HIGHER MATHEMATICS

Design & Arch
POWERED BY PHYSICSWALLAH

Practice Module

Study Material

NATA, JEE MAINS PAPER 2, JEE AAT

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Relations

1. Let A = {1, 2, 3} and B = {x : x ∈ N, x is a prime numbers less than 12}, then n(A × B) is equal to
(*a*) 12 (*b*) 15
(*c*) 32 (*d*) 36

2. If A = {2, 3} then A × A × A
(*a*) {(2, 2, 2), (2, 3, 3), (2, 3, 2), (2, 3, 3), (3, 2, 2) (3, 2, 3), (3, 3, 2)(3, 3, 3)}
(*b*) {(2, 2, 2), (2, 2, 3), (2, 3, 2), (2, 3, 3), (3, 2, 2) (3, 2, 3), (3, 3, 2)(3, 1, 3)}
(*c*) {(2, 2, 2), (2, 2, 3), (2, 3, 2), (2, 3, 3), (3, 2, 2) (3, 2, 3), (3, 1, 2)(3, 3, 3)}
(*d*) {(2, 2, 2), (2, 2, 3), (2, 3, 2), (2, 3, 3), (3, 2, 2) (3, 2, 3), (3, 3, 2)(3, 3, 3)}

3. If set A and B has 12 elements in common, then the number of elements in (A × B) ∩ (B × A) is
(*a*) 12 (*b*) 24
(*c*) 100 (*d*) 144

4. Let X = {1, 2, 3, 4, 5} and Y = {1, 3, 5, 7, 9}. Which of the following is not a relation from X to Y ?
(*a*) R_1 = {(x, y) : y = x + 2, x ∈ X, y ∈ Y}
(*b*) R_2 = {(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)}
(*c*) R_3 = {(1, 1), (1, 3), (3, 5), (3, 7), (5, 7)}
(*d*) R_4 = {(1, 3), (2, 5), (2, 4), (7, 9)}

5. Let ℤ be the set of integers. A relation ℝ on ℤ is defined as ℝ = {(x, y) : xy < 0, x, y, ∈ ℤ}. Then, which one of the following is correct?
(*a*) R is symmetric but not reflexive
(*b*) R is reflexive but not symmetric
(*c*) R is symmetric and reflexive, but not transitive
(*d*) R is an equivalence relation

6. Let A = {1, 2, 3, 4, 5}. The domain of the relation on A defined by R = {(x, y) : y = 2x – 1} is
(*a*) {1, 2, 3} (*b*) {1, 2}
(*c*) {1, 3, 5} (*d*) {2, 4}

7. A relation on the set of straight lines is such that two lines are related if they are parallel. This relation is
(*a*) Symmetric but not reflexive and transitive.
(*b*) Reflexive but not symmetric and transitive.
(*c*) Symmetric and reflexive but not transitive.
(*d*) Equivalence relation

8. If P and Q are two relations on a set A and both are Reflexive Relations, then P ∩ Q is
(*a*) Reflexive relation
(*b*) Symmetric relation
(*c*) Both reflexive and symmetric relations
(*d*) None of these

9. A relation R is defined on the set Z of integers as follows: mRn ⇔ m + n is odd. Then R is
(*a*) Reflexive and transitive
(*b*) Reflexive and symmetric
(*c*) Symmetric and transitive
(*d*) Symmetric

10. Let the relation R be defined in N by aRb if 2a + 3b = 30. Then R is equal to
(*a*) {(8, 3), (6, 6), (9, 4)}
(*b*) {(1, 3), (4, 9), 5, 10), (12, 12)}
(*c*) {(3, 4), (6, 5), (9, 9), (12, 2)}
(*d*) {(3, 8), (6, 6), (9, 4), (12, 2)}

11. If A = {(x, y) : (x + 3, 5) = (6, 2x + y)}
B = {(x, y) : (x + 1, 1) = (3, y – 2)}
Find n (A × B).
(*a*) 1 (*b*) 2
(*c*) 4 (*d*) 6

12. Which one of the following relations on the set of real numbers R is an equivalence relation?
(*a*) $aR_1\, b \Leftrightarrow |a| = |b|$
(*b*) $aR_2\, b \Leftrightarrow a \geq b$
(*c*) $aR_3\, b \Leftrightarrow$ a divides b
(*d*) $aR_4\, b \Leftrightarrow a < b$

13. Let X be the set of all citizens of India. Elements x, y in X are said to be related if the difference in their age is 5 years. Which one of the following is correct?

(a) The relation is an equivalence relation on X
(b) The relation is symmetric but neither reflexive nor transitive
(c) The relation is reflexive but neither symmetric nor transitive
(d) None of the above

14. If A = $\{x^2 + y^2 = 1, x\,y \in R\}$
Then, A is
(a) Reflexive (b) Symmetric
(c) Transitive (d) Anti-symmetric

15. If R be a relation from A = {1, 2, 3, 4} to B = {1, 3, 5}, i.e., $(a, b) \in R \Leftrightarrow a < b$ then RoR^{-1} is:
(a) {(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)}
(b) {(3, 1)(5, 1), (3, 2), (5, 2), (5, 3), (5, 4)}
(c) {(3, 3), (3, 5), (5, 3), (5, 5)}
(d) {(3, 3)(3, 4), (4, 5)}

16. If we define a relation R on the set N × N as (a, b) R (c, d) $\Leftrightarrow$ a + d = b + c for all (a, b), (c, d) $\in$ N × N, then the relation is
(a) Symmetric only
(b) Symmetric and transitive only
(c) Equivalence relation
(d) Reflexive only

17. If A = {e, f, g, h} and R = {(e, e), (e, f), (f, f), (f, g), (g, g), is a (h, h), (h, e)} is a binary relation on A then which of the following is correct?
(a) R is reflexive but neither symmetric nor transitive
(b) R is reflexive, symmetric, but not transitive
(c) R is reflexive, transitive, but not symmetric
(d) R is reflexive, symmetric, and transitive

18. Let R be the set of real numbers. Consider the following subsets of R × R.
S = {(x, y) : y = x + 1 and 0 < x < 2}, T = {(x, y) : x – y is an integer}. Which one of the following is true?
(a) Neither S nor T is an equivalence relation on R.
(b) Both S and T are equivalence relations on R.
(c) S is an equivalence relation on R, but T is not.
(d) T is an equivalence relation on R, but S is not.

19. Let S = {1, 2, 3...}. A relation R on S × S is defined by xRy if $\log_a x > \log_a y$ and when $a = \frac{1}{2}$. Then the relation is
(a) Reflexive only
(b) Symmetric only
(c) Transitive only
(d) Both symmetric and transitive

20. For real numbers x and y, we write $_xR_y \Leftrightarrow x - y + \sqrt{2}$ is an irrational number. Then, the relation R is
(a) Reflexive (b) Symmetric
(c) Transitive (d) None of these

21. On Q, the set of rational numbers, define a relation R as follows: aRb if a cos 15° + b sin 15° is an irrational number, then:
(a) Domain of R is Q
(b) Domain of R is Q – Z
(c) Domain of R is Q – N
(d) Domain of R is Q – A where A is a singleton set

22. R is a relation over the set of integers and it is given by $(x, y) \in R, \Leftrightarrow |x - y| \le 1$. Then R is
(a) Reflexive and transitive
(b) Reflexive and symmetric
(c) Symmetric and transitive
(d) An equivalence relation

23. If R = $\{(x, y) : x, y \in Z, x^2 + y^2 \le 4\}$ is a relation on Z, then the domain of R is
(a) {0, 1, 2} (b) {–2, 2, 0}
(c) {–2, –1, 0, 1, 2} (d) None of these

24. A relation ϕ from C to R is defined by $x\phi y \Leftrightarrow |x| = y$. Which one is correct?
(a) (2 + 3i) ϕ 13 (b) 3ϕ (–3)
(c) (1 + i) ϕ 2 (d) i ϕ 1

