



# 47 JEE ADVANCED YEARS (2024 TO 1978)

## CHAPTER-WISE & TOPIC-WISE SOLVED PAPERS

### FIRST TIME EVER!

Analyse the question-wise difficulty level in real-time with Correct (C), Wrong (W) and Unattempted (UA) questions response tagging provided by IIT-JEE



# PHYSICS

ANSWER KEY VERIFIED FROM OFFICIAL WEBSITE OF JEE ADVANCED

# JEE ADVANCED 2024 Paper Analysis

## Paper 1

Section	Question Type	No. of Questions	Marks per Question	Negative marks per question	Total Marks (Section wise)
1	Single Correct MCQs	4	3	-1	12
2	More than One Correct MCQs	3	4	-2	12
3	Non Negative Integer Type Questions	6	4	0	24
4	Matrix Type Questions	4	3	-1	12
	Total Questions per subject	17			60

## Paper 2

Section	Question Type	No. of Questions	Marks per Question	Negative marks per question	Total Marks (Section wise)
1	Single Correct MCQs	4	3	-1	12
2	More than One Correct MCQs	3	4	-2	12
3	Non Negative Integer Type Questions	6	4	0	24
4	(Numerical Value Based Questions)	4	3	-1	12
	Total Questions per subject	17			60

Total No. of Questions in Part 1 = 51 Questions

Total No. of Questions in Part 2 = 51 Questions

Each Subject Carries = 60 Marks

Total Marks = 180

## Paper Analysis

Paper Difficulty Compared to Last Year	Easier Compared to 2023 Paper
Overall Difficulty Level of JEE 2024	
Easy	18%
Moderate	49%
Difficult	33%

Physics Topics	Paper 1 Difficulty Level	Paper 2 Difficulty Level
Mechanics	Total 3 questions in paper 1 from Mechanics out which 1 question from fluid mechanics was difficult 1 question from Rotational mechanics was easy and 1 moderate level question from rotational mechanics	Total 3 questions in paper 2 were asked from Mechanics out which all of them were moderate level
Thermodynamics	Total 2 questions were asked from thermodynamics in paper 1 one question was easy and the other was moderate	No questions asked in paper 2
Electrodynamics	3 Moderate level Questions	Total 6 questions were asked out of which 1 Question from EMI was difficult and rest were moderate level. Rest were moderate
Optics	3 moderate level question from optics out which 2 were from Ray optics and 1 was from wave optics	3 moderate level question in paper 2 out which 2 were from wave optics and 1 was from ray optics
Modern Physics	1 moderate level question	1 easy level question from x rays
Oscillations & waves	Total 3 moderate level question out of which 2 question from waves and 1 question from shm	1 Question from SHM was difficult
Mathematical tools, Errors & measurement	1 question from mathematical tools were asked in paper 1	1 question from error was asked in paper. Level of question was easy

## Overall Analysis

Paper Difficulty Compared to Last Year	
<ul style="list-style-type: none"> <li>- Paper was overall moderate</li> <li>- 6 Questions were easy, 3 Questions were Difficult and rest were moderate</li> <li>- Compared to the other topic, Electrodynamics has the maximum weightage</li> <li>- Weightage of 12th was slightly more compared to 11th topics</li> <li>- In paper 1, 8 out of 17 questions were from 11th topics and 9 out of 17 questions were from 12th</li> <li>- In paper 2, 7 out of 17 were from 11th and 10 out of 17 were from 12th.</li> </ul>	<ul style="list-style-type: none"> <li>- Total questions from mechanics - 6</li> <li>- Total questions from thermodynamics - 2</li> <li>- Total questions from Electrodynamics - 9</li> <li>- Total questions from Optics - 6</li> <li>- Total questions from Waves &amp; oscillations - 4</li> <li>- Total questions from mathematical tools, errors and measurement - 2</li> <li>- Questions were based on fundamental concepts</li> <li>- Language of questions was on tougher side</li> <li>- Questions were not lengthy.</li> </ul>



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# HINTS AND SOLUTIONS

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# 2024

# JEE ADVANCED SOLVED PAPER

## Physics Paper-1

### SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:  
*Full Marks* : +3 If **ONLY** the correct option is chosen;  
*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);  
*Negative Marks* : -1 In all other cases.

1. A dimensionless quantity is constructed in terms of electronic charge  $e$ , permittivity of free space  $\epsilon_0$ , Planck's constant  $h$ , and speed of light  $c$ . If the dimensionless quantity is written as  $e^\alpha \epsilon_0^\beta h^\gamma c^\delta$  and  $n$  is a non-zero integer, then  $(\alpha, \beta, \gamma, \delta)$  is given by

- (a)  $(2n, -n, -n, -n)$  (b)  $(n, -n, -2n, -n)$   
 (c)  $(n, -n, -n, -2n)$  (d)  $(2n, -n, -2n, -2n)$

2. An infinitely long wire, located on the  $z$ -axis, carries a current  $I$  along the  $+z$ -direction and produces the magnetic field  $\vec{B}$ . The magnitude of the line integral  $\int \vec{B} \cdot d\vec{l}$  along a straight line from the point  $(-\sqrt{3}a, a, 0)$  to  $(a, a, 0)$  is given by

[  $\mu_0$  is the magnetic permeability of free space.]

- (a)  $7\mu_0 I/24$  (b)  $7\mu_0 I/12$   
 (c)  $\mu_0 I/8$  (d)  $\mu_0 I/6$

3. Two beads, each with charge  $q$  and mass  $m$ , are on a horizontal, frictionless, non-conducting, circular hoop of radius  $R$ . One of the beads is glued to the hoop at some point, while the other one performs small oscillations about its equilibrium position along the hoop. The square of the angular frequency of the small oscillations is given by [ $\epsilon_0$  is the permittivity of free space.]

- (a)  $q^2 / (4\pi\epsilon_0 R^3 m)$  (b)  $q^2 / (32\pi\epsilon_0 R^3 m)$   
 (c)  $q^2 / (8\pi\epsilon_0 R^3 m)$  (d)  $q^2 / (16\pi\epsilon_0 R^3 m)$

4. A block of mass 5 kg moves along the  $x$ -direction subject to the force  $F = (-20x + 10)\text{N}$ , with the value of  $x$  in metre. At time  $t = 0\text{s}$ , it is at rest at position  $x = 1\text{ m}$ . The position and momentum of the block at  $t = (\pi/4)\text{s}$  are

- (a)  $-0.5\text{ m}, 5\text{ kg m/s}$  (b)  $0.5\text{ m}, 0\text{ kg m/s}$   
 (c)  $0.5\text{ m}, -5\text{ kg m/s}$  (d)  $-1\text{ m}, 5\text{ kg m/s}$

### SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:  
*Full Marks* : +4 **ONLY** if (all) the correct option(s) is(are) chosen;  
*Partial Marks* : +3 If all the four options are correct but **ONLY** three options are chosen;  
*Partial Marks* : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;  
*Partial Marks* : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option  
*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);  
*Negative Marks*: -2 In all other cases.
- For example, in a question, if (a), (b) and (d) are the **ONLY** three options corresponding to correct answers, then  
 choosing **ONLY** (a), (b) and (d) will get +4 marks;  
 choosing **ONLY** (a) and (b) will get +2 marks;  
 choosing **ONLY** (a) and (d) will get +2 marks;  
 choosing **ONLY** (b) and (d) will get +2 marks;  
 choosing **ONLY** (a) will get +1 mark;  
 choosing **ONLY** (b) will get +1 mark;  
 choosing **ONLY** (d) will get +1 mark;  
 choosing no option (i.e. the question is unanswered) will get 0 marks; and choosing any other combination of options will get -2 marks.

