SHAURYA

NDA/NA

National Defence Academy & Naval Academy

MATHEMATICS

- Precise Chapterwise Theory
- Solved Examples
- Practice Exercises with Solutions
- Handpicked PYQs Covered Till Date
- Chapter-Wise Weightage Analysis of Past 5 Years Papers

As Per Latest UPSC Exam Pattern

NDA Past 5 Year Paper Analysis

MATHEMATICS

		2021-I	2021-II	2022-I	2022-II	2023-I	2023-II	2024-I	2024-II	2025-I	2025-II
1.	Basic Maths	0	0	1	0	3	0	0	5	0	0
2.	Logarithm	1	1	0	1	0	0	3	2	0	2
3.	Sets	1	1	4	8	0	1	1	3	0	3
4.	Quadratic Equations	4	4	4	2	5	4	3	7	2	6
5.	Trigonometric Functions	13	13	10	9	6	8	9	7	11	9
6.	Properties of Triangle	2	2	1	4	6	2	3	3	5	0
7.	Height and Distance	2	2	2	3	2	0	0	0	3	0
8.	Binomial Theorem	3	3	2	3	3	4	4	1	3	2
9.	Sequence and Series	6	6	5	3	5	6	2	5	4	6
10.	Permutations and Combinations	4	4	8	7	9	2	6	4	6	5
11.	Complex Number	5	5	2	3	4	6	5	6	3	3
12.	Basics of Coordinate Geometry	1	1	1	0	0	0	1	4	2	1
13.	Straight Lines	5	5	6	2	2	5	6	1	3	6
14.	Circles	1	1	1	2	0	2	3	1	2	1
15.	Parabola and Ellipse	1	1	2	3	2	5	2	2	2	1
16.	Hyperbola and Rectangular Hyperbola	1	1	0	0	0	1	0	0	1	1
17.	Relation and Function	5	5	5	8	6	11	7	9	7	6
18.	Inverse Trigonometric Functions	2	2	2	1	0	3	2	2	3	2
19.	Matrices	2	2	5	3	3	4	2	6	5	3
20.	Determinants	8	8	5	6	9	7	8	1	6	6
21.	Methods of Differentiation	3	3	4	5	5	2	1	0	4	4
22.	Limits	2	2	2	4	4	5	1	1	5	3
23.	Continuity and Differentiability	3	3	2	0	2	4	2	4	2	2
24.	Application of Derivatives	5	5	4	4	3	3	4	4	4	6
25.	Indefinite Integration	3	3	2	3	0	0	4	3	1	3
26.	Definite Integration	2	2	2	3	5	3	7	4	2	2
27.	Area Under the Curve	1	1	2	3	3	1	1	2	3	2
28.	Differential Equations	5	5	4	2	3	2	4	2	0	3
29.	Vector Algebra	8	8	5	6	6	5	5	5	5	7
30.	3-D Geometry	2	2	5	3	4	5	5	6	5	4
31.	Probability	5	5	7	12	8	14	12	14	11	16
32.	Statistics	14	14	13	7	12	5	7	6	9	4
33.	Binary Numbers	0	0	2	0	0	0	0	0	1	1

CONTENT

CHAPTER

1

Basic Maths

NUMBERS

A number is an arithmetic value used for representing a quantity and it is used in making mathematical calculations.

Any number can be formed with the help of 10 digits.

i.e., 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

BASE OF A NUMBER SYSTEM

- The number of different types of digits involved in a number system is called the base of that number system.
- If a number system involves only two digits 0 and 1, then it base is 2.
- A number system in which 10 digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are used, is a system with base 10.
- If a number system involves 8 digits then it is called octal number system.

TYPES OF NUMBERS

NATURAL NUMBERS:

- Counting numbers (also called natural numbers): The set of numbers beginning 1, 2, 3, 4, ... and going on infinitely.
- Set of natural number is denoted by $N = \{1, 2, 3, 4, ...\}$.
- The sum of two natural numbers is always a natural number.

  For Example: 5 + 3 = 8 which is also a natural number.
- The product of two natural numbers is also a natural number.

  For Example: 2 × 3 = 6 which is also a natural number.
- But when a natural number is subtracted from another natural number, the result is not always a natural number. This implies that natural numbers are not closed under subtraction.

  For Example: 3 – 5 = –2 and 3 – 3 = 0.

WHOLE NUMBERS

- This group has all of the Natural Numbers in it with the number 0.
- Whole number is denoted by W.
- Set of whole number is: $W = \{0, 1, 2, 3.......\}$.

Closure Property of Whole Numbers

- **Addition of whole numbers** always gives a whole number. Thus whole numbers are closed under addition. **For Example:** 3 + 8 = 11
- **Subtraction of whole numbers** does not always give a whole number. Thus whole numbers are not closed under subtraction.

  For Example: 5 – 7 = –2 .... , which is a negative integer and not a whole number.

  For Example: 9 – 6 = 3 which is whole number.
- **Multiplication of whole numbers** always gives a whole number. Thus whole numbers are closed under multiplication.

  For Example: 5 × 8 = 40
- **Dividing a whole number** by another does not always give a whole number. Thus whole numbers are not closed under division.

  For Example: $4 \div 7 = \frac{4}{7}$, which is a fraction, thus it will not be a whole number.

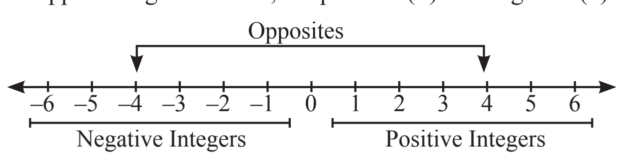
INTEGERS

- Integer is set of counting numbers, zero, and negative of counting numbers.
- Set of integers is denoted by Z.
- Set of integers is: $Z = \{0, \pm 1, \pm 2, \pm 3, \pm 4.......\}$

Integers are of Two Types

- **Negative integers**
  - Negative integers are the set of integers before 0. They do not have any fractional or decimal part.

    For Example: –1, –2, –3....
- **Positive integers/Whole numbers**
  - Positive integers/whole numbers are the set of natural numbers including zero. They do not have any fractional or decimal part.
  - To distinguish between the positive and negative sides of 0, opposite signs are used, i.e. positive (+) and negative (–).



Additive Inverse

- An additive inverse of a number is defined as the value, which on adding with the original number results in zero value.

  For Example: 3 + (–3) = 0. So 3 is the additive inverse of –3 and vice versa.
- Each integer on the number line, except 0, consists of its mirror image of the opposite sign.

For example: Mirror image of 6 will be –6, that of –1 will be 1 etc.

What is Zero?

Zero is the only whole number which is not a natural number. It is represented by 0.

- **Properties of Zero (0)**
  - Zero added to any number gives the number itself. Hence, 0 is the additive identity for all numbers.
  - Multiply any number by 0 is always be 0.
  - Zero divided by any number (except 0) gives 0.
  - Any number divided by 0 is not defined.
  - Zero to the power 0, (0^0) is not defined

NATURAL NUMBERS CAN BE DIVIDED INTO DIFFERENT TYPES

- Even and odd Numbers
- Prime Numbers and Composite Numbers
- Relatively Prime/co-prime numbers

EVEN NUMBERS

- An even number is any number that is exactly divisible by 2.

  For example: 12 is exactly divisible by 2, so 12 is even.
- Any number whose last digit is 0,2,4,6 or 8 is an even number.
- Other examples of even numbers are 58, 44884, 998632, 98, 48, and 10000000.
- A number n is even if there exist a number k, such that n = 2 where k is an integer.

ODD NUMBERS

- Odd number is a number, when divided by 2 does not yield an integer.