25. A relation defined on a set A = [–1, 1] as $R = \left\{(x, y) : \sin^{-1}x + \cos^{-1}y = \frac{\pi}{2}\right\}$ is
(a) Symmetric and reflexive but not transitive
(b) Reflexive and transitive but not symmetric
(c) Transitive and symmetric but not reflexive
(d) An equivalence relation

26. Let R be a relation over the set N × N and it is defined by (a,b)R(c,d) $\Rightarrow$ a + d = b + c then R is
(a) Reflexive only (b) Symmetric only
(c) Transitive only (d) An equivalence relation

27. For any two real numbers a and b, we define aRb if and only if $\sin^2 a + \cos^2 b = 1$. The relation R is:
(a) Reflexive but not symmetric
(b) Symmetric but not transitive
(c) Transitive but not reflexive
(d) An equivalence relation

28. If R = $\{(x, y) : x, y \in Z, x^2 + 3y^2 \le 8\}$ is a relation on the set of integers Z, then the domain of R^{-1} is
(a) {0, 1} (b) {–2, –1, 1, 2}
(c) {–1, 0, 1} (d) {–2, –1, 0, 1, 2}

29. The relation R defined on the set A = {1, 2, 3, 4, 5} by R = {(x, y) : $|x^2 - y^2| < 16$}, then:

(*a*) R = {(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)}

(*b*) R = {(2, 2), (3, 2), (4, 2), (2, 4)}

(*c*) R = {(3, 3), (4, 3), (5, 4), (3, 4)}

(*d*) None of these

30. The linear relation between the components of the ordered pairs of the relation R given by: R = {(0, 2), (-1, 5), (2, -4), ...} is

(*a*) x + y = 2 (*b*) 3x – y = 1

(*c*) x + 3y = 2 (*d*) 3x + y = 2

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ANSWER KEY

1. (*b*)	**2.** (*d*)	**3.** (*d*)	**4.** (*d*)	**5.** (*a*)	**6.** (*a*)	**7.** (*d*)	**8.** (*a*)	**9.** (*d*)	**10.** (*d*)
11. (*a*)	**12.** (*a*)	**13.** (*b*)	**14.** (*b*)	**15.** (*c*)	**16.** (*c*)	**17.** (*a*)	**18.** (*d*)	**19.** (*c*)	**20.** (*a*)
21. (*a*)	**22.** (*b*)	**23.** (*c*)	**24.** (*d*)	**25.** (*d*)	**26.** (*d*)	**27.** (*d*)	**28.** (*c*)	**29.** (*d*)	**30.** (*d*)

SOLUTION

1. (*b*) We have, A = {1, 2, 3} and B = {2, 3, 5, 7, 11}

$\Rightarrow n(A \times B) = n(A) \times n(B) = 3 \times 5 = 15$

2. (*d*) A = {2, 3}

A × A = {2, 3} × {2, 3}

A × A = {(2, 2), (2, 3), (3, 2), (3, 3)}

A × A × A = { (2, 2), (2, 3), (3, 2), (3, 3)} × {2, 3}

= {(2, 2, 2), (2, 2, 3), (2, 3, 2), (2, 3, 3), (3, 2, 2) (3, 2, 3), (3, 3, 2)(3, 3, 3)}

3. (*d*) We know

$n(A \times B) \cap n(B \times A) = [n(A \cap B)]^2$

$\Rightarrow n(A \times B) \cap n(B \times A) = n(A \cap B) \times n(B \cap A)$

= 12 × 12

= 144

4. (*d*) R_4 is not a relation from X to Y because (2, 4) (7, 9) $\notin$ X × Y.

5. (*a*) Given R = {(x, y) : xy < 0, x, y, ∈ Z}

(i) Reflexive

$\because x, y \in Z \Rightarrow x \cdot x = x^2 \geq 0$

Hence, (x, x) $\notin$ R

R is not reflexive

(ii) Symmetric

Let (x, y) ∈ R ⇒ xy < 0

⇒ yx < 0 ⇒ (y, x) ∈ R

Hence, R is symmetric.

(iii) Transitive

Let (–3, 2), (2, –1) ∈ R but (–3, –1) $\notin$ R as –3 × –1 = 3 > 0

∴ R is not transitive.

6. (*a*) Since, A = {1, 2, 3, 4, 5}

∴ Relation from A to A

R = {(x, y) : y = 2x – 1} = {(1, 1), (2, 3), (3, 5)}

⇒ Domain = {1, 2, 3}

7. (*d*) 1. Reflexive :

All the lines are parallel to itself, hence it is reflexive.

2. Symmetric:

If the line l is parallel to m ⇒ m || l

Hence, the relation is symmetric.

3. Transitive:

(l, m), (m, n) ∈ R

⇒ l || m and m || n

⇒ l || n ⇒ (l, n) ∈ R

Hence, this relation is also transitive.

Hence, this is an equivalence relation.

8. (*a*) It is a standard result that if P and Q are two relations on a set A and both are Reflexive relations, then

P ∩ Q It is also a Reflexive Relation on the set A and also P ∪ Q is also a Reflexive Relation on A.

Since P is reflexive, for any a ∈ A, (a, a) ∈ P.

Since Q is reflexive, for any a ∈ A, (a, a) ∈ Q.

⇒ (a, a) ∈ P ∩ Q

9. (*d*) ∵ R is a relation defined on the set Z of integers as mRn ⇔ m + n is odd

(1) Then, mRm = 2m it is not odd. Hence, it is not reflexive.

(2) Let mRn ⇔ m + n it be odd. Hence, nRm ⇔ n + m it is odd.

$\therefore$ This relation is symmetric.

(3) Let $(2, 3), (3, 4) \in R$ but $2 + 4 = 6 \Rightarrow (2, 4) \notin R$,

Hence, this relation is not transitive.

10. (*d*) Given that aRb: $2a + 3b = 30$

$\Rightarrow 3b = 30 - 2a$

$\Rightarrow b = \frac{30 - 2a}{3}$

for $a = 3, b = 8$

$a = 6, b = 6$

$a = 9, b = 4$

$a = 12, b = 2$

Hence, $R = \{(3, 8), (6, 6), (9, 4), (12, 2)\}$

11. (*a*) $A = \{(x, y) : (x + 3, 5) = (6, 2x + y)\}$

$x + 3 = 6; 5 = 2x + y$

$x = 3 ; y = -1$

$A = \{(3, -1)\}$

$B = \{(x, y) : (x + 1, 1) = (3, y-2)\}$

$x + 1 = 3; 1 = y-2;$

$x = 2; y = 3;$

$B = \{(2, 3)\}$

$n(A \times B) = n(A) \times n(B)$

$= 1 \times 1 = 1$

12. (*a*) (i) Reflexive:

$a \in R, aR_1a$

$\Rightarrow |a| = |a|$

(ii) Symmetric:

$a, b \in R$

$aR_1a \Rightarrow |a| = |b|$

$\Rightarrow |b| = |a|$

$\Rightarrow bR_1 a$

(iii) Transitive:

$a, b, c \in R$

$aR_1b \Rightarrow |a| = |b|, bR_1c$

$\Rightarrow |b| = |c|$. So, $|a|=|c|$

$aR_1c = R_1$ is an equivalence relation on R.

13. (*b*) We have, X = {All citizens of India }

and $R = \{(x, y): x, y \in X , |x - y| = 5\}$

For reflexive:-

$|x - x| = 0 \neq 5$

$\therefore (x, x) \in R$

So , R is not reflexive

For symmetric:

let $xRy \Rightarrow |x - y| = 5 \Rightarrow |y - x| = 5$

$\Rightarrow yRx$

So, R is symmetric

For transitive:

Let $x, y, z \in X$ such that

$xRy \Rightarrow |x - y| = 5$

and $yRz \Rightarrow |y - z| = 5$

But $|x - z| \neq 5$

So, R is not transitive.