5. A particle of mass  $m$  is moving in a circular orbit under the influence of the central force  $F(r) = -kr$ , corresponding to the potential energy  $V(r) = kr^2/2$ , where  $k$  is a positive force constant and  $r$  is the radial distance from the origin. According to the Bohr's quantization rule, the angular momentum of the particle is given by  $L = n\hbar$ , where  $\hbar = h/(2\pi)$ ,  $h$  is the Planck's constant, and  $n$  a positive integer. If  $v$  and  $E$  are the speed and total energy of the particle, respectively, then which of the following expression(s) is(are) correct?

- (a)  $r^2 = n\hbar \sqrt{\frac{1}{mk}}$  (b)  $v^2 = n\hbar \sqrt{\frac{k}{m^3}}$   
 (c)  $\frac{L}{mr^2} = \sqrt{\frac{k}{m}}$  (d)  $E = \frac{n\hbar}{2} \sqrt{\frac{k}{m}}$

## JEE-Advanced

## UNITS, SYSTEM OF UNITS

## Single Correct

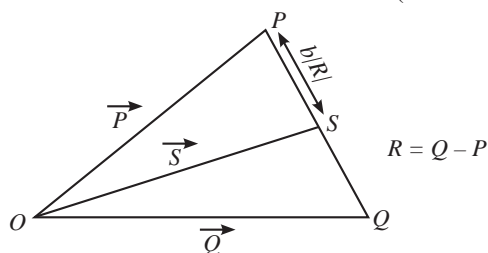
1. Consider an expanding sphere of instantaneous radius  $R$  whose total mass remains constant. The expansion is such that the instantaneous density  $\rho$  remains uniform throughout the volume. The rate of fractional change in density  $\left(\frac{1}{\rho} \frac{d\rho}{dt}\right)$  is constant. The velocity  $v$  of any point of the surface of the expanding sphere is proportional to

C-26.78 W-30.86 UA-42.37 (JEE Adv. 2017)

- (a)  $R$  (b)  $\frac{1}{R}$  (c)  $R^3$  (d)  $R^{\frac{2}{3}}$

2. Three vectors  $P$ ,  $Q$  and  $R$  are shown in the figure. Let  $S$  be any point on the vector  $R$ . The distance between the points  $P$  and  $S$  is  $b|R|$ . The general relation among vectors  $P$ ,  $Q$  and  $S$  is

C-46.13 W-17.08 UA-36.79 (JEE Adv. 2017)



- (a)  $S = (1 - b^2)P + bQ$  (b)  $S = (b - 1)P + bQ$   
 (c)  $S = (1 - b)P + bQ$  (d)  $S = (1 - b)P + b^2Q$

3. A cube has a side of length  $1.2 \times 10^{-2}$  m. Calculate its volume.

(IIT-JEE 2003)

- (a)  $1.7 \times 10^{-6} \text{ m}^3$  (b)  $1.73 \times 10^{-6} \text{ m}^3$   
 (c)  $1.70 \times 10^{-6} \text{ m}^3$  (d)  $1.732 \times 10^{-6} \text{ m}^3$

## Subjective

4. Write the dimensions of the following in terms of mass, time, length and charge. (IIT-JEE 1982)

A. Magnetic flux B. Rigidity modulus

5. Give the MKS units for each of the following quantities.

(IIT-JEE 1980)

1. Young's modulus 2. Magnetic induction  
 3. Power of a lens

## Multiple Correct

6. A physical quantity  $\vec{S}$  is defined as  $\vec{S} = (\vec{E} \times \vec{B}) / \mu_0$ , where  $\vec{E}$  is electric field,  $\vec{B}$  is magnetic field and  $\mu_0$  is the permeability of free space. The dimensions of  $\vec{S}$  are the same as the dimensions of which of the following quantity (ies)?

C-40.2 W-27.78 UA-22.17 PC-9.84 (JEE Adv. 2021)

- (a)  $\frac{\text{Energy}}{\text{Charge} \times \text{Current}}$  (b)  $\frac{\text{Force}}{\text{Length} \times \text{Time}}$   
 (c)  $\frac{\text{Energy}}{\text{Volume}}$  (d)  $\frac{\text{Power}}{\text{Area}}$

7. The SI unit of the inductance, the henry can be written as

(IIT-JEE 1998)

- (a) weber/ampere (b) volt-second/ampere  
 (c) joule/(ampere)<sup>2</sup> (d) ohm-second

## Match the Column

8. Some physical quantities are given in Column-I and some possible SI units in which these quantities may be expressed are given in Column-II. Match the physical quantities in Column-I with the units in Column-II. (IIT-JEE 2007)

Column-I		Column-II	
A.	$GM_e M_s G$ - universal gravitational constant, $M_e$ - mass of the earth, $M_s$ - mass of the sun,	p.	(volt) (coulomb) (meter)
B.	$\frac{3RT}{M}$ $R$ - universal gas constant, $T$ - absolute temperature, $M$ - molar mass,	q.	(kilogram) (meter) <sup>3</sup> (second) <sup>-2</sup>
C.	$\frac{F^2}{q^2 B^2}$ $F$ - force, $q$ - charge, $B$ - magnetic field,	r.	(meter) <sup>2</sup> (second) <sup>-2</sup>
D.	$\frac{GM_e}{R_e}$ $G$ - universal gravitational constant, $M_e$ - mass of the earth, $R_e$ - radius of the earth,	s.	(farad) (volt) <sup>2</sup> (kg) <sup>-1</sup>



9. Column-I gives three physical quantities. Select the appropriate units for the choices given in Column-II. Some of the physical quantities may have more than one choice. (IIT-JEE 1990)

Column-I	Column-II
Capacitance	ohm-second
Inductance	Coulomb <sup>2</sup> -joule <sup>-1</sup>
Magnetic induction	Coulomb (volt) <sup>-1</sup>
	Newton (ampere metre) <sup>-1</sup>
	volt-second (ampere) <sup>-1</sup>