  For example: 25 cannot be divided by 2, so 25 is odd.
- Any number whose last digit is not 0, 2, 4, 6, or 8 is an odd number.
- Other examples of odd numbers are 53, 881, 238637, 99, 45, and 100000023.
- A number n is odd if there exist a number k, such that $n = 2k + 1k$ where k is an integer.
- This is a formal way of saying that if an odd number n is divided by 2, we always get a quotient k with a remainder of 1.

  Having a remainder of 1 means that n cannot be exactly divisible by 2.

BASICOPERATIONSWITHEVENANDODDNUMBERS

- **Addition:**
  - Even + even = even

    For example: 2 + 4 = 6, Which is also even.
  - Even + odd = odd

    For example: 2 + 1 = 3, which is odd
  - Odd + odd = even : For example: 5 + 3 = 8
- **Subtraction:**
  - Even – even = even

    For example: 8 – 4 = 4
  - Even – odd = odd = odd – even

    For example: 6 – 3 = 3
  - Odd – odd = even

    For example: 9 – 5 = 4
- **Multiplication**
  - Even × even = even

    For example: 2 × 4 = 8
  - Even × odd = even

    For example: 4 × 3 = 12
  - Odd × odd = odd

    For example: 5 × 3 = 15

PRIME NUMBERS

- A Prime Number is a positive number that is divided evenly only by 1 and itself.

  And it must be a whole number greater than 1.

  For example: If we list the factors of 28, we have1, 2, 4, 7, 14, and 28. That's six factors. If we list the factors of 29, we only have 1 and 29. That's two factors. So we say that 29 is a prime number, but 28 isn't.
- Note that the definition of a prime number doesn't allow 1 to be a prime number: 1 only has one factor, namely 1. Prime numbers have exactly two factors, not "at most two" or anything like that. When a number has more than two factors it is called a composite number.

Some Important Points

- The only even prime number is 2.
- If the sum of a number's digits is a multiple of 3, that number can be divided by 3.
- No prime number greater than 5 ends in a 5. Any number greater than 5 that ends in a 5 can be divided by 5.
- Zero and 1 are not considered prime numbers.

COMPOSITE NUMBERS

Any positive integers than has at least one factor other than 1 and itself.

For example: 4,10,26 are composite numbers as, 4 has three factors 1,2,4; 10 has four factors 1,2,5,10 and 26 has four factors, 1,2,13,26.

Note: 0 & 1 are neither prime nor composite.

CO-PRIME NUMBERS

- A set of numbers which do not have any other common factor other than 1 are called co-prime or relatively prime numbers.
- This means those numbers whose HCF is 1.

  For example: 8 and 9 have no other common factor other than 1 so they are coprime numbers.

Example 1: 21 and 22

The factors of 21 and 1, 3, 7 and 21

The factors 22 are 1, 2, 11 and 22

Here 21 and 22 have only one common factor that is 1. Hence, they are co-prime.

Example 2: 21 and 27

21 and 27:

The factors of 21 are 1, 3, 7 and 21.

The factors of 27 are 1, 3, 9 and 27.

Here, 21 and 27 have two common factors 1 and 3, hence their HCF is 3 and they are not co-prime.

RATIONAL NUMBERS

- Any number which can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is known as rational numbers.

  For Example: $\frac{2}{3}, -\frac{3}{4}, -\frac{4}{7}, 0, 4, -2$
- The term rational is derived from the word 'ratio' because the rational numbers are figures which can be written in the ratio form.
- All the fractions are rational numbers as they are already in the form $\frac{p}{q}$, where p & q are integers and $q \neq 0$.
- All integers are rational number as then can be written in form were p = the integer and $q = 1$.
- Rational number may or may not be integers.

NUMERATOR AND DENOMINATOR

- If $\frac{a}{b}$ is a rational number, then the integer a is known as its numerator and the integer b is called its denominator.

  For Example: $\frac{2}{3}$ here 2 is numerator and 3 is denominator.

ZERO IS A RATIONAL NUMBER

- Since, we can write 0 in anyone of the forms $\frac{0}{1}, \frac{0}{-1}, \frac{0}{2}, \frac{0}{-2}$ and so on. Thus 0, can be expressed as $\frac{p}{q}$, where $p = 0$ and q is any non-zero integer. Hence 0 is a rational number.

  Positive Rational Number:
  - A rational number is said to be positive if its numerator and denominator are either both positive or both negative.

    For Example: $\frac{5}{7}, \frac{-6}{-5}$ etc.

    Negative Rational Number: Every negative integer is a negative rational number.

    For Example: –1, –2, –3 and so on, which may be expressed as $-1 = \frac{-1}{2}, -2 = \frac{-2}{1}$, ... are all negative rational numbers.

    The rational number 0 is neither positive nor negative.

IRRATIONAL NUMBER

- An irrational number is any number that is not rational. It is a number that cannot be written as a ratio of two integers (or cannot be expressed as a fraction).
- An irrational number has endless non-repeating digits to the right of the decimal point.

  For Example: $\sqrt{2}, \pi, 3.6843...., \sqrt{14}$

SOME USEFUL LAWS OF EXPONENTS

1. $x^m \times x^n = x^{m+n}$

   For example: $2^2 \times 2^3 = 2^{2+3} = 2^5$
2. $x^m \div x^n = x^{m-n}$

   For example: $2^2 \div 2^3 = 2^{2-3} = 2^{-1} = 1/2$
3. $(x^m)^n = x^{mn}$

   For example: $(2^2)^3 = 2^6$
4. $x^n \times y^n = (x \times y)^n$

   For example: $2^2 \times 3^2 = (2 \times 3)^2 = (6)^2 = 36$
5. $x^n \div y^n = \left(\frac{x}{y}\right)^n$

   For example: $2^2 \div 3^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$
6. $x^0 = 1$

   For example: $2^0 = 1$

ALGEBRAIC EXPRESSIONS

- An algebraic expression (or) a variable expression is a combination of terms which are combined with the operations such as addition, subtraction, multiplication or division.
- **Terms**
  - Several parts of an algebraic expression separated by + or – signs are called the terms of the expression

POLYNOMIALS

- A polynomial is an algebraic expression consisting of one or more terms, in each of which exponent of the variable is zero or a positive integer.

DEGREE OF A POLYNOMIAL

- Degree of a polynomial is the highest power of the variable in the polynomial.

TYPES OF POLYNOMIALS

- **Linear Polynomial:**
  - Polynomial having degree one is called linear polynomial and its general form is $ax + b$.

    Examples: $x, 3x, 4x + 1, x + z$ etc.
- **Quadratic Polynomial:**
  - Polynomial having degree two is called quadratic polynomial and its general form is $ax^2 + bx + c$.

    Examples: $x^2, x^2 - 3, xy + 1, y^2 + y + 1$ etc.
- **Cubic Polynomial:**
  - Polynomial having degree three is called cubic polynomial and its general form is $ax^3 + bx^2 + cx + d$.

    Examples: $x^3, y^3, x^3 + 2, x^3 + z^2, x^3 + x^2 + x + 1$ etc.
- **Monomial:**
  - A polynomial consisting of a single term is called a monomial.

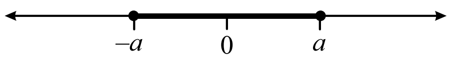
Note: $|f(x)| + |g(x)| = |f(x) + g(x)|$

$\Rightarrow f(x) \cdot g(x) \geq 0$

INEQUALITIES INVOLVING ABSOLUTE VALUE

(*i*) $|x| \leq a$ (where $a > 0$)

It implies those values of x on real number line which are at distance a or less than a from zero.



$\Rightarrow -a \leq x \leq a$

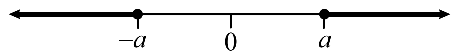
e.g. $|x| \leq 2 \Rightarrow -2 \leq x \leq 2$

$|x| < 3 \Rightarrow -3 < x < 3$

In general, $|f(x)| \leq a$ (where $a > 0$) $\Rightarrow -a \leq f(x) \leq a$.