The relation is symmetric but neither reflexive nor transitive.

14. (*b*) We have,

$A = \{(x, y) \mid x^2 + y^2 = 1, x, y \in R\}$

Reflexive Relation

If $(x, x) \in A$ then

$\Rightarrow x^2 + x^2 = 1$

$\Rightarrow 2x^2 = 1$

$\Rightarrow x = \pm\frac{1}{\sqrt{2}}$

does not hold for all R

Therefore it's not a Reflexive

Symmetric Relation

If $(x, y) \in A$

$x^2 + y^2 = 1$

$\Rightarrow y^2 + x^2 = 1$

$\Rightarrow (y, x) \in A$

$\because (x, y) \in A$ then $(y, x) \in A$

Therefore, A is Symmetric

Transitive Relation

If $(x, y) \in R$ and $(y,z) \in R$

$x^2 + y^2 = 1$ and $y^2 + z^2 = 1$

Subtract the above two equations

$x^2 - z^2 = 0$

$\Rightarrow x = \pm z$

$\therefore$ does not hold $\forall x, z \in R$

Therefore, A is not transitive

From the above definition, it is clear that A is Symmetric only

15. (*c*) Given, $A = \{1, 2, 3, 4\}$

$B = \{1, 3, 5\}$

$R = \{(a, b) : a < b\}$

$R = \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)\}$

$\Rightarrow R^{-1} = \{(3, 1), (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)\}$

$(3, 1) \in R^{-1}, (1, 3) \in R$

$(3, 3) \in RoR^{-1}$

$(3, 2) \in R^{-1}, (2, 5) \in R$

$(3, 5) \in RoR^{-1}$

$(5, 1) \in R^{-1}, (1, 3) \in R$

$(5, 3) \in RoR^{-1}$

$(5, 1) \in R^{-1}$, $(1, 5) \in R$

$(5, 5) \in RoR^{-1}$

Hence, $RoR^{-1} = \{(3, 3), (3, 5), (5, 3), (5, 5)\}$

16. (*c*) It is given that:

$(a, b) R (c, d) \Leftrightarrow a + d = b + c$

Let $(a, b) \in N \times N$.

$\Rightarrow (a + b) = (b + a)$

$\Rightarrow (a, b) R (a, b)$

Therefore, R is reflexive.

Let (a, b) R (c, d) where $(a, b), (c, d) \in N \times N$.

$\Rightarrow (a + d) = (b + c)$

$\Rightarrow (b + c) = (a + d)$

$\Rightarrow (c + b) = (a + d)$

$\Rightarrow (c, d) R (a, b)$

Therefore, R is symmetric.

It is given that (a, b) R (c, d).

$\Rightarrow (a + d) = (b + c)$

$\Rightarrow (a - b) = (c - d)$ ...(1)

Let (c, d) R (e, f).

$\Rightarrow (c + f) = (d + e)$

$\Rightarrow (c - d) = (e - f)$ ...(2)

From equations (1) and (2):

$\Rightarrow (a - b) = (e - f)$

$\Rightarrow (a + f) = (e + b)$

$\Rightarrow (a, b) R (e, f)$

Therefore, R is transitive.

The relation R satisfies reflexive, symmetric, and transitive.

Therefore, R is an equivalence relation.

17. (*a*) A relation R on a set A is reflexive when :

{(a, a) R}, ∀ aA

So in the given case R = {(e, e), (e, f), (f, f), (f, g), (g, g), (h, h), (h, e)} {(e, e), (f, f), (g, g), (h, h)} R.

Therefore, the relation is reflexive

A relation R on a set A is symmetric when:

{(a, b) R, (b, a) R}, ∀ a, bA

But in the given relation,

(e, f) R but (f, e) R, therefore not symmetric

A relation R on a set A is transitive if and only if:

$\{(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R\}$, $\forall$ a, b, $c \in A$

But in the given relation

$(e, f) \in R$ and $(f, g) \in R$ but $(e, g) \notin R$, therefore not transitive

18. (*d*) Since, $(1, 2) \in S$ but $(2,1) \notin S$

$\therefore$ S is not symmetric.

Hence, S is not an equivalence relation.

Given, $T = \{(x, y) : (x - y) \in I\}$

Now $x - x = 0 \in I$, it is a reflexive relation.

Again, now $(x - y) \in I$

$\Rightarrow y - x \in I$ It is a symmetric relation.

Let,

$x - y = I_1$ and $y - z = I_2$

Now,

$x - z = (x - y) + (y - z) = I_1 + I_2 \in I$

$\therefore$ T is also transitive.

Hence, T is an equivalence relation.

19. (*c*) Here it is given that: $xRy = \log_a x > \log_a y$ and $a = \frac{1}{2}$.

Then, we get $\log_{\frac{1}{2}}(x) > \log_{\frac{1}{2}}(y) \Rightarrow x < y$

The connection is not reflexive because, in the case of reflexive, $x < x$ is false.

In the case of symmetry, if $x < y$ then $y < x$ is likewise false.

If $x < y$ and $y < z$, then $x < z$, then the relation is transitive exclusively.

20. (*a*) ${}_xR_y \Leftrightarrow x - y + \sqrt{2}$ is an irrational number

Let $(x, x) \in R$

Then, $x - x + \sqrt{2}$ which is an irrational number.

$\therefore$ *R* is an reflexive relation

$(\sqrt{2}, 0) \in R$ but $(0, \sqrt{2})$ does not belong to R

$\therefore$ *R* is not a symmetric relation

$(\sqrt{2}, 0) \in R$, $(0, 2\sqrt{2}) \in R$ but $(\sqrt{2}, 2\sqrt{2})$ does not belong to R.

$\therefore$ *R* is not a transitive relation

21. (*a*) Given, acos15° + bsin15° is an irrational number.

$\cos(15°) = \cos(45° - 30°)$

$= \cos 45° \cos 30° + \sin 45° \sin 30°$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$\sin(15)° = \sin(45° - 30°)$

$= \sin 45° \cos 30° - \cos 45° \sin 30°$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

$a\cos 15° + b\sin 15° = a\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) + b\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$

$= \frac{a\sqrt{3}}{2\sqrt{2}} + \frac{a}{2\sqrt{2}} + \frac{b\sqrt{3}}{2\sqrt{2}} - \frac{b}{2\sqrt{2}}$

$= \frac{(a+b)\sqrt{3}}{2\sqrt{2}} + \frac{a-b}{2\sqrt{2}}$

Hence, the domain is Q.

22. (*b*) Let, x be an element in Z

then, $|x - x| = 0 \le 1$

So, every element of Z is related to itself.

$\Rightarrow$ R is a reflexive relation.

Now, let the two elements in Z be x, y

Therefore, $|x - y| \le 1$ then $|y - x| \le 1$

So, $xRy \Leftrightarrow yRx$

$\Rightarrow$ R is a symmetric relation

Also, R is not a transitive relation because $(2, 1) \Rightarrow |2 - 1| \le 1 \in R$ and $(1, 0) \Rightarrow |1 - 0| \le 1 \in R$ but $(2, 0) \Rightarrow |2-0| \ge 1 \notin R$.

23. (*c*) The relation states that $x^2 + y^2$ it must be less than or equal to 4.

This describes a circle with radius 2 centered at the origin.

Since $x \in Z$ that satisfies the inequality: $x^2 \le 4$

This implies:

$-2 \le x \le 2$

if $x = \pm 1, y = \pm\sqrt{3}$ (not an integer)

Therefore, the domain of R is : $\{-2,-1,0,1,2\}$

24. (*d*) We have $\phi : C \to R$ $x \phi y \Leftrightarrow |x| = y$.

Now,

(A) $(2 + 3i) \phi 13$

$|(2+3i)| = \sqrt{2^2 + 3^2} \ne 13.$

So (2 + 3i) and 13 are not related.