### Fill in the Blanks

10. Two vectors  $A$  and  $B$  are defined as  $A = a\hat{i}$  and  $B = a(\cos \omega t \hat{i} + \sin \omega t \hat{j})$ , where  $a$  is a constant and  $\omega = \pi/6 \text{ rad s}^{-1}$ . If  $|A + B| = \sqrt{3}|A - B|$  at time  $t = \tau$  for the first time, the value of  $\tau$ , in seconds, is ..... C-35.57 W-49.68 UA-14.75 (JEE Adv.2018)

## DIMENSION, FINDING DIMENSIONAL FORMULA

### Single Correct

11. Which of the following sets have different dimensions? (IIT-JEE 2005)
- Pressure, Young's modulus, Stress
  - Emf, Potential difference, Electric potential
  - Heat, Work done, Energy
  - Dipole moment, Electric flux, Electric field
12. In the relation,  $p = \frac{\alpha}{\beta} e^{-\frac{az}{k\theta}}$  is pressure,  $z$  is distance,  $k$  is Boltzmann's constant and  $\theta$  is the temperature. The dimensional formula of  $\beta$  will be (IIT-JEE 2004)
- $[M^0L^2T^0]$
  - $[ML^2T]$
  - $[ML^0T^{-1}]$
  - $[M^0L^2T^{-1}]$
13. A quantity  $X$  is given by  $\epsilon_0 L \frac{\Delta V}{\Delta t}$ , where  $\epsilon_0$  is the permittivity of free space,  $L$  is a length,  $\Delta V$  is a potential difference and  $\Delta t$  is a time interval. The dimensional formula for  $X$  is the same as that of (IIT-JEE 2001)
- Resistance
  - Charge
  - Voltage
  - Current
14. The dimensions of  $\frac{1}{2}\epsilon_0 E^2$  ( $\epsilon_0$ : permittivity of free space;  $E$ : electric field) is: (IIT-JEE 2001)
- $[MLT]$
  - $[ML^2T^{-2}]$
  - $[ML^{-1}T^{-2}]$
  - $[ML^2T^{-1}]$
15. Let  $[\epsilon_0]$  denote the dimensional formula of the permittivity of vacuum. If  $M = \text{mass}$ ,  $L = \text{length}$ ,  $T = \text{Time}$  and  $A = \text{electric current}$ , then (IIT-JEE 1998)
- $[\epsilon_0] = [M^{-1}L^{-3}T^2A]$
  - $[\epsilon_0] = [M^{-1}L^{-3}T^4A^2]$
  - $[\epsilon_0] = [M^{-2}L^2T^{-1}A^{-2}]$
  - $[\epsilon_0] = [M^{-1}L^2T^{-1}A^2]$

### Multiple Correct

16. A length-scale ( $l$ ) depends on the permittivity ( $\epsilon$ ) of a dielectric material, Boltzmann's constant ( $k_B$ ), the absolute temperature ( $T$ ), the number per unit volume ( $n$ ) of certain charged particles and the charge ( $q$ ) carried by each of the particles. Which of the following expression(s) for  $l$  is (are) dimensionally correct?

C-17.85 W-20.89 UA-49.13 PC-12.13 (JEE Adv. 2016)

$$(a) l = \sqrt{\left(\frac{nq^2}{\epsilon k_B T}\right)} \quad (b) l = \sqrt{\left(\frac{Ek_B T}{nq^2}\right)}$$

$$(c) l = \sqrt{\left(\frac{q^2}{\epsilon h^{2/3} k_B T}\right)} \quad (d) l = \sqrt{\left(\frac{q^2}{\epsilon n^{1/3} k_B T}\right)}$$

17. Planck's constant  $h$ , speed of light  $c$  and gravitational constant  $G$  are used to form a unit of length  $L$  and a unit of mass  $M$ . Then, the correct options is/are (JEE Adv. 2015)
- $M \propto \sqrt{c}$
  - $M \propto \sqrt{G}$
  - $L \propto \sqrt{h}$
  - $L \propto \sqrt{G}$
18. In terms of potential difference  $V$ , electric current  $I$ , permittivity  $\epsilon_0$ , permeability  $\mu_0$  and speed of light  $c$  the dimensionally correct equations is/are (JEE Adv. 2015)
- $\mu_0 I^2 = E_0 V^2$
  - $\epsilon_0 I = \mu_0 V$
  - $I = E_0 c V$
  - $\mu_0 c I = \epsilon_0 V$
19. Let  $[\epsilon_0]$  denote the dimensional formula of the permittivity of the vacuum and  $[\mu_0]$  that of the permeability of the vacuum. If  $M = \text{mass}$ ,  $L = \text{length}$ ,  $T = \text{time}$  and  $I = \text{electric current}$ . Then, (IIT-JEE 1998)
- $[\epsilon_0] = [M^{-1}L^{-3}T^2A]$
  - $[\epsilon_0] = [M^{-1}L^{-3}T^4A^2]$
  - $[\mu_0] = [MLT^{-2}A^{-2}]$
  - $[\mu_0] = [ML^2T^{-1}I]$
20. The pairs of physical quantities that have the same dimensions is (are) (IIT-JEE 1995)
- Reynolds number and coefficient of friction
  - Curie and frequency of a light wave
  - Latent heat and gravitational potential
  - Planck's constant and torque
21. The dimensions of the quantities in one (or more) of the following pairs are the same. Identify the pair (s). (IIT-JEE 1986)
- Torque and work
  - Angular momentum and work
  - Energy and Young's modulus
  - Light year and wavelength
22.  $L$ ,  $C$  and  $R$  represent the physical quantities inductance, capacitance and resistance, respectively. The combinations which have the dimensions of frequency are (IIT-JEE 1984)
- $\frac{1}{RC}$
  - $\frac{R}{L}$
  - $\frac{1}{\sqrt{LC}}$
  - $\frac{C}{L}$

### Match the Column

23. Match the physical quantities given in Column-I with dimensions expressed in terms of mass ( $M$ ), length ( $L$ ), time ( $T$ ) and charge ( $Q$ ) given in Column-II and write the correct answer against the matched quantity in a tabular form in your answer book. (IIT-JEE 1990)

Column-I	Column-II
A. Angular momentum	p. $[ML^2T^{-2}]$
B. Latent heat	q. $[ML^2Q^{-2}]$
C. Torque	r. $[ML^2T^{-1}]$
D. Capacitance	s. $[ML^3T^{-1}Q^{-2}]$
E. Inductance	t. $[M^{-1}L^{-2}T^2Q^2]$
F. Resistivity	u. $[L^2T^{-2}]$

### Subjective

24. Planck's constant has dimensions: (IIT-JEE 1985)

## ERRORS IN MEASUREMENT

### Single Correct

36. A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is  $\delta T = 0.01$  s and he measures the depth of the well to be  $L = 20$  m. Take the acceleration due to gravity  $g = 10 \text{ ms}^{-2}$  and the velocity of sound is  $300 \text{ ms}^{-1}$ . Then the fractional error in the measurement