(*ii*) $|x| \geq a$ (where $a > 0$)

It implies those values of x on real number line which are at distance a or more than a from zero



$\Rightarrow x \leq -a$ or $x \geq a$

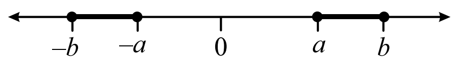
e.g. $|x| \geq 3 \Rightarrow x \leq -3$ or $x \geq 3$

$|x| > 2 \Rightarrow x < -2$ or $x > 2$

In general, $|f(x)| \geq a \Rightarrow f(x) \leq -a$ or $f(x) \geq a$

(*iii*) $a \leq |x| \leq b$ (where $a, b > 0$)

It implies those value of x on real number line whose distance from zero is equal to a or b or lies between a and b



$\Rightarrow [-b, -a] \cup [a, b]$

e.g. $2 \leq |x| \leq 4 \Rightarrow x \in [-4, -2] \cup [2, 4]$

(*iv*) If $|x + y| = |x| + |y|$, $xy \geq 0$

If $|x - y| = |x| + |y|$, $xy \leq 0$

If $|x + y| = ||x| - |y||$, $xy \leq 0$

If $|x - y| = ||x| - |y||$, $xy \geq 0$

SOLVED EXAMPLES

1. $\left(\frac{2}{3}\right)^4 \div \left(\frac{2}{3}\right)^4$ is equal to

(*a*) 0 (*b*) 1 (*c*) –1 (*d*) $\frac{2}{3}$

Ans. (*b*) $\left(\frac{2}{3}\right)^4 \div \left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^{4-4} = \left(\frac{2}{3}\right)^0 = 1$

2. $\left(-\frac{4}{5}\right)^3$ is equal to

(*a*) $\frac{125}{64}$ (*b*) $\frac{64}{125}$ (*c*) $\frac{-64}{125}$ (*d*) $\frac{-125}{64}$

Ans. (*c*) $\left(\frac{-4}{5}\right)^3 = \left(\frac{-4}{5}\right) \times \left(\frac{-4}{5}\right) \times \left(\frac{-4}{5}\right) = \frac{(-4)^3}{5^3} = \frac{-64}{125}$

3. Express $\left(\frac{-2}{3}\right)^3$ as a rational number:

(*a*) $\frac{4}{27}$ (*b*) $\frac{8}{27}$ (*c*) $\frac{-8}{27}$ (*d*) $\frac{-8}{9}$

Ans. (*c*) $\left(\frac{-2}{3}\right)^3 = \frac{(-2)^3}{(3)^3} = \frac{(-2)\times(-2)\times(-2)}{3\times3\times3} = \frac{-8}{27}$

4. Express $\left(\frac{-3}{4}\right)^{-3}$ as a rational number:

(*a*) $\frac{27}{64}$ (*b*) $\frac{-27}{64}$ (*c*) $\frac{-64}{27}$ (*d*) $\frac{64}{27}$

Ans. (*c*) $\left(\frac{-3}{4}\right)^{-3} = \left(-\frac{4}{3}\right)^3 = -\frac{(4)^3}{(3)^3} = -\frac{4\times4\times4}{3\times3\times3} = -\frac{64}{27}$

5. Express $\frac{27}{125}$ as a power of rational number:

(*a*) $\left(\frac{3}{5}\right)^3$ (*b*) $\left(\frac{3}{5}\right)^{-3}$ (*c*) $\left(\frac{5}{3}\right)^3$ (*d*) $\left(-\frac{5}{3}\right)^3$

Ans. (*a*) $\frac{27}{125} = \frac{3\times3\times3}{5\times5\times5} = \frac{3^3}{5^3} = \left(\frac{3}{5}\right)^3$

6. Value of $\left(\frac{-2}{3}\right)^3 \times \left(\frac{-3}{2}\right)^2$ is equal to

(*a*) $\frac{-3}{2}$ (*b*) $\frac{2}{3}$ (*c*) $\frac{3}{2}$ (*d*) $\frac{-2}{3}$

Ans. (*d*) $\left(\frac{-2}{3}\right)^3 \times \left(\frac{-3}{2}\right)^2 = \frac{(-2)^3}{(3)^3} \times \frac{(-3)^2}{(2)^2} = -\frac{2}{3}$

7. Value of $\left[\left(-\frac{2}{3}\right)^2\right]^3$ is equal to

(*a*) $\frac{-64}{729}$ (*b*) $\frac{64}{729}$ (*c*) $-\frac{35}{243}$ (*d*) $\frac{35}{243}$

Ans. (*b*) $\left[\left(-\frac{2}{3}\right)^2\right]^3 = \left(-\frac{2}{3}\right)^6 = \frac{(-2)^6}{(3)^6} = \frac{64}{729}$

EXERCISE

1. In $10 \times 20 \times 30 \times ...... \times 1000$, how many trailing zeros are present?
   (a) 100 (b) 124
   (c) 120 (d) 150

2. In a class of 100 students, 50 students passed in Mathematics and 70 passed in English, 5 students failed in both Mathematics and English. How many students passed in both the subjects?
   (a) 50 (b) 40 (c) 35 (d) 25

3. What are the values of x and y in the following pair of equations?
   $x^2y - 2y = 42$
   $x + 2 = 5$
   (a) $x = -2, y = -6$ (b) $x = 2, y = 6$
   (c) $x = -3, y = -6$ (d) $x = 3, y = 6$

4. The smallest integer to be added to 9547×9545 to make the sum a perfect square number, is
   (a) 0 (b) 1 (c) 2 (d) 3

5. Find the square root of the perfect square made by multiplying 4050 with a least positive integer.
   (a) 80 (b) 90 (c) 85 (d) 95

6. The multiplicative inverse of additive inverse of $\frac{2}{x}$ is
   (a) $\frac{-2}{x}$ (b) $\frac{2}{x}$ (c) $\frac{-x}{2}$ (d) $\frac{x}{2}$

7. Rohit multiplies a number by 2 instead of dividing the number by 2. Resultant number is what percentage of the correct value?
   (a) 200% (b) 300% (c) 50% (d) 400%

8. One fruit salad recipe requires 1/2 cup of sugar. Another recipe for the same fruit salad requires 2 tablespoons of sugar. If 1 tablespoon is equivalent to 1/16 cup, how much more sugar does the first recipe require?
   (a) 1/8 cup (b) 5/16 cup
   (c) 3/8 cup (d) 7/16 cup

9. The sum of two numbers is 5 times their difference. If the smaller number is 24, find the larger number.
   (a) 30 (b) 32 (c) 36 (d) 48

10. In a town, 55% people watch news on television, 40% read a newspaper, 25% neither read a newspaper nor watch news on television and 4000 people do both, read a newspaper and watch news on television. Find the population of the town.
    (a) 22,000 (b) 20,000
    (c) 150,000 (d) 18,000

11. The sum of three consecutive multiples of 5 is 285. Find the largest number.
    (a) 75 (b) 100 (c) 120 (d) 90

12. In a group 40% people like only milk, 20% like only tea and 20% like only coffee, if 2% like all the three drinks and 8% like both milk and coffee, 7% like both coffee and tea and 9% like both tea and milk, then how many percent of people like only two drinks?
    (a) 22% (b) 7% (c) 18% (d) 15%

13. If $a = 4011$ and $b = 3989$ then value of ab.
    (a) 15999879 (b) 15899879
    (c) 15989979 (d) 15998879

14. The price of 12 kg of sugar is equal to that of 6 kg of rice. The price of 10 kg of sugar and 8 kg of rice is ₹ 1040. Find the price of 1 kg of sugar.
    (a) ₹ 80 (b) ₹ 40 (c) ₹ 70 (d) ₹ 60