(B) $3\phi (-3)$

$|3| = 3 \ne -3.$

So, 3 and –3 are not related.

(C) $(1 + i) \phi 2$

$|(1+i)| = \sqrt{1^2 + 1^2} \ne 2$

So, 1 + i and 2 are not related.

(D) $i \phi 1$

$|i| = 1 = 1$

So, i and 1 are related.

Hence, option (*d*) is correct.

25. (*d*) $\sin^{-1}x + \cos^{-1}y = \frac{\pi}{2}$

$(x, y) \in R$

$\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$\Rightarrow (x, x) \in R$

$\Rightarrow$ R is reflexive

Let $\sin^{-1}x + \cos^{-1}y = \frac{\pi}{2}$

$\Rightarrow \frac{\pi}{2} - \sin^{-1}x + \frac{\pi}{2} - \cos^{-1}y = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}x + \sin^{-1}y = \frac{\pi}{2}$

$\Rightarrow (y, x) \in R \Rightarrow$ R is symmetric

Let $(x, y) \in R$ $\sin^{-1}x + \cos^{-1}y = \frac{\pi}{2}$ and

$(y, z) \in R \Rightarrow \sin^{-1}y + \cos^{-1}z = \frac{\pi}{2}$

$\sin^{-1}x + \cos^{-1}z = \frac{\pi}{2} \Rightarrow (x,z) \in R$

$\Rightarrow$ R is transitive

$\Rightarrow$ R is an equivalence relation

26. (*d*) We have, $(a, b)R(c, d) \Rightarrow a + d = b + c$

Reflexive:

$(a, b)R(a, b) \Rightarrow a + b = b + a$

$= a + b = a + b$

This is true always, so R is Reflexive

Symmetric:

$(a, b)R(c, d) \Rightarrow a + d = b + c$

$(c, d)R(a, b) \Rightarrow c + b = d + a$

$\Rightarrow a + d = b + c$

Which is also true, so R is symmetric

Transitive:

If (a, b)R(c, d) and (c, d)R(e, f) that is

$a + d = b + c$ and $c + f = d + e$

Adding both equations

$a + d + c + f = b + c + d + e$

$a + f = b + e$

$(a, b)R(e, f) \in R$

The relation is also transitive.

27. (*d*) Let the given relation be defined as

$R = \{(a, b) \mid \sin^2 a + \cos^2 b = 1\}$

For reflexive,

$(\sin^2 a + \cos^2 a = 1, \forall \theta \in R)$

$(\sin^2 \theta + \cos^2 \theta = 1, \forall \theta \in R)$

$= aRa$

$= (a, a) \in R$

R is reflexive.

For symmetric,

$\sin^2 a + \cos^2 b = 1$

$\Rightarrow 1 - \cos^2 a + 1 - \sin^2 b = 1$

$\Rightarrow \sin^2 b + \cos^2 a = 1$

$\Rightarrow bRa$

Hence R is symmetric.

For transitive

Let aRb, bRc

$\Rightarrow \sin^2 a + \cos^2 b = 1$

and $\sin^2 b + \cos^2 c = 1$

On adding eqs. (i) and (ii), we get

$\sin^2 a + (\sin^2 b + \cos^2 b) + \cos^2 c = 2$

$\Rightarrow \sin^2 a + \cos^2 c + 1 = 2$ ...(i)

$\Rightarrow \sin^2 a + \cos^2 c = 1$ ...(ii)

Hence R is transitive also.

Therefore, relation R is an equivalence relation.

28. (*c*) $x^2 + 3y^2 \le 8, x, y \in Z$

For $y = -1 \Rightarrow x = -2, -1, 0, 1, 2$

For $y = 0 \Rightarrow x = -2, -1, 0, 1, 2$

For $y = 1 \Rightarrow x = -2, -1, 0, 1, 2$

Domain of $R = \{-2, -1, 0, 1, 2\}$

$\Rightarrow$ Range of R = Domain of $R^{-1} = \{-1, 0, 1\}$

29. (*d*) Given that, $R = \{(x, y) : |x^2 - y^2| < 16\}$ and the set, $A = \{1, 2, 3, 4, 5\}$.

Now, let x = 1,

$\Rightarrow |x^2 - y^2| < 16$

$\Rightarrow |1 - y^2| < 16$

$\Rightarrow |y^2 - 1| < 16$

$\Rightarrow y = 1, 2, 3, 4$

Let x = 2,

$\Rightarrow |4 - y^2| < 16$

$\Rightarrow |y^2 - 4| < 16$

$\Rightarrow y = 1, 2, 3, 4$

Let x = 3,

$\Rightarrow |9 - y^2| < 16$

$\Rightarrow |y^2 - 9| < 16$

$\Rightarrow y = 1, 2, 3, 4$

Let x = 4,

$\Rightarrow |16 - y^2| < 16$

$\Rightarrow |y^2 - 16| < 16$

$\Rightarrow y = 1, 2, 3, 4, 5$

Let

x = 5,

$\Rightarrow |25 - y^2| < 16$

$\Rightarrow |y^2 - 25| < 16$

$\Rightarrow y = 4, 5$

Therefore, the relation R defined on the set $A = \{1, 2, 3, 4, 5\}$ by

$R = \{(x, y) : |x^2 - y^2| < 16$, then

R={(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (5, 4), (5, 5).

30. (*d*) Given $R = \{(0, 2), (-1, 5), (2, -4), ...\}$.

Let y = ax + b be the linear relation between the components of R.

Since $(0, 2) \in R$

$\Rightarrow y = ax + b$

$\Rightarrow 2 = b$

Also $(-1, 5) \in R$

$\Rightarrow 5 = -a + b$

Substituting b = 2 in the above equation, we get

$\Rightarrow 5 = -a + 2$

$\Rightarrow -a = 5 - 2$

$\Rightarrow a = -3$

Substituting these values of a and b in y = ax + b, we get

$y = -3x + 2$

$\Rightarrow 3x + y = 2$

Therefore, the required linear relation between the components of the relation

$R = \{(0, 2), (-1, 5), (2, -4), ...\}$ is $3x + y = 2$.

3

Function

1. Let f : R → R be defined by $f(x) = x^2 + 1$. Then, the pre-image of 17 is

(*a*) 4 (*b*) 3

(*c*) 2 (*d*) None

2. The domain of the function $f(x) = \frac{(x^2+1)}{x^2-3x+3}$ is

(*a*) R – {1, 2} (*b*) R – {1, 4}

(*c*) R (*d*) R – {1}

3. Find the range of f(x) = 1 +3 cos 2x

(*a*) [2, 3] (*b*) [2, 4]

(*c*) [–2, 4] (*d*) None of the above

4. If f(x) = 3 – x, –4 ≤ x ≤ 4, then the domain of $\log_e (f(x))$ is

(*a*) [–4, 4] (*b*) (–∞, 3]

(*c*) (–∞, 3) (*d*) [–4, 3)

5. The composite mapping fog of the maps f : R → R, f(x) = sin x, g : R → R, $g(x) = x^2$ is

(*a*) $\sin x + x^2$ (*b*) $(\sin x)^2$

(*c*) $\sin x^2$ (*d*) $\frac{\sin x}{x^2}$

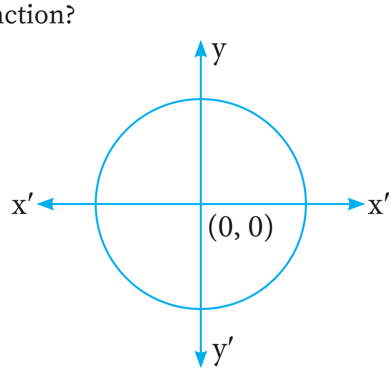
6. Find the inverse for $h(x) = \frac{1+9x}{4-x}$.