$\frac{\delta L}{L}$  is closest to **C-17.92 W-40 UA-42.08 (JEE Adv. 2017)**

- (a) 1% (b) 5% (c) 3% (d) 0.2%

37. The current voltage relation of diode is given by  $I = (e^{1000V/T} - 1) \text{ mA}$ , where the applied voltage  $V$  is in volt and the temperature  $T$  is in kelvin. If a student makes an error measuring  $\pm 0.01 \text{ V}$  while measuring the current of  $5 \text{ mA}$  at  $300 \text{ K}$ , what will be the error in the value of current in mA? **(JEE Adv. 2014)**

- (a) 0.2 mA (b) 0.02 mA (c) 0.5 mA (d) 0.05 mA

38. In the determination of Young's modulus  $\left(Y = \frac{4MLg}{\pi d^2 l}\right)$  by using Searle's method, a wire of length  $L = 2 \text{ m}$  and diameter  $d = 0.5 \text{ mm}$  is used. For a load  $M = 2.5 \text{ kg}$ , an extension  $l = 0.25 \text{ mm}$  in the length of the wire is observed. Quantities  $d$  and  $l$  are measured using a screw gauge and a micrometer, respectively. They have the same pitch of  $0.5 \text{ mm}$ . The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the  $Y$  measurement is **C-10.96 W-38.26 UA-50.78 (IIT-JEE 2012)**

- (a) Due to the errors in the measurements of  $d$  and  $l$  are the same  
(b) Due to the error in the measurement of  $d$  is twice that due to the error in the measurement of  $l$   
(c) Due to the error in the measurement of  $l$  is twice that due to the error in the measurement of  $d$   
(d) Due to the error in the measurement of  $d$  is four times that due to the error in the measurement of  $l$

39. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is  $0.5 \text{ mm}$  and there are 50 divisions on the circular scale. The reading on the main scale is  $2.5 \text{ mm}$  and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is **(IIT-JEE 2011)**

- (a) 0.9% (b) 2.4% (c) 3.1% (d) 4.4%

40. A student uses a simple pendulum of exactly  $1 \text{ m}$  length to determine  $g$ , the acceleration due to gravity. He uses a stop watch with the least count of  $1 \text{ s}$  for this and records  $40 \text{ s}$  for 20 oscillations. For this observation, which of the following statement(s) is/are true? **(IIT-JEE 2010)**

- (a) Error  $\Delta T$  in measuring  $T$ , the time period, is  $0.05 \text{ s}$   
(b) Error  $\Delta T$  in measuring  $T$ , the time period, is  $1 \text{ s}$   
(c) Percentage error in the determination of  $g$  is 5%  
(d) Percentage error in the determination of  $g$  is 2.5%

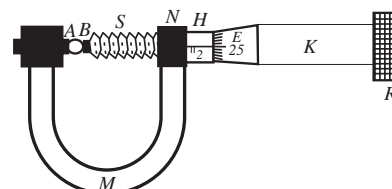
41. A student performs an experiment to determine the Young's modulus of a wire, exactly  $2 \text{ m}$  long, by Searle's method. In a particular reading,

the student measures the extension in the length of the wire to be  $0.8 \text{ mm}$  with an uncertainty of  $\pm 0.05 \text{ mm}$  at a load of exactly  $1.0 \text{ kg}$ . The student also measures the diameter of the wire to be  $0.4 \text{ mm}$  with an uncertainty of  $\pm 0.01 \text{ mm}$ . Take  $g = 9.8 \text{ m/s}^2$  (exact). The Young's modulus obtained from the reading is close to

**(IIT-JEE 2007)**

- (a)  $(2.0 \pm 0.3) \times 10^{11} \text{ N/m}^2$  (b)  $(2.0 \pm 0.2) \times 10^{11} \text{ N/m}^2$   
(c)  $(2.0 \pm 0.1) \times 10^{11} \text{ N/m}^2$  (d)  $(2.0 \pm 0.05) \times 10^{11} \text{ N/m}^2$

42. The circular scale of a screw gauge has 50 divisions and pitch of  $0.5 \text{ mm}$ . Find the diameter of the sphere. Main scale reading is 2. **(IIT-JEE 2006)**



- (a)  $1.2 \text{ mm}$  (b)  $1.25 \text{ mm}$  (c)  $2.20 \text{ mm}$  (d)  $2.25 \text{ mm}$

43. A wire has a mass  $(0.3 \pm 0.003) \text{ g}$ , radius  $(0.5 \pm 0.005) \text{ mm}$  and length  $(6 \pm 0.06) \text{ cm}$ . The maximum percentage error in the measurement of its density is **(IIT-JEE 2004)**

- (a) 1 (b) 2 (c) 3 (d) 4

### Comprehension Based/Passage Based

**Paragraph (Q. 44-45):** If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation  $z = x/y$ . If the errors in  $x$ ,  $y$  and  $z$  are  $\Delta x$ ,  $\Delta y$  and  $\Delta z$  respectively, then

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y} \left( 1 \pm \frac{\Delta x}{x} \right) \left( 1 \pm \frac{\Delta y}{y} \right)^{-1}$$

The series expansion for  $\left( 1 \pm \frac{\Delta y}{y} \right)^{-1}$ , to first power in  $\Delta y/y$ , is  $1 \mp (\Delta y/y)$ .

The relative errors in independent variables are always added. So the error in  $z$  will be  $\Delta z = z \left( \frac{\Delta x}{x} + \frac{\Delta y}{y} \right)$ . The above derivation makes the assumption that  $\Delta x/x \ll 1$ ,  $\Delta y/y \ll 1$ . Therefore, the higher powers of these quantities are neglected. **(JEE Adv. 2018)**

44. Consider the ratio  $r = \frac{(1-a)}{(1+a)}$  to be determined by measuring a dimensionless quantity  $a$ . If the error in the measurement of  $a$  is  $\Delta a$  ( $\Delta a/a \ll 1$ ), then what is the error  $\Delta r$  in determining  $r$ ? **C-34.7 W-33.81 UA-31.49 (JEE Adv. 2018)**

- (a)  $\frac{\Delta a}{(1+a)^2}$  (b)  $\frac{-2\Delta a}{(1+a)^2}$  (c)  $\frac{2\Delta a}{(1-a)^2}$  (d)  $\frac{2a\Delta a}{(1-a^2)}$

45. In an experiment, the initial number of radioactive nuclei is 3000. It is found that  $1000 \pm 40$  nuclei decayed in the first 10s. For  $|x| < 1$ ,  $\ln(1+x) = x$  up to first power in  $x$ . The error  $\Delta \lambda$ , in the determination of the decay constant  $\lambda$  in  $\text{s}^{-1}$ , is **C-13.32 W-38.29 UA-48.39 (JEE Adv. 2018)**