15. The number 494 has
    (a) two prime factors (b) three prime factors
    (c) four prime factors (d) no prime factors

16. The greatest four digit number which is exactly divisible by 12,15, 20 and 54 is:
    (a) 9990 (b) 9346
    (c) 9828 (d) 9720

17. In a school of 350 students, 50% student passed in science, 30% students passed in social and studies, 20% student passed in both the subjects. What is the number of students who passed in at least one subject?
    (a) 250 (b) 210 (c) 190 (d) 300

18. The scale of the map is 20 km for 1 cm. What area of the country will be represented on the map whose area is 60,000 km^2 ?
    (a) 150 cm^2 (b) 100cm^2
    (c) 400 cm^2 (d) None of the above

19. In an examination, 40% of students failed in English, 20% failed in math. If 10% of the students failed in both these subjects, then what percentage of students passed in both the subject?
    (a) 50% (b) 40%
    (c) 30% (d) None of these

20. If $1^2 + 2^2 + 3^2 + ... + 14^2 = 1015$, then $3^2 + 6^2 + 9^2 + ... + 42^2$ is equal to
    (a) 9135 (b) 9325
    (c) 9315 (d) 9235

21. What is the average of first 150 natural numbers?
    (a) 70 (b) 70.5 (c) 75 (d) 75.5

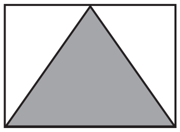
22. What is the average of the numbers: 0, 0, 4, 10, 5, and 5?
    (a) 2 (b) 3 (c) 4 (d) 5

23. What is the rate of discount if a car whose price was 4,000 was sold for 3,200?
    (a) 14% (b) 16% (c) 18% (d) 20%

24. $|-4| + |4| - 4 + 4 = ?$
    (a) 0 (b) 2 (c) 4 (d) 8

25. What is the value of x in the equation $3x - 15 - 6 = 0$?
    (a) 7 (b) 8 (c) 9 (d) –9

26. What is the area in cm^2 of the shaded region in the diagram below?



(*a*) 6 (*b*) 7
(*c*) 8 (*d*) 9

27. If *A* completes a particular work in 8 days and *B* completes the same work in 24 days. How many days will it take if they work together?

(*a*) 4 (*b*) 5
(*c*) 6 (*d*) 7

28. What comes next in the sequence: 1,3,11,43,______?

(*a*) 161 (*b*) 171
(*c*) 181 (*d*) 191

29. What is the distance traveled by a car which traveled at a speed of 80 km / hr for 3 hours and 30 minutes?

(*a*) 275 km (*b*) 280 km
(*c*) 285 km (*d*) 290 km

30. In a class of 40 students 20% are girls. How many boys are there in the class?

(*a*) 26 (*b*) 28
(*c*) 30 (*d*) 32

31. $2 + 2 - 2 \times 2 \div 2 = ?$

(*a*) 0 (*b*) 1
(*c*) 2 (*d*) 4

32. $|2| + |-2| + (2)^2 + (-2)^2 = ?$

(*a*) 6 (*b*) 8
(*c*) 10 (*d*) 12

33. If $x = -1$, then what is the value of the function $f(x) = x^3 + 4x + 12$

(*a*) 7 (*b*) 9
(*c*) 11 (*d*) 13

34. If $\frac{x}{2} + 4 = \frac{7}{2}$ then $x = ?$

(*a*) –2 (*b*) –1
(*c*) 1 (*d*) 2

35. What comes next in the sequence: 2, 4, 10, 28, __?

(*a*) 64 (*b*) 70
(*c*) 76 (*d*) 82

36. Find the value of *x* for which $\frac{(x+1)^4(x-2)(x-3)^3(x-4)^2}{x^3-36x} \geq 0.$

(*a*) $(-6, 0) \cup [2, 3] \cup (6, \infty) \cup \{4\}$
(*b*) $[-6, 0] \cup [2, 3) \cup [6, \infty) \cup \{4\}$
(*c*) $(-6, 0) \cup [2, 3) \cup (6, \infty) \cup \{4\}$
(*d*) $[-6, 0) \cup (2,3] \cup (6, \infty) \cup \{4\}$

37. Find the number of integer values of variable *x* satisfying the following pair of inequalities.

$$\frac{(x-1)(x+4)}{x-3} < 0 \;\&\; x^2 + 6x - 27 \leq 0$$

(*a*) 5 (*b*) 6
(*c*) 0 (*d*) 4

38. The solution of the inequality $2x - 1 \leq x^2 + 3 \leq x - 1$ is

(*a*) $x \in R$ (*b*) $(-2, 2]$
(*c*) $(-2, 2)$ (*d*) $x \in \phi$

39. The real solutions of the equation where $|x|^2 - 3|x| + 2 = 0$ where $x_1 < x_2 < x_3 < x_4$ then

(*a*) $|x_1| = |x_3|$ (*b*) $|x_1| = |x_3|$
(*c*) $x_1 + x_4 = x_2 + x_3$ (*d*) $-x_2 + x_4 = x_1 - x_3$

40. Find the value of *x* for which $||x - 2| - 1| = 2$.

(*a*) {3, 1} (*b*) {5, –1}
(*c*) { –1, 5, 1} (*d*) { –1, 1, 3, 5}

41. Find the value of *x* for which $|||x - 5| - 4| - 3| = 2$.

(*a*) {14, –4} (*b*) {0, 2}
(*c*) {8, 10} (*d*) All of these

42. Solve $||x-1|-2| < 5$

(*a*) (–2, 4) (*b*) (–2, 8)
(*c*) (–6, 4) (*d*) (–6, 8)

43. Solve $|x^2 - 2x| + |x - 4| > |x^2 - 3x + 4|$.

(*a*) $(0,2) \cup (4, \infty)$ (*b*) $(-2, 4) \cup [7, \infty)$
(*c*) $(-\infty, 0) \cup (-2, 4]$ (*d*) $(0, 2] \cup (-4, \infty)$

ANSWER KEY

EXERCISE

1. (*b*)	**2.** (*d*)	**3.** (*d*)	**4.** (*b*)	**5.** (*b*)	**6.** (*c*)	**7.** (*d*)	**8.** (*c*)	**9.** (*c*)	**10.** (*b*)
11. (*b*)	**12.** (*c*)	**13.** (*a*)	**14.** (*b*)	**15.** (*b*)	**16.** (*d*)	**17.** (*b*)	**18.** (*a*)	**19.** (*a*)	**20.** (*a*)
21. (*d*)	**22.** (*c*)	**23.** (*d*)	**24.** (*d*)	**25.** (*a*)	**26.** (*a*)	**27.** (*c*)	**28.** (*b*)	**29.** (*b*)	**30.** (*d*)
31. (*c*)	**32.** (*d*)	**33.** (*a*)	**34.** (*b*)	**35.** (*d*)	**36.** (*a*)	**37.** (*b*)	**38.** (*d*)	**39.** (*c*)	**40.** (*b*)
41. (*d*)	**42.** (*d*)	**43.** (*a*)							

EXPLANATION

EXERCISE

1. (*b*) According to the question given that,

$10 \times 20 \times 30 \times ... \times 1000$

$= (10 \times 1) \times (10 \times 2) \times (10 \times 3) \times (10 \times 4) ..... \times (10 \times 100)$

$= 10^{100} \times (1 \times 2 \times 3 \times ... \times 100) = 10^{100} \times (100!)$

Number of zeroes = 100 + {(100)/ 5 + (20)/5}= 124

∴ The number of trailing zeroes in $10 \times 20 \times 30 \times ... \times 1000$ is 124.