(*a*) $\frac{9x-1}{9+4x}$ (*b*) $\frac{4x+1}{9-x}$

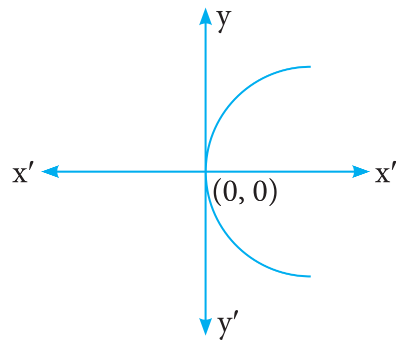
(*c*) $\frac{4x-1}{9+x}$ (*d*) None of these

7. Which of the following graphs represents a function?

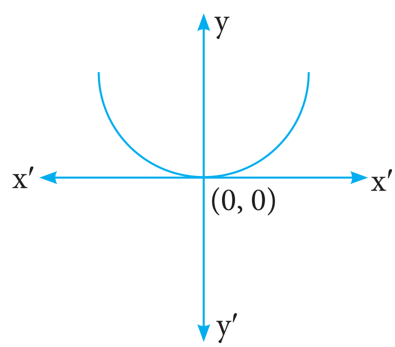
(*a*)



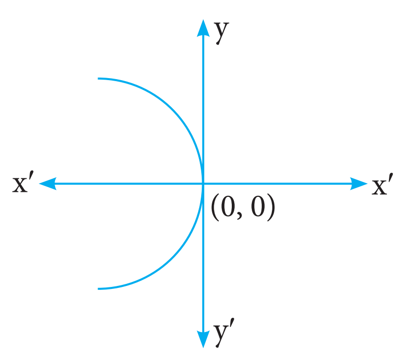
(*b*)



(*c*)



(*d*)



8. The function f : R – {–1} → R – {1} defined by $f(x) = \frac{x}{x+1}$ is

(*a*) Both one-one and onto

(*b*) Only one-one

(*c*) Only onto

(*d*) Neither one-one nor onto

9. Find the inverse for $g(x) = 4(x - 3)^5 + 21$.

(a) $3+\sqrt[5]{\frac{1}{4}(x-21)}$

(b) $3-\sqrt[5]{\frac{1}{4}(x+21)}$

(c) $\frac{1}{3}+\sqrt[5]{4(x+21)}$

(d) None of these

10. Which of the following functions are invertible?

(a) One-one Into Functions

(b) One-one Onto Functions

(c) Many-one Onto Functions

(d) Many-one Into Functions

11. What is the domain of the function $f(x)=\sqrt{1-(x-1)^2}$

(a) (0, 1) (b) [–1, 1]

(c) (0, 2) (d) [0, 2]

12. If $f(x) = x^n$, $n \in N$ and $(g \circ f)(x) = ng(x)$ then g(x) can be

(a) $n|x|$ (b) e^x

(c) $3\cdot\sqrt[3]{x}$ (d) $\log|x|$

13. Let $f(x)=\sqrt{x^4+15}$, then the graph of the function y = f(x) is symmetrical about

(a) The x-axis (b) The line x = y

(c) The y-axis (d) The origin

14. The range of the function $f(x) = \sin[x]$, $-\frac{\pi}{4}\le x\le\frac{\pi}{4}$, where [x] denotes the greatest integer ≤ x is

(a) {0} (b) {0, –1}

(c) {0, ±sin1} (d) {0, –sin1}

15. What is the period of $\frac{|\sin x|}{\sec x}, \sec x \neq 0$?

(a) $\frac{\pi}{3}$ (b) π

(c) $\frac{\pi}{4}$ (d) Not periodic

16. Which of the following is an even function?

(a) $x\left(\frac{a^x-1}{a^x+1}\right)$ (b) tan x

(c) $\frac{a^x-a^{-x}}{2}$ (d) $\frac{a^x+1}{a^x-1}$

17. Find the domain and range of the following function $f(x)=\frac{(x^2-1)}{(x-1)}$.

(a) Domain = Range

(b) Domain = [0, 4], Range = (–∞, –1) ∪ (1, 4]

(c) Domain = R – {1}, Range = R – {2}

(d) None of the above

18. Let f(x)f(y) = f(xy) for all real x, y. If f(2) = 4, then what is the value of $f\left(\frac{1}{2}\right)$?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) 1 (d) 4

19. If x, y ∈ {1, 2, 3, 4}, then f = {(x, y) : xy = x – y} is

(a) a one-one function.

(b) a onto function.

(c) a many one function

(d) not a function

20. If f : [1, ∞) → [2, ∞) is given by $f(x)=x+\frac{1}{x}$ then $f^{-1}(x)$ equals

(a) $\frac{x+\sqrt{x^2-4}}{2}$ (b) $\frac{x}{1+x^2}$

(c) $\frac{x-\sqrt{x^2-4}}{2}$ (d) $1+\sqrt{x^2-4}$

21. Let f(x) and g(x) be two functions such that g(x) = ln(x) and $f(g(x)) = e^x - e^{-x}$, determine f(x).

(a) $e^{e^x}-e^{-e^x}$ (b) $e^{e^x}+e^{-e^x}$

(c) $e^{-e^{-x}}-e^{-e^{-x}}$ (d) $e^{e^x}-e^{e^{-x}}$

22. The functional equation f(xy) = f(x) + f(y) holds for:

(a) All real numbers x, y

(b) Only positive values of x, y

(c) Only negative values of x, y

(d) All integers x, y

23. If $f(x)=\frac{\sin 3x}{\sin x}$ then the range of values of f(x) for real values of x is

(a) [–1, 3] (b) (∞, –1]

(c) (3, ∞) (d) [–1, 3)

24. The range of a function $f(x)=\sqrt{2-x}+\sqrt{1+x}\ \forall x\in[-1,2]$ is

(a) [3, 6] (b) $[\sqrt{3}, \sqrt{6}]$

(c) $[0, \sqrt{6}]$ (d) $[0, \sqrt{3}]$

25. If $f(x)=\log_e\left(\frac{1+x}{1-x}\right), g(x)=\frac{3x+x^3}{1+3x^2}$ and $g \circ f(t)$ = g(f(t)), then what is $g\circ f\left(\frac{e-1}{e+1}\right)$ equal to?

(*a*) 2 (*b*) 1

(*c*) 0 (*d*) 0.5

26. If $f(x)=\log\left(\frac{1+x}{1-x}\right)$ and $g(x)=\frac{3x+x^3}{1+3x^2}$, then $f^\circ g(x) = k.f(x)$, then the value of k is

(*a*) k = 3 (*b*) k = 2

(*c*) k = 0 (*d*) k = 8

27. The range of the function $f(x)=x^2+\frac{1}{x^2+1}$ is

(*a*) [1, +∞) (*b*) $\left[\frac{3}{2},+\infty\right)$

(*c*) [2, +∞) (*d*) None of these

28. Let $f(x)=\log_{x^2}25$ and $g(x) = \log_x 5$ then f(x) = g(x) holds for x belonging to

(*a*) R (*b*) (0, 1) ∪ (1, +∞)

(*c*) ϕ (*d*) None of these

29. If x takes all real values, then the range of $f(x)=\frac{x^2-2x+4}{x^2+2x+4}$ is:

(*a*) [3, ∞) (*b*) $\left(-\infty,\frac{1}{3}\right]$

(*c*) $(-\infty,\frac{1}{3}]\cup[3,\infty)$ (*d*) $\left[\frac{1}{3},3\right]$

30. Which of the following pairs of functions is non-identical?

(*a*) f(x) = |x|, g(x) = x sin (x)