- (a) 0.04 (b) 0.03  
(c) 0.02 (d) 0.01



## Subjective

46. The energy of a system as a function of time  $t$  is given as  $E(t) = A^2 \exp(-\alpha t)$ , where  $\alpha = 0.2 \text{ s}^{-1}$ . The measurement of  $A$  has an error of 1.25%. If the error in the measurement of time is 1.50%, the percentage error in the value of  $E(t)$  at  $t = 5 \text{ s}$  is

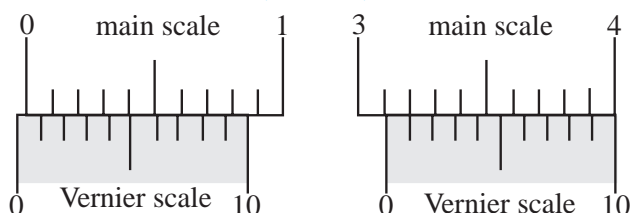
(JEE Adv. 2015)

## MEASURING INSTRUMENTS

### Single Correct

47. The smallest division on the main scale of a Vernier calipers is 0.1 cm. Ten divisions of the Vernier scale correspond to nine divisions of the main scale. The figure below on the left shows the reading of this caliper with no gap between its two jaws. The figure on the right shows the reading with a solid sphere held between the jaws. The correct diameter of the sphere is

C-17.89, W-46.36, UA-35.75 (JEE Adv. 2021)



- (a) 3.07 cm (b) 3.11 cm  
(c) 3.15 cm (d) 3.17 cm

48. A steel wire of diameter 0.5 mm and Young's modulus  $2 \times 10^{11} \text{ Nm}^{-2}$  carries a load of mass  $m$ . The length of the wire with the load is 1.0 m. A vernier scale with 10 divisions is attached to the end of this wire. Next to the steel wire is a reference wire to which a main scale, of least count 1.0 mm, is attached. The 10 divisions of the vernier scale correspond to 9 divisions of the main scale. Initially, the zero of vernier scale coincides with the zero of main scale. If the load on the steel wire is increased by 1.2 kg, the vernier scale division which coincides with a main scale division is ( $g = 10 \text{ ms}^{-2}$  and  $\pi = 3.2$ ).

C-10.09 W-63.21 UA-26.7 (JEE Adv. 2018)

- (a) 1 (b) 3  
(c) 2 (d) 4

49. There are two vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The vernier scale of one of the calipers ( $C_1$ ) has 10 equal divisions that correspond to 9 main scale divisions. The vernier scale of the other caliper ( $C_2$ ) has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers  $C_1$  and  $C_2$  respectively, are

C-28.16 W-43.44 UA-28.4 (JEE Adv. 2016)

- (a) 2.87 and 2.87 (b) 2.87 and 2.83  
(c) 2.85 and 2.82 (d) 2.87 and 2.86

50. The diameter of a cylinder is measured using a vernier calipers with no zero error. It is found that the zero of the vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The vernier scale has 50 division equivalent to 2.45 cm. The 24<sup>th</sup> division of the vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is

C-36.72 W-54.07 UA-9.21 (JEE Adv. 2013)

- (a) 5.112 cm (b) 5.124 cm  
(c) 5.136 cm (d) 5.148 cm

51. A vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the vernier scale which match with 16 main scale divisions. For this vernier calipers, the least count is

(IIT-JEE 2010)

- (a) 0.02 mm (b) 0.05 mm  
(c) 0.1 mm (d) 0.2 mm

52. Students I, II and III perform an experiment for measuring the acceleration due to gravity ( $g$ ) using a simple pendulum. They use different lengths of the pendulum and/or record time for different number of oscillations. The observations are shown in the table. Least count for length = 0.1 cm. Least count for time = 0.1 s

Student	Length of the pendulum	Number of for ( $n$ ) Oscillations ( $n$ )	Total time period Oscillation (cm)	Time(s)
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If  $E_I$ ,  $E_{II}$  and  $E_{III}$  are the percentage errors in  $g$  i.e.,  $\left(\frac{\Delta_g}{g} \times 100\right)$  for

students I, II and III, respectively

(IIT-JEE 2008)

- (a)  $E_I = 0$  (b)  $E_I$  is minimum  
(c)  $E_I = E_2$  (d)  $E_3$  is maximum

### Multiple Correct

53. Consider a vernier caliper in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the vernier calipers, 5 divisions of the vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then

(JEE Adv. 2015)

- (a) If the pitch of the screw gauge is twice the least count of the vernier caliper, the least count of the screw gauge is 0.01 mm  
(b) If the pitch of the screw gauge is twice the least count of the Vernier caliper, the least count of the screw gauge is 0.05 mm  
(c) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier calipers, the least count of the screw gauge is 0.01 mm  
(d) If the least count of the linear scale of the screw gauge is twice the least count of the vernier caliper, the least count of the screw gauge is 0.005 mm

### Numerical Types/Integer Types

54. During Searle's experiment, zero of the vernier scale lies between  $3.20 \times 10^{-2} \text{ m}$  and  $3.25 \times 10^{-2} \text{ m}$  of the main scale. The 20<sup>th</sup> division of the vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the vernier scale still lies between  $3.20 \times 10^{-2} \text{ m}$  and  $3.25 \times 10^{-2} \text{ m}$  of the main scale but now the 45<sup>th</sup> division of vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is  $8 \times 10^{-7} \text{ m}^2$ . The least count of the vernier scale is  $1.0 \times 10^{-5} \text{ m}$ . The maximum percentage error in the Young's modulus of the wire is

C-16.38 W-67.42 UA-16.2 (JEE Adv. 2014)

55. The edge of a cube is measured using a vernier caliper. (9 divisions of the main scale is equal to 10 divisions of vernier scale and 1 main scale division is 1 mm). The main scale division reading is 10 and 1 division of vernier scale was found to be coinciding with the main scale. The mass of the cube is 2.736 g. Calculate the density in  $\text{g/cm}^3$  upto correct significant figures. (IIT-JEE 2005)
56. The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. While measuring the diameter of a wire, the linear scale reads 1 mm and 47<sup>th</sup> division on the circular scale coincides with the reference line. The length of the wire is 5.6 cm. Find the curved surface area (in  $\text{cm}^2$ ) of the wire in appropriate number of significant figures (IIT-JEE 2004)

### Subjective

57.  $N$  divisions on the main scale of a vernier calipers coincide with  $(N+1)$  divisions on the vernier scale. If each division on the main scale is of  $a$  units, determine the least count of instruments. (IIT-JEE 2003)

## FUNCTION, DIFFERENTIATION AS A RATE MEASUREMENT

### Numerical Types/Integer Types

58. A person of height 1.6 m is walking away from a lamp post of height 4 m along a straight path on the flat ground. The lamp post and the person are always perpendicular to the ground. If the speed of the person is  $60 \text{ cm s}^{-1}$ , the speed of the tip of the person's shadow on the ground with respect to the person is \_\_\_\_\_  $\text{cm s}^{-1}$ .

