a)
28.	(a)	29.	(a)		

Environmental Issues

1.	(c)	2.	(a)	3.	(a)
4.	(a)	5.	(d)	6.	(a)
7.	(a)	8.	(a)	9.	(c)
10.	(d)	11.	(d)	12.	(a)
13.	(d)	14.	(a)	15.	(a)
16.	(d)	17.	(a)	18.	(a)
19.	(a)	20.	(c)		

Climate Change

1.	(a)	2.	(c)	3.	(a)
4.	(c)	5.	(a)	6.	(c)
7.	(a)	8.	(a)	9.	(a)
10.	(a)	11.	(a)	12.	(a)
13.	(d)	14.	(c)	15.	(a)
16.	(d)	17.	(a)	18.	(d)

7.	(a)	8.	(c)	9.	(a)
10.	(a)	11.	(a)	12.	(a)
13.	(d)	14.	(a)	15.	(a)
16.	(b)	17.	(c)	18.	(b)
19.	(a)	20.	(b)	21.	(a)
22.	(d)	23.	(b)	24.	(c)
25.	(b)	26.	(b)	27.	(b)
28.	(a)	29.	(b)	30.	(a)
31.	(a)	32.	(a)	33.	(a)
34.	(a)	35.	(a)	36.	(a)
37.	(c)	38.	(a)	39.	(a)

SOLUTIONS

Ecology and Environment

1. **(a)**

A. **Pyramid of Numbers – 3:**

This can be upright (grasslands) or inverted (Parasitic food chain, where one tree supports many parasites).

B. **Pyramid of Biomass – 2:**

In forests, biomass decreases at higher trophic levels (can be inverted). In aquatic ecosystems, producers (phytoplankton) have less biomass than consumers – so inverted.

C. **Pyramid of Energy – 1:**

Always upright because energy is lost at each trophic level (10% law).

D. **Pyramid of Numbers (Grassland) – 4:**

A classic example of upright pyramid: many grasses → fewer herbivores (e.g., grasshoppers) → even fewer carnivores.

2. **(a)**

A. **Forest ecosystem – 2 (Upright):** Producers (trees) have large biomass, decreasing toward top carnivores.

B. **Aquatic ecosystem – 1 (Inverted):** Tiny phytoplankton (low biomass) support larger biomass of zooplankton.

C. **Grassland ecosystem – 3 (Generally upright):** Herbaceous plants have more biomass than herbivores/carnivores.

D. **Oceanic food chain – 4:** Due to fast turnover of phytoplankton, biomass of consumers may exceed producers.

3. **(a)**

A. **Producers – 1:** Form the base of all ecological pyramids by converting solar energy into biomass.

B. **Primary Consumers – 2:** Herbivores feeding directly on producers.

C. **Secondary Consumers – 3:** Feed on primary consumers (e.g., frogs, small carnivores).

D. **Decomposers – 4:** Operate at all levels, breaking down organic matter and recycling nutrients.

4. **(a)**

A. **Primary Succession – 1:** Begins on barren areas (e.g., lava rocks, sand dunes) with no initial biotic community.

B. **Secondary Succession – 2:** Occurs where a biological community has been removed due to disturbance (e.g., forest fire, agriculture).

C. **Autogenic Succession – 3:** Driven by biotic components of the system (e.g., shade-tolerant plants replacing pioneers).

D. **Allogenic Succession – 4:** Caused by abiotic external forces (e.g., flood, drought, volcanic eruption).

5. **(a)**

A. **Nudation – 1:** First stage where a bare area is formed due to volcanic eruption, landslide, retreat of glaciers, etc.

B. **Invasion – 2:** Dispersal and colonization by pioneer species such as lichens, algae, or grasses.

C. **Competition & Coaction – 3:** Organisms compete for light, nutrients, space, influencing further succession.

D. **Stabilization (Climax) – 4:** A stable, self-sustaining climax community is formed (e.g., deciduous forest in humid regions).