(*b*) $f(x)=\sqrt{x^2}, g(x)=|x|$

(*c*) $f(x)=\sqrt{x^2-6x+9}, g(x)=x-3$

(*d*) f(x) = [{x}], g(x) = {[x]}

❖ ❖ ❖

ANSWER KEY

1. (*a*)	**2.** (*c*)	**3.** (*c*)	**4.** (*d*)	**5.** (*c*)	**6.** (*c*)	**7.** (*c*)	**8.** (*a*)	**9.** (*a*)	**10.** (*b*)
11. (*d*)	**12.** (*d*)	**13.** (*c*)	**14.** (*d*)	**15.** (*d*)	**16.** (*a*)	**17.** (*c*)	**18.** (*a*)	**19.** (*d*)	**20.** (*a*)
21. (*a*)	**22.** (*b*)	**23.** (*a*)	**24.** (*b*)	**25.** (*b*)	**26.** (*a*)	**27.** (*a*)	**28.** (*b*)	**29.** (*d*)	**30.** (*c*)

SOLUTION

1. (*a*) We have, $f(x) = x^2 + 1$

$\Rightarrow f^{-1}(17) = x$

$\Rightarrow f(x) = 17$

$\Rightarrow x^2 + 1 = 17$

$\Rightarrow x^2 = 16$

$\Rightarrow x = \pm 4$

2. (*c*) We have, $f(x)=\frac{(x^2+1)}{x^2-3x+3}$

$\Rightarrow (x^2 + 1)$ is defined for all $x \in R$

Also, $x^2 - 3x + 3 = 0$ has imaginary roots as D < 0.

∴ $x^2 - 3x + 3$ is also defined for all real numbers, i.e., $x \in R$

3. (*c*) We have, f(x) = 1 + 3 cos 2x

∵ $-1 \le \cos 2x \le 1$

$\Rightarrow -3 \le 3\cos 2x \le 3$

$\Rightarrow 1 - 3 \le 1 + 3\cos 2x \le 3 + 1$

$\Rightarrow -2 \le f(x) \le 4$

∴ Range = [–2,4]

4. (*d*) We have, f(x) = 3 – x, here $-4 \le x \le 4$

For $\log_e \{f(x)\}$ to be defined,

$3 - x > 0 \Rightarrow 3 > x$

$\Rightarrow x < 3$

$\Rightarrow$ Domain = [–4, 3)

5. (*c*) We have,

$(fog)(x) = f(g(x)) = f(x^2) = \sin x^2$

6. (*c*) Let $y = h^{-1}(x) \Rightarrow x = h(y)$

$\Rightarrow x=\frac{1+9y}{4-y}$

$\Rightarrow x(4 - y) = 1 + 9y$

$\Rightarrow 4x - xy = 1 + 9y$

$\Rightarrow 4x - 1 = 9y + xy$

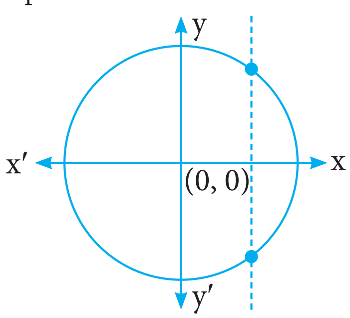
$\Rightarrow 4x - 1 = (9 + x)y$

$$\Rightarrow y = \frac{4x-1}{9+x}$$

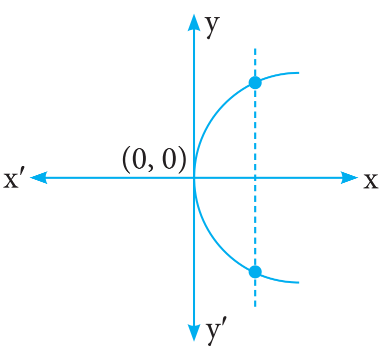
$$\Rightarrow h^{-1}(x) = \frac{4x-1}{9+x}$$

7. (*c*) **We know, function:** It is a special type of relation in which each and every element of its domain is the preimage of a unique value in its co-domain.

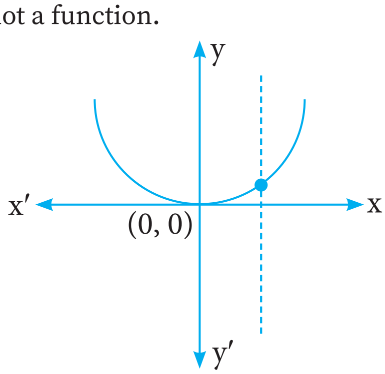
Graphical way to Identify: Draw a line parallel to y-axis if it cuts at one point then that graph represents a function and if it cuts at more than one point then it is not a function.

(i) 

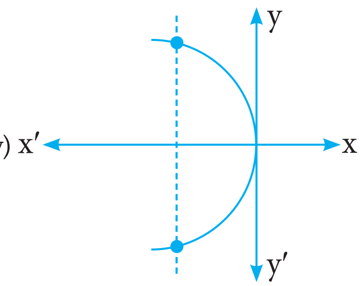
The line cuts the graph at two points, hence it is not a function.

(ii) 

The line cuts the graph at two points, hence it is not a function.

(iii) 

The line cuts the graph at only one point, hence it is a function.

(iv) 

The line cuts the graph at two points, hence it is not a function.

8. (*a*) We have, $f(x) = \frac{x}{x+1}$

$$\because f'(x) = \frac{1}{(x+1)^2} > 0$$

$\therefore$ f is one-one.

Let $y \in$ co-dom(f)

$$\Rightarrow y = \frac{x}{x+1} \Rightarrow x = \frac{y}{1-y} \in \text{dom}(f)$$

So, for every y in the codomain of f there exists a x in the domain of f

$\therefore$ f is onto

9. (*a*) Let $y = g^{-1}(x) \Rightarrow g(y) = x$

$\Rightarrow 4(y - 3)^5 + 21 = x$

$\Rightarrow x - 21 = 4(y - 3)^5$

$$\Rightarrow \frac{1}{4}(x-21) = (y-3)^5$$

$$\Rightarrow \sqrt[5]{\frac{1}{4}(x-21)} = y-3$$

$$\Rightarrow 3 + \sqrt[5]{\frac{1}{4}(x-21)} = y$$

$$\Rightarrow y = 3 + \sqrt[5]{\frac{1}{4}(x-21)}$$

$$\Rightarrow g^{-1}(x) = 3 + \sqrt[5]{\frac{1}{4}(x-21)}$$

10. (*b*) By Definition we know that only One-one Onto Functions or Bijective Functions are invertible rest other functions are not invertible.

11. (*d*) $y = f(x) = \sqrt{1-(x-1)^2}$ is defined if $1 - (x - 1)^2 \geq 0$

$1 - (x - 1)^2 \geq 0$

$\Rightarrow (x - 1)^2 - 1 \leq 0$

$\Rightarrow x^2 + 1 - 2x - 1 \leq 0$

$\Rightarrow x^2 - 2x \leq 0$

$\Rightarrow x(x - 2) \leq 0$

$x \in [0, 2]$

Hence, the correct answer is $x \in [0, 2]$.

12. (*d*) We have, $f(x) = x^n$, $n \in N$ and $(g \circ f)(x) = ng(x)$.

Since $f(x) = x^n$

Taking log on both sides.

$\log f(x) = n \log x$ ...(i)

Also, it is given that $(g \circ f)(x) = ng(x)$.

$g(f(x)) = ng(x)$ ...(ii)

Comparing (i) and (ii), we get

$g(x) = \log x$

Or we can say

$g(x) = \log |x|$

As the argument of log is > 0

Thus, the g(x) can be log |x|.

13. (*c*) Since an even function is a function for which f(–x) = f(x) for all x in its domain.

And the graph of an even function has even symmetry, which means that it is symmetric about the y-axis.

We have, $f(x) = \sqrt{x^4 + 15}$

$\Rightarrow f(-x) = \sqrt{(-x)^4 + 15}$

$\Rightarrow f(-x) = \sqrt{x^4 + 15}$

$\Rightarrow f(-x) = f(x)$

So, f(x) is an even function and hence, it is symmetrix about the y-axis.