C-12.69 W-65.37 UA-21.94 (JEE Adv. 2023)

## ANSWER KEY

- |                                              |           |           |               |            |                            |               |             |               |
|----------------------------------------------|-----------|-----------|---------------|------------|----------------------------|---------------|-------------|---------------|
| 1. (a)                                       | 2. (c)    | 3. (a)    | 6. (b,d)      | 7. (a,c,d) | 8. A-p,q B-r,s C-r,s D-r,s | 11. (d)       | 12. (a)     | 13. (d)       |
| 14. (c)                                      | 15. (b)   | 16. (b,d) | 17. (a, c, d) | 18. (a,c)  | 19. (b,c)                  | 20. (a, b, c) | 21. (a, d)  | 22. (a, b, c) |
| 23. A-(r), B-(u), C-(p), D-(t), E-(q), F-(s) | 27. (a)   | 28. (c)   | 29. (d)       | 30. (c)    | 31. (b)                    | 32. (a, b)    | 33. (a,b,d) |               |
| 34. [3]                                      | 35. [4]   | 36. (a)   | 37. (a)       | 38. (a)    | 39. (c)                    | 40. (a,c)     | 41. (b)     | 42. (d)       |
| 44. (b)                                      | 45. (c)   | 47. (c)   | 48. (b)       | 49. (b)    | 50. (b)                    | 51. (d)       | 52. (d)     | 53. (b,c)     |
| 55. [2.66]                                   | 56. [2.6] | 58. [40]  |               |            |                            |               |             | 54. [4]       |

1. (a) The velocity  $v$  any point of the surface of the expanding sphere is proportional to  $R$ .

The density is given as

$$\rho = \frac{\text{Mass}}{\text{Volume}}$$

Mass =  $\rho \times \text{volume} = \text{constant}$

On differentiating,

$$\Rightarrow V \frac{d\rho}{dt} + \rho \frac{dV}{dt} = 0$$

$$\Rightarrow \frac{4}{3}\pi R^3 \times \frac{d\rho}{dt} + \rho \times \frac{d}{dt}\left(\frac{4}{3}\pi R^3\right) = 0$$

$$\Rightarrow \frac{1}{\rho} \frac{d\rho}{dt} = -\frac{3}{R} \frac{dR}{dt}$$

$$\Rightarrow R^3(-3R^{-4}) \frac{dR}{dt} = \text{constant}$$

$$\frac{dR}{dt} \propto R$$

2. (c) The general relation among vectors  $P$ ,  $Q$  and  $S$  is  $S = (1-b)P + bQ$ .

From triangular law of vector addition, we get  $\vec{OP} + \vec{PS} = \vec{OS}$

$$\therefore \vec{P} + b\left|\vec{R}\right|\frac{\vec{R}}{\left|\vec{R}\right|} = \vec{S}$$

$$\vec{P} + b\vec{R} = \vec{S}$$

But  $\vec{R} = \vec{Q} - \vec{P}$  (Given)

$$\vec{P} + b(\vec{Q} - \vec{P}) = \vec{S}$$

$$\Rightarrow \vec{S} = (1-b)\vec{P} + b\vec{Q}$$

3. (a) **Given:** Length of the cube,

$$L = 1.2 \times 10^{-2} \text{ m}$$

We know, Volume of the cube,

$$V = L^3 = (1.2 \times 10^{-2}) \times (1.2 \times 10^{-2}) \times (1.2 \times 10^{-2}) \text{ m}^3$$

$$\Rightarrow V = 1.728 \times 10^{-6} \text{ m}^3$$

But as the length contains only 2 significant digits, the volume must also contain 2 significant digits. Hence, volume of the cube is  $1.7 \times 10^{-6} \text{ m}^3$ .

4. (A) The dimensions of the magnetic flux and rigidity modulus in terms of mass, length, time and charge are  $[\phi] = [M^1 L^2 T^{-1} Q^{-1}]$  and  $[n] = ML^{-1} T^{-2}$ . Dimension of magnetic field,  $[B] = [M^1 L^0 Q^{-1}]$  and Dimension of surface Area,  $[A] = [L^2]$ .

The magnetic flux is given as the product of the magnetic field and surface area;  $\phi = BA$ .

The dimension of magnetic flux in term of mass, time, length and charge,

$$[\phi] = [M^1 L^2 T^{-1} Q^{-1}]$$

- (B) Shear stress =  $\eta$  (Shear strain),

where,  $\eta$  = modulus of rigidity

According to the formula, we get the dimensions of ' $\eta$ ' same as that of pressure as of shear strain is dimensionless.

$$\therefore [\eta] = ML^{-1} T^{-2}$$

5.

1. Young's modulus =  $Nm^{-2}$  = Newton per meter<sup>2</sup>
2. Magnetic induction =  $W \cdot m^{-2}$  = Tesla
3. Power of lens =  $m^{-1}$  = diopter

6. (b,d)

$$\vec{S} = [\vec{E} \times \vec{B}] \frac{1}{\mu_0}$$

$\vec{S}$  is pointing vector denotes flow of energy per unit area per unit time

$$\vec{S} = \frac{\text{watt}}{m^2}$$

7. (a,c,d)

Using the formula of emf;

$$e = \frac{d\phi}{dt} = L \frac{dI}{dt}$$

$$\Rightarrow [\text{volt}] = \frac{[\text{weber}]}{[\text{second}]} = [L] \frac{[\text{ampere}]}{[\text{second}]}$$

And Energy dissipated ( $E$ ) =  $VI t$

Putting the units of all the above values,

$$\Rightarrow [\text{Joule}] = [\text{volt}][\text{Ampere}][\text{second}]$$

$$\Rightarrow [\text{volt-second}] = \frac{[\text{Joule}]}{[\text{Ampere}]}$$

Thus, we can now evaluate the unit of self inductance, units of self inductance  $L$  is,

$$[L] = \frac{\text{weber}}{\text{Ampere}} = \frac{\text{volt-second}}{\text{Ampere}} = \frac{\text{Joule}}{\text{Ampere}^2}$$

$d$  also correct

$$\frac{1}{2} Li^2 = i^2 Rt \Rightarrow [L] = [Rt] = \text{Ohm-second}$$

8. A-p,q B-r,s C-r,s D-r,s

Given:  $G$  is the universal gravitational constant,  $R$  is the universal gas constant,  $T$  is the absolute temperature and  $B$  is the magnetic field.

(A)  $GM_e M_x$  where,  $M_e$  is the mass of the earth and  $M_x$  is the mass of the Sun. From the Gravitational force formula,  $GM_e M_x = Fr^2$

Taking the units of each value and deriving its dimensions:

SI unit of Force is (kilogram)(metre) $s^{-2}$ .