2. (*d*) According to question given that Total number of students (∪) = 100

Number of students who passed in mathematics (*A*) = 50

No. of students received a passing grade in English (*B*) = 70

Number of students who failed in both subjects.

$(\cup - (A \cup B)) = 5$

The difference between the number of students required for no overlap and the total is used to calculate the number of students who passed both subjects. Students who passed in both the subjects

$= 70 + 50 - (100 - 5) = 120 - 95 = 25$

∴ The total number of students who passed both subjects is 25.

3. (*d*) Given, $x + 2 = 5 \Rightarrow x = 3$

and given, $x^2y - 2y = 42$

$\Rightarrow (3)^2y - 2y \Rightarrow 7y = 42 \Rightarrow y = 6$

4. (*b*) We know that, $(a + b)(a - b) = a^2 - b^2$

According to question given product 9547 × 9545 we find that the difference between 9547 – 9545 is 2 .

Then after 9545 and before 9547, there is one integer 9546 .

Then we can write 9547 = 9546 + 1 and 9545 = 9546 – 1.

Then, $9547 \times 9545 = (9546 + 1)(9546 - 1) = 9546^2 - 1^2$.

∴ When 1 is added to the product 9547 × 9545, it is clear that the result will be a perfect square.

5. (*b*) Factorize any number to find its square root.

$4050 = 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5$

When we multiply by 2 in $4050 \Rightarrow 4050 \times 2 = 8100$

Then, $\sqrt{8100} = 90$

∴ Multiply 4050 by two to get 8100, the square root of which is 90.

6. (*c*) Additive inverse of $2/x = -2/x$

$\Rightarrow$ Multiplicative inverse of $-2/x = 1/(-2/x)$

∴ Multiplicative inverse of $-2/x = -x/2$

7. (*d*) According to question given that a number is multiplied by 2 instead of dividing the number by 2. If the number be *x*.

Correct value = $x/2$

Resultant number = $2 \times x$

Required percentage $= \dfrac{\text{resultant number}}{\text{correct value}} \times 100$

$\Rightarrow$ Required percentage $= \left[\dfrac{(2x)}{(x/2)}\right] \times 100\ \% = 400\%$

8. (*c*) According to question given that 1 tablespoon is the same as $\dfrac{1}{16}$ cup.

Sugar required in one fruit salad recipe. = 1/ 2 cup

Another recipe for the same fruit salad requires sugar = 2 tablespoons = 2 × (1/16) = 1/ 8 cup

More sugar required by the first recipe $= \dfrac{1}{2} - \dfrac{1}{8} = \dfrac{3}{8}$ cup

9. (*c*) Given that smaller no. = 24

Let the larger no. be = *a*

Then, $(a + 24) = 5(a - 24) \Rightarrow 4a = 144 \Rightarrow a = 36$

10. (*b*) 55% watch television, 40% read a newspaper,

25% neither read a newspaper nor watch television & 4000 people do both.

Now, we consider the total population is *x*.

According to question given: $n(A \cap B) = 4000$, $n(A) = 0.55\,x$, $n(B) = 0.40\,x$

Out of 100%, 25% neither read a newspaper nor watch television.

$\Rightarrow$ The percentage of people who read a newspaper or watch television.

$\Rightarrow 100 - 25 = 75\%$

$\Rightarrow n(A \cup B) = 0.75x$

$\because\ n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow 0.75x = 0.55x + 0.40x - 4000 \Rightarrow x = 20000$

∴ Total population of a town is 20,000

11. (*b*) According to given question, 285 is the sum of three consecutive multiples of 5.

Then, Three consecutive multiples of 5 are $5x$, $5(x + 1)$ and $5(x + 2)$

285 is the sum of three consecutive multiples of 5.

$\therefore\ 5x + 5x + 5 + 5x + 10 = 285 \Rightarrow x = 18$

Then, largest number $= 5(x + 2) = 5 \times 20 = 100$

2025-II NDA/NA Solved Paper

1. If $p^x = q^y = r^z$, where x, y and z are in GP, then consider the following statements:

   I. p, q and r are in AP.

   II. $ln\ p$, $ln\ q$ and $ln\ r$ are in GP.

   Which of the statements given above is/are correct?

   (*a*) I only (*b*) II only

   (*c*) Both I and II (*d*) Neither I nor II

2. If A and B are non-empty subsets of a set, and A^c and B^c represent their complements, then which of the following is/are correct?

   I. $A - B = B^c - A^c$

   II. $A - B^c = A^c - B$

   Select the answer using the code given below.

   (*a*) I only (*b*) II only

   (*c*) Both I and II (*d*) Neither I nor II

3. Let $y = x!$ and $z = (2x)!$. If $(z/y) = 120$, then what is the value of $(3x)!$?

   (*a*) 362880 (*b*) 181440 (*c*) 90720 (*d*) 45360

4. Let n be a natural number. The number of consecutive zeros at the end of the expansion of $n!$ is exactly 2. How many values of n are possible?

   (*a*) 3 (*b*) 4 (*c*) 5 (*d*) More than 5

5. If $(10 + \log_{10}x)$, $(10 + \log_{10}y)$ and $(10 + \log_{10}z)$ are in AP, then consider the following statements:

   I. The GM of x and z is y^2 .

   II. The AM of $\log_{10}x$ and $\log_{10}z$ is $\log_{10}y$.

   Which of the statements given above is/are correct?

   (*a*) I only (*b*) II only

   (*c*) Both I and II (*d*) Neither I nor II

6. How many terms of the series 1 + 3 + 5 + 7 +... amount to a sum equal to 12345678987654321?

   (*a*) 11111111 (*b*) 110000011

   (*c*) 111101111 (*d*) 111111111

7. How many terms are identical in the two APs 19, 21, 23,... up to 110 terms and 19, 22, 25, 28, ... up to 75 terms?

   (*a*) 35 (*b*) 36

   (*c*) 37 (*d*) 38

8. If $\alpha = \dfrac{-1+\sqrt{-3}}{2}$

   then what is the value of

   $(1 + \alpha^{19} - \alpha^{35})^{100} - (1 - 3\alpha^{25} + \alpha^{38})^{50}$?

   (*a*) – 2 (*b*) –1

   (*c*) 0 (*d*) 2

9. What is the remainder when 5^{99} is divided by 13?

   (*a*) 10 (*b*) 9

   (*c*) 8 (*d*) 6

10. What is the value of the determinant of the inverse of the matrix

    $\begin{bmatrix} -4 & -5 \\ 2 & 2 \end{bmatrix}$?

    (*a*) $\frac{1}{2}$ (*b*) 1 (*c*) 2 (*d*) 4

11. In a class of 45 students, 34 like to play cricket and 26 like to play football. Further, each student likes to play at least one of the two games. How many students like to play exactly one game?

    (*a*) 45 (*b*) 30 (*c*) 25 (*d*) 15

12. The system of equations

    $2x - 3y - 5 = 0$, $15y - 10x + 50 = 0$

    (*a*) has a unique solution

    (*b*) has infinitely many solutions

    (*c*) is inconsistent

    (*d*) is consistent and has exactly two solutions

13. If $\left(\dfrac{1-i}{1+i}\right)^{2m}\left(\dfrac{1+i}{1-i}\right)^{2n} = 1$

    where $i = \sqrt{-1}$, then what is the smallest positive value of $(m - n)$?

    (*a*) 1

    (*b*) 2

    (*c*) 4

    (*d*) 8

14. In obtaining the solution of the system of equations $x + y + z = 7$, $x + 2y + 3z = 16$ and $x + 3y + 4z = 22$ by Cramer's rule, the value of y is obtained by dividing D by D_2, where

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{vmatrix}$$

What is the value of the determinant D_2?