14. (*d*) $-\frac{\pi}{4} \le x \le \frac{\pi}{4} \Rightarrow -\frac{\pi}{4} \le x < 0 \text{ or } 0 \le x \le \frac{\pi}{4}$

$\Rightarrow -0.7857 \le x \le 0 \text{ or } 0 \le x \le 0.7857$

$\Rightarrow [x] = -1 \text{ or } [x] = 0$

$\Rightarrow \sin[x] = \sin(-1) \text{ or } \sin 0.$

$\Rightarrow \sin[x] = -\sin 1 \text{ or } 0.$

$\therefore R(f) = \{-\sin 1, 0\}.$

15. (*d*) We have, $\frac{|\sin x|}{\sec x}$

$f(x) = \frac{|\sin x|}{\sec x} = \pm \sin x \cos x$

$= \pm \frac{1}{2} \times 2 \sin x \cos x$

$= \pm \frac{1}{2} \sin 2x$

We know, sin 2x is periodic with $\frac{2\pi}{2} = \pi$ but

$\sec x \ne 0 \Rightarrow x \ne (2n+1)\frac{\pi}{2}$

Hence, $\frac{|\sin x|}{\sec x}$ is periodic with 2π

16. (*a*) Since an even function is a function for which f(–x) = f(x) for all x in its domain.

We have,

$f(x) = x\left(\frac{a^x - 1}{a^x + 1}\right)$

$\Rightarrow f(-x) = -x\left(\frac{a^{-x} - 1}{a^{-x} + 1}\right)$

$\Rightarrow f(-x) = -x\left(\frac{\frac{1}{a^x} - 1}{\frac{1}{a^x} + 1}\right)$

$\Rightarrow f(-x) = -x\left(\frac{1 - a^x}{1 + a^x}\right)$

$\Rightarrow f(-x) = x\left(\frac{a^x - 1}{a^x + 1}\right)$

$\Rightarrow f(-x) = f(x)$

So, f(x) is an even function.

17. (*c*) f(x) can take all real values for the value of x except x = 1

$\therefore$ Domain = R – {1}

$\therefore f(x) = (x^2 - 1)/(x - 1)$ does not exist for x = 1

$\therefore f(x) = ((x - 1)(x + 1))/(x - 1)$

$\Rightarrow f(x) = x + 1$

Since, f(x) doest not exist at x = 1

$\Rightarrow f(1) = 1 + 1 = 2$

Hence, Range can not take the value 2.

$\therefore$ Range = R – {2}

18. (*a*) We have,

$f(x)f(y) = f(xy)$ ...(i)

for all real x, y and f(2) = 4

Putting x = 2, y = 1 in eq(i),

$\Rightarrow f(2)f(1) = f(2)$

$\Rightarrow f(1) = 1$

Now, putting $x = \frac{1}{2}, y = 2$

$\Rightarrow f\left(\frac{1}{2}\right) f(2) = f(1)$

$\Rightarrow f\left(\frac{1}{2}\right) \times 4 = 1$

$\Rightarrow f\left(\frac{1}{2}\right) = \frac{1}{4}$

19. (*d*) Given,

$f = \{(x,y) : xy = x - y\} \Rightarrow f(xy) = x - y$

Put $x = 1, y = 1 \Rightarrow f(1 \cdot 1) = 1 - 1 = 0 \notin \{1,2,3,4\}$

Hence, here f(1) has no image so it is not a function.

20. (*a*) **Step 1: Bijective function:**

Given that $f:[1,\infty)\to[2,\infty]$ and $f(x)=x+\frac{1}{x}$

f is bijective,

For one-one

$f(x_1)=f(x_2)$

$\Rightarrow x_1+\frac{1}{x_1}=x_2+\frac{1}{x_2}$

$\Rightarrow x_1-x_2=\frac{x_1-x_2}{x_1\ x_2}$

$\Rightarrow x_1=x_2$

For onto

$x+\frac{1}{x}\ge 2\sqrt{x\times\frac{1}{x}}\ge 2$

So, range is equal to co-domain.

Step 2: Inverse function:

Therefore inverse exists.

$y=f(x)=x+\left(\frac{1}{x}\right)$

$\Rightarrow x^2-xy+1=0$

$\Rightarrow x=\frac{\{y\pm\sqrt{y^2-4}\}}{2}$

$\Rightarrow x=\frac{\{y+\sqrt{y^2-4}\}}{2},\frac{\{y-\sqrt{y^2-4}\}}{2}$

Since the values of x and y are positive.

As we know that value of x should be >1, so $\frac{\{y-\sqrt{y^2-4}\}}{4}$ is not valid

$\Rightarrow x=\frac{\{y+\sqrt{y^2-4}\}}{2}$

$\Rightarrow f^{-1}(x)=\frac{\{x+\sqrt{x^2-4}\}}{2}$

Hence, the value of $f^{-1}(x)$ is $\frac{\{x+\sqrt{x^2-4}\}}{2}$.

21. (*a*) We have, $g(x)=\ln(x)$ and $f(g(x))=e^x-e^{-x}$

$f(g(x))=f(\ln x)=e^x-e^{-x}$ ...(i)

Let $y=\ln x$, so $x=e^y$

Substituting $x=e^y$ in eq (i),

$f(y)=e^{e^y}-e^{-e^y}$

Thus, the required function is:

$f(x)=e^{e^x}-e^{-e^x}$

22. (*b*) We have, $f(xy)=f(x)+f(y)$

This equation is the defining property of logarithmic functions.

Let $f(x)=\log_a(x)$, where $a>0$ and $a\neq 1$

Then, $f(xy)=\log_a(xy)=\log_a(x)+\log_a(y)$

Therefore, $f(xy)=f(x)+f(y)$ is true for logarithmic functions.

And we know, the logarithmic function

$\log_a(x)$, $\log_a(y)$ is defined only for positive values of x, y.

23. (*a*) Given that,

$y=\frac{\sin 3x}{\sin x}$

$y=\frac{3\sin x-4\sin^3 x}{\sin x}$

$y=3-4\sin^2 x$

$\sin^2 x=\frac{3-y}{4}$

We know that,

$0\le\sin^2 x\le 1$

$0\le\frac{3-y}{4}\le 1$

$0\le 3-y\le 4$

$-3\le -y\le 1$

$-1\le y\le 3$

24. (*b*) We have, $f(x)=\sqrt{2-x}+\sqrt{1+x}\ \forall x\in[-1,2]$

Let $y=f(x)=\sqrt{2-x}+\sqrt{1+x}$

Squaring both sides,

$\Rightarrow y^2=2-x+1+x+2\sqrt{2-x}\sqrt{1+x}$

$\Rightarrow y^2=3+2\sqrt{(2-x)(1+x)}$

$\Rightarrow y^2=3+2\sqrt{-(x^2-x-2)}$

$\Rightarrow y^2=3+2\sqrt{-(x^2-x-2)}$

$\Rightarrow y^2=3+2\sqrt{\frac{9}{4}-\left(x-\frac{1}{2}\right)^2}$ ...(i)

$\because -\infty<x-\frac{1}{2}<\infty$

$\Rightarrow 0\le\left(x-\frac{1}{2}\right)^2<\infty$

$\Rightarrow -\infty<-\left(x-\frac{1}{2}\right)^2\le 0$

$\Rightarrow -\infty<\frac{9}{4}-\left(x-\frac{1}{2}\right)^2\le\frac{9}{4}$

$\Rightarrow 0\le\sqrt{\frac{9}{4}-\left(x-\frac{1}{2}\right)^2}\le\frac{3}{2}$

$$\Rightarrow 0 \le 2\sqrt{\frac{9}{4}-\left(x-\frac{1}{2}\right)^2} \le 3$$

$$\Rightarrow 3 \le 3+2\sqrt{\frac{9}{4}-\left(x-\frac{1}{2}\right)^2} \le 6$$

$$\Rightarrow 3 \le y^2 \le 6$$

$$\Rightarrow \sqrt{3} \le y \le \sqrt{6}$$

25. (*b*) To find $(g\circ f)\left(\frac{e-1}{e+1}\right) = g\left(f\left(\frac{e-1}{e+1}\right)\right)$