SI unit of  $r^2$  is  $m^2$ .

$$= Nm^2 kg \frac{m}{s^2} \times m^2 = kgm^3 s^{-2}$$

Also, (volt)(coulomb)(meter) has the same dimensions.

(Joule)(meter)

$$= kg \left(\frac{m}{s^2}\right) \cdot m \cdot m \Rightarrow \frac{dn}{N} = -td$$

$$= \left(1 - \frac{9}{10}\right) \times 1mm = kg m^3 s^{-2}$$

- (B)  $\frac{3RT}{M}$  where,  $R$  is the universal gas constant,  $T$  is the

absolute temperature.

Taking root mean square velocity formula:

$$v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow v_{rms}^2 = \frac{3RT}{M}$$

Unit of  $\frac{3RT}{M}$  is  $m^2 s^{-2}$

Also, (farad)(volt)<sup>2</sup>(kg)<sup>-1</sup> has the same dimensions.

(farad)(volt)<sup>2</sup>(kg)<sup>-1</sup>

= (Joule)(kg)<sup>-1</sup>

$$= kg \left(\frac{m}{s^2}\right) \cdot m (kg)^{-1}$$

$$= m^2 s^{-2}$$

- (C)  $\frac{F^2}{q^2 B^2}$

Force in the magnetic field;

$$F = qvB \Rightarrow v^2 = \frac{F^2}{q^2 B^2}$$

unit of  $v^2$  is  $m^2 s^{-2}$  which is also equal to  $FVkg^{-1}$ .

Hence, (farad)(volt)<sup>2</sup>(kg)<sup>-1</sup> has the same dimensions.

- (D)  $\frac{GM_e}{R_e}$

We know, Escape velocity,

$$V_e = \sqrt{\frac{2GM}{R}} \Rightarrow V_e^2 = \frac{2GM}{R}$$

So the unit of  $\frac{GM}{R}$  is  $m^2 s^{-2}$

9. Let us first derive Capacitance, Inductance and Magnetic Induction in terms of given units. Capacitance is the charge per unit volt. And we know, Volt has unit of work per unit charge.

So capacitance has units of charge square per unit energy.

Inductance is defined as volt per unit rate of change of current and voltage per unit current is resistance, so inductance is ohm second.

Magnetic Induction is defined as the property of the material by which it gets magnetized by an external magnetic field.

Equation for magnetic induction:

For a moving rod,  $N = 1$  and the flux  $\Phi = BA \cos \theta$ ,  $\theta = 0^\circ$  and  $\cos \theta = 1$ , a  $B$  is perpendicular to  $A$ .

10.  $A = a \hat{i}$  and  $B = a \cos \omega t \hat{i} + a \sin \omega t \hat{j}$

$$A + B = (a + \cos \omega t) \hat{i} + \sin \omega t \hat{j}$$

$$A - B = (a - \cos \omega t) \hat{i} + a \sin \omega t \hat{j}$$

$$|A + B| = \sqrt{3} |A - B|$$

$$\sqrt{(a + a \cos \omega t)^2 + (a \sin \omega t)^2} = \sqrt{3}$$

$$\sqrt{(a + a \cos \omega t)^2 + (a \sin \omega t)^2}$$

$$\Rightarrow 2 \cos \frac{\omega t}{2} = \pm \sqrt{3} \times 2 \sin \frac{\omega t}{2}$$

$$\Rightarrow \tan \frac{\omega t}{2} = \pm \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{\omega t}{2} = n\pi \pm \frac{\pi}{6} \Rightarrow \frac{\pi}{12} t = n\pi \pm \frac{\pi}{6}$$

$$\Rightarrow t = (12n \pm 2) s = 2s, 10s, 14s \text{ and so on.}$$

11. (d) Dipole moment, electric flux and electric field have different sets of dimensions.

The dimensions of the dipole moment, electric flux and electric field are given as;

Electric Dipole moment = Charge  $\times$  distance between the charges =  $[AT] \times [L] = [ALT]$

Electric flux = field strength  $\times$  area =  $[MLT^{-3}A^{-1}][L^2] = [ML^3T^{-3}A^{-1}]$

And Electric field = Force per unit charge =  $[MLT^{-3}A^{-1}]$

Since, all have different dimensions.

12. (a) The dimensional formula for  $\beta$  will be  $[M^0L^2T^0]$

Given: The given relation:

$$p = \frac{\alpha}{\beta} e^{-\left(\frac{\alpha z}{k\theta}\right)}, \text{ where } p \text{ is the pressure,}$$

$k$  is the Boltzmann's constant and  $\theta$  is the temperature.

In given equation,  $\frac{\alpha z}{k\theta}$  should be dimensionless

$$\therefore \alpha = \frac{k\theta}{z}$$

$$\Rightarrow \alpha = \frac{ML^2T^{-2}K^{-1} \times K}{L} = MLT^{-2}$$

$$\text{And } p = \frac{\alpha}{\beta}$$

Now, the dimension for  $\beta$  is given as:

$$\Rightarrow \beta = \frac{\alpha}{p} = \frac{MLT^{-2}}{ML^{-1}T^{-2}} = M^0L^2T^0$$

13. (d) The dimensional formula for  $X$  is the same as that of Current.

Given:  $X$  is given as  $\epsilon_0 L \frac{\Delta V}{\Delta t}$ , where,  $\epsilon_0$

is the permittivity of free space,  $L$  is a length,  $\Delta V$  is a potential difference and  $\Delta t$  is a time interval.

Dimensional formula for  $\epsilon_0 = [M^{-1}L^{-3}T^4A^2]$

Dimensional formula for  $L = [L]$

Dimensional formula for  $\Delta V = [ML^2T^{-3}I^{-1}]$

Dimensional formula for  $\Delta t = [T]$

Therefore, dimensional formula for  $X$  is: Therefore, it's the same as that of current.

14. (c) Given:  $\epsilon_0$  is the permittivity of free space and  $E$  is the given electric field.

Taking the dimensions of Electric field and permittivity of space;

Dimension of  $\epsilon_0 : M^{-1}L^{-3}T^4A^2$

Dimension of  $E$  (Electric field)

$$= MLT^{-2}/AT = MLT^{-3}A^{-1}$$

Dimension of

$$\left[ \frac{1}{2} \epsilon_0 E^2 \right] = [M^{-1}L^{-3}T^4A^2][MLT^{-3}A^{-1}]^2$$

$$= ML^{-1}T^{-2}$$

15. (b) The dimensional formula for the permittivity of vacuum is

$$[\epsilon_0] = [M^{-1}L^{-3}T^4A^2].$$

Given: Mass, length, time and electric current are  $M$ ,  $L$ ,  $T$  and  $A$ .