(*a*) –13 (*b*) –3 (*c*) 3 (*d*) 13

15. Consider the following in respect of non-singular matrices A and B:

I. $(AB)^{-1} = A^{-1} B^{-1}$

II. $(BA)(AB)^{-1} = I$, where I is the identity matrix

III. $(AB)^T = A^T B^T$

How many of the above are correct?

(*a*) None (*b*) One (*c*) Two (*d*) All three

16. The value of the determinant

$$\begin{vmatrix} a & b & c \\ l & m & n \\ p & q & r \end{vmatrix}$$

is equal to

(*a*) $\begin{vmatrix} a & b & c \\ p & q & r \\ l & m & n \end{vmatrix}$ (*b*) $\begin{vmatrix} l & m & n \\ a & b & c \\ p & q & r \end{vmatrix}$

(*c*) $\begin{vmatrix} p & q & r \\ a & b & c \\ l & m & n \end{vmatrix}$ (*d*) $\begin{vmatrix} a & p & l \\ b & q & m \\ c & r & n \end{vmatrix}$

17. Let 1, ω, ω^2 be three cube roots of unity. If $x = a + b$, $y = a\omega + b\omega^2$, $z = a\omega^2 + b\omega$, then what is $x^2 + y^2 + z^2$ equal to?

(*a*) $6ab$ (*b*) $3ab$ (*c*) $a^2 + b^2$ (*d*) 1

18. How many 4-digit numbers that are divisible by 4 can be formed using the digits 1, 2, 3 and 4 (repetition of digits is not allowed)?

(*a*) 3 (*b*) 6 (*c*) 9 (*d*) 12

19. If a, b, c are the sides of a triangle ABC and p is the perimeter of the triangle then what is $\begin{vmatrix} p+c & a & b \\ c & p+a & b \\ c & a & p+b \end{vmatrix}$ equal to

(*a*) p^3 (*b*) $2p^3$ (*c*) $3p^3$ (*d*) $4p^3$

20. Which one of the following is the greatest coefficient in the expansion of $(1 + x)^{100}$?

(*a*) The coefficient of x^{100}

(*b*) The coefficient of x^{99}

(*c*) The coefficient of x^{51}

(*d*) The coefficient of x^{50}

For the following **two (02)** items:

Let α and β be the roots of the quadratic equation

$x^2 + (\log_{0.5}(a^2))x + (\log_{0.5}(a^2))^4 = 0$

where $a^2 \neq 1$ and $\log_{0.5}(a^2) > 0$. Further, $\beta^2 = \alpha(\log_a 2(0.5))$.

21. What is β equal to?

(*a*) $\log_a 2(0.5)$ (*b*) $\log_{0.5}(a^2)$

(*c*) $2(\log_a 2(0.5))$ (*d*) $2(\log_{0.5}(a^2))$

22. What is the relation between α and β?

(*a*) $\alpha = 2\beta$ (*b*) $2\alpha = \beta$ (*c*) $\alpha = -2\beta$ (*d*) $2\alpha = -\beta$

For the following **two (02)** items:

Let $p = \sum_{j=1}^{n} \log_{10} 2^j$ and $q = \sum_{j=1}^{n} \log_{10} 5^j$

23. If $p + q = 66$, then which one of the following is correct?

(*a*) $n < 7$ (*b*) $7 < n < 9$

(*c*) $9 < n < 12$ (*d*) $n > 12$

24. If $p + q = 15$, then what is $q - p$ equal to?

(*a*) $\log_{10} 2.5$ (*b*) $5\log_{10} 2.5$

(*c*) $10 \log_{10} 2.5$ (*d*) $15\log_{10} 2.5$

For the following **two (02)** items:

Let $\sin A + \sin B = p$ and $\cos A + \cos B = q$.

25. What is $\frac{p}{q}$ equal to?

(*a*) $\tan\left(\frac{A-B}{2}\right)$ (*b*) $\cot\left(\frac{A-B}{2}\right)$

(*c*) $\tan\left(\frac{A+B}{2}\right)$ (*d*) $\cot\left(\frac{A+B}{2}\right)$

26. What is $\frac{p^2 - q^2}{p^2 + q^2}$ equal to?

(*a*) $\cos(A + B)$ (*b*) $\cos(A - B)$

(*c*) $\cos\left(\frac{\pi}{2} - A - B\right)$ (*d*) $\cos(\pi - A - B)$

For the following **two (02)** items:

Let $p = \operatorname{cosec} 20°$ and $q = \operatorname{cosec} 70°$

27. What is $\left(\frac{\sqrt{3}p}{4} - \frac{q}{4}\right)$ equal to?

(*a*) –1 (*b*) 0

(*c*) 1 (*d*) 2

28. What is $\frac{p^2 + q^2}{p^2 q^2}$ equal to

(*a*) $\frac{1}{2}$ (*b*) 1

(*c*) $\frac{3}{2}$ (*d*) 2

For the following **two (02)** items:

Let $\cos(2x + 3y) = \frac{1}{2}$ and $\cos(3x + 2y) = \frac{\sqrt{3}}{2}$

where $-\pi < (2x + 3y) < \pi$ and $-\pi < (3x + 2y) < \pi$.

29. How many values does $(x + y)$ have?

(*a*) Two (*b*) Three

(*c*) Four (*d*) More than four

30. How many values does $(y - x)$ have?

(*a*) Two (*b*) Three

(*c*) Four (*d*) More than four

For the following **two (02)** items:

Consider the equation

$abx^2 + bcx + ca = cax^2 + abx + bc$

31. If the roots of the equation are equal, then which one of the following is correct?

(*a*) $ac = b^2$ (*b*) $a + c = 2b$

(*c*) $\frac{1}{a}+\frac{1}{c}=\frac{1}{2b}$ (*d*) $\frac{1}{a}+\frac{1}{c}=\frac{2}{b}$

32. If the roots of the equation are equal, then *a*, *b*, *c* are in

(*a*) AP (*b*) GP

(*c*) HP (*d*) None of the above

For the following **two (02)** items:

Let (6 + 10 + 14 +........up to *m* terms)

= (1 + 3 + 5 + 7 +.......up to *n* terms)

where $m < 25$ and $n < 25$.

33. What is the relation between *m* and *n*?

(*a*) $n^2 = m(m+1)$ (*b*) $n^2 = m(m+2)$

(*c*) $n^2 = 2m(m+1)$ (*d*) $n^2 = 2m(m+2)$

34. How many values of *m* are possible?

(*a*) None (*b*) One

(*c*) Two (*d*) More than two

For the following **two (02)** items:

There are 8 points on *a* plane out of which 4 points are collinear.

35. How many triangles can be formed by joining these points?

(*a*) 56 (*b*) 54

(*c*) 53 (*d*) 52

36. How many quadrilaterals can be formed by joining these points?

(*a*) 70 (*b*) 69

(*c*) 53 (*d*) None of the above

For the following **two (02)** items:

Let $f(x) = ax^2 + bx + c$ be *a* quadratic polynomial such that $f(1) = f(4) = 2$. Further, 2 is *a* root of $f(x) = 0$.

37. What is the other root of $f(x) = 0$?

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) Cannot be determined.

38. What is $(a + b + c)$ equal to?

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) Cannot be determined.

For the following **two (02)** items:

Let $A=\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}$

39. What is the value of the determinant of the matrix A^4?

(*a*) 0 (*b*) 1

(*c*) $\cos 4\theta - \sin 4\theta$ (*d*) $\cos^2 4\theta - \sin^2 4\theta$

40. What is $[\text{Adj } A]^{-1}$ equal to?

(*a*) $-A$ (*b*) $-A^T$

(*c*) A (*d*) A^T

41. What is the sum of the binary number $[101101101]_2$ and $[100011]_2$?

(*a*) $(110010000)_2$ (*b*) $(110001000)_2$

(*c*) $(110000100)_2$ (*d*) $(100100000)_2$

42. Set *X* contains $3n$ elements and set *Y* contains $2n$ elements, and they have *n* elements in common. How many elements does $(X - Y) \times (Y - X)$ have?