Firstly $f\left(\frac{e-1}{e+1}\right) = \log_e\left(\frac{1+\frac{e-1}{e+1}}{1-\frac{e-1}{e+1}}\right)$

$$f\left(\frac{e-1}{e+1}\right) = \log_e\left(\frac{\frac{e+1+e-1}{e+1}}{\frac{e+1-e+1}{e+1}}\right)$$

$$f\left(\frac{e-1}{e+1}\right) = \log_e(e) = 1$$

Now $g\left(f\left(\frac{e-1}{e+1}\right)\right) = g(1)$

$$\Rightarrow g(1) = \frac{3(1)+(1)^3}{1+3(1)^2}$$

$$\Rightarrow g(1) = 1$$

26. (*a*) Given that

$f(x) = \log\left(\frac{1+x}{1-x}\right)$.....(1) and $g(x) = \frac{3x+x^3}{1+3x^2}$

$\Rightarrow$ fog(x) = f(g(x))

$$= f\left(\frac{3x+x^3}{1+3x^2}\right)$$

$$= \log\left(\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\left(\frac{3x+x^3}{1+3x^2}\right)}\right)$$

$$\Rightarrow fog(x) = \log\left(\frac{1+3x^2+3x+x^3}{1+3x^2-3x-x^3}\right)$$

$$\Rightarrow fog(x) = \log\left(\frac{(1+x)^3}{(1-x)^3}\right)$$

$$\Rightarrow fog(x) = \log\left(\frac{1+x}{1-x}\right)^3$$

$$\Rightarrow f\ o\ g\ (x) = 3\log\left(\frac{1+x}{1-x}\right)$$

[$\log a^m = m \log a$]

$\Rightarrow$ kf(x) = 3f(x)

[Given (fog)(x) = kf(x) and using (1)

$\Rightarrow$ k = 3

27. (*a*) Given,

$$f(x) = x^2 + \frac{1}{(x^2+1)}$$

$$\Rightarrow f(x) = x^2 + \frac{1}{(x^2+1)} + 1 - 1$$

$$\Rightarrow f(x) = (x^2+1) + \frac{1}{(x^2+1)} - 1$$

Let the term $(x^2+1)+\frac{1}{(x^2+1)}$ be g(x)

For, g(x)

A.M $\ge$ G.M

$$\Rightarrow \frac{(x^2+1)+\frac{1}{(x^2+1)}}{2} \ge \sqrt{(x^2+1)\times\frac{1}{x^2+1}}$$

$$\Rightarrow \frac{(x^2+1)+\frac{1}{(x^2+1)}}{2} \ge 1$$

$$\Rightarrow (x^2+1)+\frac{1}{(x^2+1)} \ge 2$$

$\Rightarrow g(x) \ge 2$

But, f(x) = g(x) – 1

So, $f(x) \ge 1$

Hence, the range is $[1, \infty)$.

28. (*b*) Given: $f(x) = \log_{x^2} 5^2$

domain of f(x), $x^2 > 0$, $x^2 \ne 1$

$x \ne 1, -1$, $x \in R - \{1, -1, 0\}$

$$f(x) = \log_{x^2} 5^2 = \frac{2}{2}\log_{|x|} 5$$

$f(x) = \log_{|x|} 5$

$g(x) = \log_x 5$

Domain of g(x); $x > 0, x \ne 1$

f(x) = g(x) for $x \in (0,1) \cup (1, \infty)$

29. (*d*) Let $y = \frac{x^2-2x+4}{x^2+2x+4}$;

$\Rightarrow (y-1)x^2 + 2(y+1)x + 4(y-1) = 0$

Since x is real, hence $D \ge 0$;

$\Rightarrow 4(y+1)^2 - 16(y-1)^2 \ge 0$

$\Rightarrow (y+1)^2 - (2y-2)^2 \geq 0$

$\Rightarrow (3y-1)(-y+3) \geq 0$

$\Rightarrow (3y-1)(y-3) \leq 0$

$\Rightarrow y \in \left[\frac{1}{3}, 3\right]$

Hence, Range $= \left[\frac{1}{3}, 3\right]$

30. (*c*) Given,

Option (*a*):

$$f(x) = |x| = \begin{cases} -x, x < 0 \\ 0, x = 0 \\ x, x > 0 \end{cases}$$

$$g(x) = x\,\text{sgn}(x) = \begin{cases} x.(-1), x < 0 \\ x.0, x = 0 \\ x.x, x > 0 \end{cases} = \begin{cases} -x, x < 0 \\ 0, x = 0 \\ x, x > 0 \end{cases}$$

The functions are identical.

Option (*b*):

$$f(x) = \sqrt{x^2} = \begin{cases} -x, x < 0 \\ x, x \geq 0 \end{cases}$$

$$g(x) = |x| = \begin{cases} -x, x < 0 \\ x, x \geq 0 \end{cases}$$

Also, they are identical.

Option (*c*):

$f(x) = \sqrt{x^2 - 6x + 9}, g(x) = x - 3$

$f(x) = \sqrt{x^2 - 6x + 9} = \sqrt{x^2 - 2.3.x + 3^2} = \sqrt{(x-3)^2}$

$= |(x-3)|$

The functions are non identical.

Option (*d*):

$f(x) = [\{x\}]$

$\forall x \Rightarrow 0 \leq \{x\} < 1 \Rightarrow [\{x\}] = 0$

$g(x) = \{[x]\}$

$\forall x \Rightarrow x = \text{Integer} \Rightarrow \{[x]\} = 0$

They are identical.

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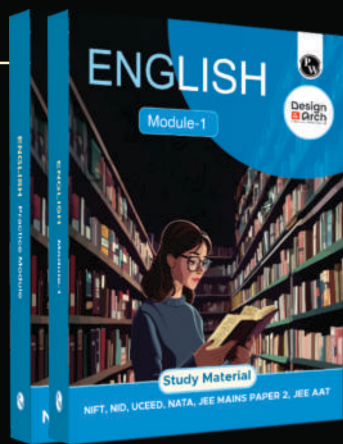

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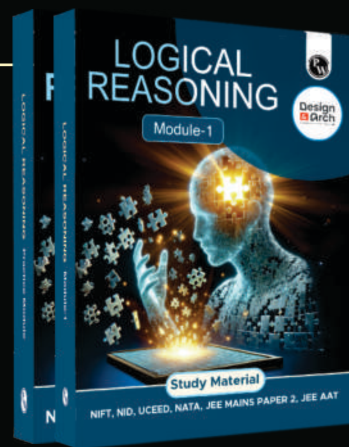

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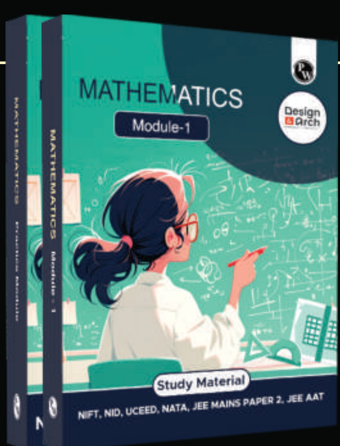

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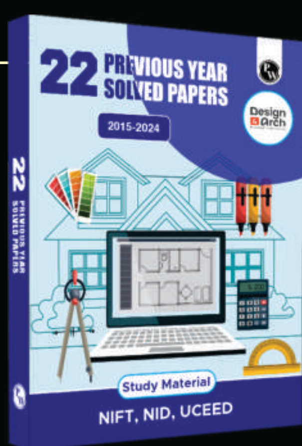

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