We can find the dimensional formula of the permittivity of free space using the force formula from Coulomb's law;

$$\text{Using Coulomb law, } F = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{R^2}$$

$$\Rightarrow \epsilon_0 = \frac{q_1q_2}{4\pi FR^2}$$

Substituting the units in the equation:

$$\text{So, } \epsilon_0 = \frac{C^2}{N \cdot m^2}$$

$$= \frac{[AT]^2}{MLT^{-2} \cdot L^2} = [M^{-1}L^{-3}T^4A^2]$$

$$16. (b,d) \quad l = \sqrt{\left(\frac{Ek_eT}{nq^2}\right)} \text{ and } l = \sqrt{\left(\frac{q^2}{\epsilon n^{1/3}k_eT}\right)}$$

are the correct dimensions.

Given: ( $\epsilon$ ) is the permittivity of a dielectric material, ( $k_e$ ) is the Boltzmann constant ( $T$ ) is the absolute temperature,  $n$  is the number of particles per unit volume and  $q$  is the charge on the particle.

The general dimension for the expression of length is given as;

$$\left[ \frac{\epsilon_0 k_e T}{q^2} \right] = \left[ \frac{q^2}{Fr^2} \frac{Fr}{q} \right] = \left[ \frac{1}{r} \right] = [L^{-1}]$$

$$\left[ \frac{q^2}{\epsilon_0 k_e T} \right] = [L], \text{ where length is written in terms of all the given values.}$$

Now, solving options (A), (B), (C) and (D) one by one:

$$\text{Option A: } \left[ \sqrt{\frac{nq^2}{\epsilon_0 k_e T}} \right] = [\sqrt{L^{-3}L}] = [L^{-1}]$$

$$\text{Option B: } \left[ \sqrt{\frac{\epsilon_0 k_e T}{nq^2}} \right] = \left[ \sqrt{\frac{L^{-1}}{L^{-3}}} \right] = [L]$$

Option C:

$$\left[ \sqrt{\frac{q^2}{\epsilon_0 k_e T n^{2/3}}} \right] = \left[ \sqrt{\frac{L}{L^{-2}}} \right] = [L^{3/2}]$$

Option D:

$$\left[ \sqrt{\frac{q^2}{\epsilon_0 k_e T n^{2/3}}} \right] = \left[ \sqrt{\frac{L}{L^{-1}}} \right] = [L]$$

Here, The dimensionally correct expressions for length are options (b) and (d).

17. (a, c, d)  $M \propto h^a c^b G^c$

$$M^1 \propto (ML^2T^{-1})^a (LT^{-1})^b (M^{-1}L^3T^{-2})^c$$

$$\propto M^{a-c} L^{2a+b+3c} T^{-a-b-2c}$$

$$a - c = 1 \quad \dots(i)$$

$$2a + b + 3c = 0 \quad \dots(ii)$$

$$a + b + 2c = 0 \quad \dots(iii)$$

On solving (i), (ii), (iii), we get

$$a = \frac{1}{2}, b = +\frac{1}{2}, c = -\frac{1}{2}$$

$\therefore M \propto \sqrt{c}$  only  $\rightarrow$  (a) is correct.

In the same way we can find that,  $L \propto h^{1/2} c^{-3/2} G^{1/2}$

$L \propto \sqrt{h}$ ,  $L \propto \sqrt{G} \rightarrow$  (c), (d) are also correct.

18. (a,c) The dimensionally correct equations are  $\mu_0 I^2 = E_0 V^2$  and  $I = E_0 c V$ .

Given:  $V$  is the potential difference,  $I$  is the Current,  $\epsilon_0$  is the permittivity,  $\mu_0$  is the permeability of free space and  $c$  is the speed of light.

Dimensional formula of potential difference is;

$$V = ML^2T^{-3}A^{-1}$$

The dimensional formula for current is;

$$\text{Current} = A$$

$$C = LT^{-1}$$

The dimensional formulas for permittivity and permeability are;

$$\mu_0 = MLT^{-2}A^{-2}$$

$$\epsilon_0 = M^{-1}L^{-3}T^4A^2$$

By equating equations we get,

$$\mu_0 I^2 = [M^1L^2T^{-2}] \text{ and } \epsilon_0 V^2 = [MLT^{-1}]$$

$$\mu_0 I^2 = \epsilon_0 V^2 \rightarrow (i)$$

Similarly,

$$I = E_0 c V \rightarrow (ii)$$

19. (b,c)  $[\epsilon_0] = [M^{-1}L^{-3}T^4A^2]$  and  $[\mu_0] = [MLT^{-2}A^{-2}]$  are the correct dimensional formulas for  $[\epsilon_0]$  and  $[\mu_0]$ .

From coulomb's law,

Dimension formula of  $\epsilon_0$

$$\epsilon_0 = \frac{1}{4\pi F} \frac{q_1 q_2}{r^2} (F = MLT^{-2})$$

$$\epsilon_0 = \frac{1}{MLT^{-2}} \frac{AT \times AT}{L^2}$$

$$= M^{-1}L^{-3}T^4A^2$$

And for the permeability of the vacuum;

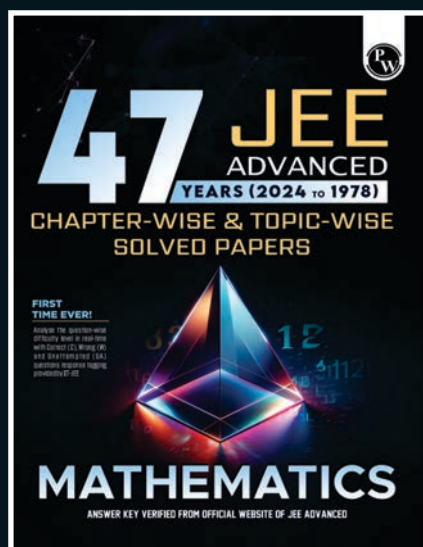
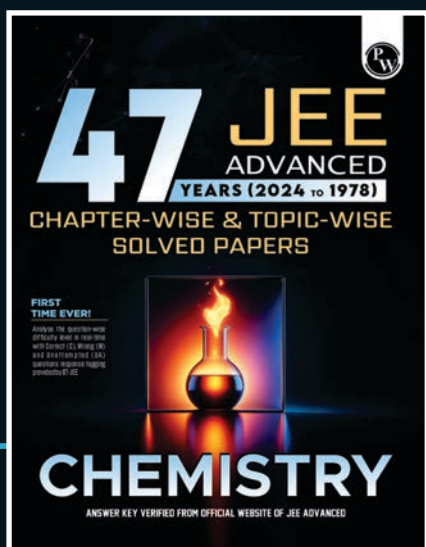
$$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\text{Dimension of } \mu_0 = \frac{1}{[\epsilon_0][\mu_0]^2}$$

# KEY FEATURES

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