(*a*) $5n^2$ (*b*) $4n^2$ (*c*) $3n^2$ (*d*) $2n^2$

43. Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$ and $B = \{0, 1, 4, 9\}$ How many elements does the subset of $A \times B$ corresponding to the relation $R = \{(x, y) : |x| < y\}$ have, where $x \in A$ and $y \in B$?

(*a*) 9 (*b*) 12 (*c*) 15 (*d*) 16

44. Consider the following statements:

Statement-I: If *X* is an $n \times n$ matrix, then $\det(mX) = m^n \det(X)$ where *m* is *a* scalar.

Statement-II: If *Y* is *a* matrix obtained from *X* by multiplying any row or column by *a* scalar *m*, then $\det(Y) = m\det(X)$.

Which one of the following is correct in respect of the above statements?

(*a*) Both Statement-I and Statement-II are correct and Statement-II explains Statement-I

(*b*) Both Statement-I and Statement-II are correct but Statement-II does not explain Statement-I

(*c*) Statement-I is correct but Statement-II is not correct

(*d*) Statement-I is not correct but Statement-II is correct

45. Consider the following statements about the matrix

$$M=\begin{bmatrix}71 & 23 & 48 \\ 57 & 28 & 29 \\ 65 & 17 & 48\end{bmatrix}$$

Statement-I: The inverse of *M* does not exist.

Statement-II: *M* is non-singular.

Which one of the following is correct in respect of the above statements?

(*a*) Both Statement-I and Statement-II are correct and Statement-II explains Statement-I

(*b*) Both Statement-I and Statement-II are correct but Statement-II does not explain Statement-I

(*c*) Statement-I is correct but Statement-II is not correct

(*d*) Statement-I is not correct but Statement-II is correct

46. What is $\cot^{-1} 9 + \operatorname{cosec}^{-1}\left(\frac{\sqrt{41}}{4}\right)$

(*a*) $\frac{\pi}{4}$ (*b*) $\frac{\pi}{3}$

(*c*) $\frac{\pi}{2}$ (*d*) π

47. How many values of θ, where $-\pi < \theta < \pi$, satisfy both the equations $\cot\theta = -\sqrt{3}$ and $\operatorname{cosec}\theta = -2$ simultaneously?

(*a*) 4 (*b*) 2 (*c*) 1 (*d*) None

48. If $x+\frac{1}{x}=2\cos\theta$, then what is $x^3+\frac{1}{x^3}$ equal to

(*a*) $\cos^3\theta$ (*b*) $\cos 3\theta$ (*c*) $2\cos 3\theta$ (*d*) $3\cos 3\theta$

ANSWER KEY

1. (*b*)	**2.** (*a*)	**3.** (*a*)	**4.** (*c*)	**5.** (*b*)	**6.** (*d*)	**7.** (*c*)	**8.** (*c*)	**9.** (*c*)	**10.** (*a*)
11. (*b*)	**12.** (*c*)	**13.** (*b*)	**14.** (*b*)	**15.** (*a*)	**16.** (*d*)	**17.** (*a*)	**18.** (*b*)	**19.** (*b*)	**20.** (*d*)
21. (*b*)	**22.** (*c*)	**23.** (*c*)	**24.** (*d*)	**25.** (*c*)	**26.** (*d*)	**27.** (*c*)	**28.** (*b*)	**29.** (*c*)	**30.** (*c*)
31. (*d*)	**32.** (*c*)	**33.** (*d*)	**34.** (*c*)	**35.** (*d*)	**36.** (*c*)	**37.** (*c*)	**38.** (*c*)	**39.** (*b*)	**40.** (*c*)
41. (*a*)	**42.** (*d*)	**43.** (*c*)	**44.** (*a*)	**45.** (*c*)	**46.** (*a*)	**47.** (*c*)	**48.** (*c*)	**49.** (*b*)	**50.** (*b*)
51. (*b*)	**52.** (*a*)	**53.** (*b*)	**54.** (*a*)	**55.** (*b*)	**56.** (*b*)	**57.** (*b*)	**58.** (*d*)	**59.** (*b*)	**60.** (*a*)
61. (*c*)	**62.** (*c*)	**63.** (*d*)	**64.** (*a*)	**65.** (*b*)	**66.** (*d*)	**67.** (*d*)	**68.** (*c*)	**69.** (*a*)	**70.** (*a*)
71. (*a*)	**72.** (*c*)	**73.** (*a*)	**74.** (*d*)	**75.** (*c*)	**76.** (*a*)	**77.** (*d*)	**78.** (*a*)	**79.** (*c*)	**80.** (*c*)
81. (*b*)	**82.** (*b*)	**83.** (*a*)	**84.** (*c*)	**85.** (*d*)	**86.** (*d*)	**87.** (*a*)	**88.** (*d*)	**89.** (*a*)	**90.** (*d*)
91. (*a*)	**92.** (*c*)	**93.** (*c*)	**94.** (*b*)	**95.** (*c*)	**96.** (*a*)	**97.** (*a*)	**98.** (*d*)	**99.** (*c*)	**100.** (*c*)
101. (*a*)	**102.** (*c*)	**103.** (*b*)	**104.** (*a*)	**105.** (*c*)	**106.** (*d*)	**107.** (*a*)	**108.** (*b*)	**109.** (*d*)	**110.** (*a*)
111. (*d*)	**112.** (*d*)	**113.** (*b*)	**114.** (*d*)	**115.** (*a*)	**116.** (*a*)	**117.** (*b*)	**118.** (*d*)	**119.** (*d*)	**120.** (*b*)

EXPLANATION

1. (*b*) We have $p^x = k \Rightarrow x = \log_p k$

$q^y = k \Rightarrow y = \log_q k$

$r^z = k \Rightarrow z = \log_r k$

Given x, y, z are in G.P. $\Rightarrow y^2 = xz$

$(\log_q k)^2 = (\log_p k).(\log_r k)$

$$\Rightarrow \left(\frac{\log k}{\log q}\right)^2 = \frac{\log k}{\log p} \times \frac{\log k}{\log r}$$

$\Rightarrow (\log q)^2 = \log p.\log r$

Hence $\log p, \log q$ and $\log r$ are in G.P.

2. (*a*) $A - B = A \cap B^C$

I. $B^C - A^C = (U - B) \cap (U - A)^C$

$= (U - B) \cap A = A \cap B^C$

Since statement I is true.

II. $A - B^C = A \cap B$

$A^C - B = A^C \cap B^C = (A \cup B)^C$

Hence $A - B^C \neq A^C - B$

Since statement (II) is false.

3. (*a*) $y = x!$, $z = (2x)!$

$$\frac{z}{y} = \frac{(2x)!}{x!} = 120$$

Check factorial ratios:

$$\frac{(2.3)!}{3!} = \frac{6!}{3!} = \frac{6\times5\times4\times3!}{3!} = 120$$

$\Rightarrow x = 3$

$\therefore (3x)! = (9)! = 362880$

4. (*c*) Number of trailing zeros $= V_5(n!) = [n/5] + [n/25] + ... = 2.$

$\because [n/25] = 0 \Rightarrow n < 25$

So $[n/5]=2 \Rightarrow 10 \leq n \leq 14 \rightarrow 5$ values.

5. (*b*) $10 + \log x$, $10 + \log y$, $10 + \log z$ are in A.P

$\Rightarrow 2\log y = \log x + \log z \Rightarrow \log(xz) = \log(y^2) \Rightarrow xz = y^2$

I. $GM(x, z) = \sqrt{xz} = \sqrt{y^2} = y$ (not y^2)

So statement I is false.

II. $AM(\log x, \log z) = \dfrac{\log x + \log z}{2}$

$$= \frac{\log(xz)}{2} = \frac{\log y^2}{2} = \log(y)$$

Hence, statement II is correct.

6. (*d*) Series: $1 + 3 + 5 + ...$ (n odd numbers).

Sum of n odd numbers $= n^2$.

So need $n^2 = 12345678987654321$.

$\sqrt{12345678987654321} = 111111111$

7. (*c*) Given two arithmetic progressions

19, 21, 23,…up to 110 (a_1=19, d_1= 2)

and 19, 22, 25,…up to 75 (a_2= 19, d_2= 3)

Common term condition:

$\Rightarrow 19 + 2(n - 1) = 19 + 3(m - 1)$

$\Rightarrow 2n - 2 = 3m - 3 \Rightarrow 2n = 3m - 1$

$$\Rightarrow n = \frac{3m-1}{2}$$

For n to be an integer, $3m - 1$ must be even, i.e., m must be odd.

The maximum term in AP1:

$19 + 109 \times 2 = 237$

The maximum term in AP2:

$19 + 74 \times 3 = 241$

So common terms ≤ 237

General common term:

$19 + 3(m - 1) \leq 237 \Rightarrow m \leq 73$

Odd m from 1 to 73

Count = $\frac{73+1}{2} = 37$

8. (c) $\alpha = \frac{-1+\sqrt{-3}}{2}$ = Cube root of unity

$(1 + \alpha^{19} - \alpha^{35})^{100} - (1 - 3\alpha^{25} + \alpha^{38})^{50}$

Since $\alpha^3 = 1$, simplify powers of α:

$\alpha^{19} = \alpha, \alpha^{35} = \alpha^2, \alpha^{25} = \alpha, \alpha^{38} = \alpha^2$

Substitute:

$(1 + \alpha - \alpha^2)^{100} - (1 - 3\alpha + \alpha^2)^{50}$

Using $\alpha^2 + \alpha + 1 = 0$:

$1 + \alpha - \alpha^2 = 2(\alpha + 1),\ 1 - 3\alpha + \alpha^2 = -4\alpha$

Simplified expression:

$(2(\alpha + 1))^{100} - (-4\alpha)^{50}$

$= (2(-\alpha^2))^{100} - 4^{50}.\ \alpha^{50} = 0$

9. (c) Since, $5^{12} = 1 \pmod{13}$,

So, $5^{99} = 5^{12 \times 8 + 3} = (5^{12})^8 \times 5^3$

We simplify to:

$5^{99} = 5^3 \pmod{13}$.

$5^3 = 125$, and $125 \div 13 = 9$ remainder 8.

10. (a) $|A| = (-4)(2) - (-5)(2) = -8 + 10 = 2$.

$\therefore |A^{-1}| = \frac{1}{|A|} = \frac{1}{2}$

11. (b) We have, $n(C) = 34$, $n(F) = 20$,

$\therefore n(C \cup F) = 34 + 26 - n(C \cap F)$

$\Rightarrow 45 = 60 - n(C \cap F) \Rightarrow n(C \cap F) = 15$

Students who like exactly one game:

$= 45 - 15 = 30$

12. (c) The system of equations is: $2x - 3y = 5$ and $15y - 10x + 50 = 0$

We have $\frac{2}{-10} = \frac{-3}{15} \neq \frac{-5}{50}$

So the system of equations is inconsistent

13. (b) Simplifying the given expression:

$$\left(\frac{1-i}{1+i}\right)^{2m}\left(\frac{1+i}{1-i}\right)^{2n} = 1$$

We get:

$\frac{1-i}{1+i} = -i$ and $\frac{1+i}{1-i} = i$

So the equation becomes:

$(-i)^{2m} \cdot i^{2n} = 1$

This simplifies to:

$i^{2m+2n} = 1$

For $i^{2m+2n} = 1$

Here $2m + 2n$ must be a multiple of 4. The smallest value of $2m + 2n = 4, \Rightarrow m + n = 2$

The smallest positive value of $m - n = 2$.

14. (b) The system of equations is:

$x + y + z = 7$

$x + 2y + 3z = 16$

$x + 3y + 4z = 22$

The given matrix for D is:

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{vmatrix}, D_2 = \begin{vmatrix} 1 & 7 & 1 \\ 1 & 16 & 3 \\ 1 & 22 & 4 \end{vmatrix}$$

$$D_2 = 1 \times \begin{vmatrix} 16 & 3 \\ 22 & 4 \end{vmatrix} - 7 \times \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} + 1 \times \begin{vmatrix} 1 & 16 \\ 1 & 22 \end{vmatrix}$$

$D_2 = 1 \times (-2) - 7 \times 1 + 1 \times 6$

$= -2 - 7 + 6 = -3$

15. (a) $A A^{-1} = I \Rightarrow (AB)^{-1} \neq A^{-1}B^{-1}$

$(BA)(BA)^{-1} = I \Rightarrow (BA)(AB)^{-1} \neq I$

$(AB)^T = B^T A^T \Rightarrow (AB)^T \neq A^T B^T$

16. (d) The given determinant is

$$\begin{vmatrix} a & b & c \\ l & m & n \\ p & q & r \end{vmatrix} = a\begin{vmatrix} m & n \\ q & r \end{vmatrix} - b\begin{vmatrix} l & n \\ p & r \end{vmatrix} + c\begin{vmatrix} l & m \\ p & q \end{vmatrix}$$

$= a(mr - nq) - b(lr - np) + c(lq - mp)$

Now, Interchange the rows in option (c) as follows:

$$\begin{vmatrix} p & q & r \\ a & b & c \\ l & m & n \end{vmatrix} R_1 \leftrightarrow R_2, R_3 \leftrightarrow R_1, R_2 \leftrightarrow R_3,$$

This will give us the matrix:

$$\begin{vmatrix} a & b & c \\ l & m & n \\ p & q & r \end{vmatrix}$$

$= a(mr - nq) - b(lr - np) + c(lq - mp)$

17. (a) We are given:

$x = a + b,\ y = a\omega + b\omega^2, z = a\omega^2 + b\omega$

$x^2 + y^2 + z^2 = (a + b)^2 + (a\omega + b\omega^2)^2 + (a\omega^2 + b\omega)^2$

$x^2 + y^2 + z^2 = (a^2 + 2ab + b^2) + (a^2\omega^2 + 2ab + b^2\omega) + (a^2\omega + 2ab + b^2\omega^2)$

$= a^2(1+\omega+\omega^2) + b^2(1+\omega+\omega^2) + 2ab(1+1+1)$

$= 6ab$ [Since $\omega^3 = 1$ and $\omega^2 + \omega + 1 = 0$]

18. (b) Divisibility by 4: Last two digits must be divisible by 4.

Possible last two digits: 12, 32, 24.

First two digits: For each valid pair, 2 digits remain, and they can be arranged in 2! = 2 ways.

Total numbers: 2 (for 12) + 2 (for 32) + 2 (for 24) = 6.

19. (b) The given determinant is:

$$\begin{vmatrix} p+c & a & b \\ c & p+a & b \\ c & a & p+b \end{vmatrix}$$

Apply $C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} p+(a+b+c) & a & b \\ p+(a+b+c) & p+a & b \\ p+(a+b+c) & a & p+b \end{vmatrix}$$

$$= \{p + (a+b+c)\}\begin{vmatrix} 1 & a & b \\ 1 & p+a & b \\ 1 & a & p+b \end{vmatrix}$$