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ENGINEERING MATHEMATICS & GENERAL APTITUDE

Topics Covered

- Linear Algebra
- Basic Calculus
- Probability and Statistics
- Vector Calculus
- Complex Analysis
- Differential Equations
- Laplace Transform & Fourier Series
- Numerical Methods
- Quantitative Aptitude
- Analytical Aptitude
- Spatial Aptitude

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ENGINEERING MATHEMATICS

GENERAL APTITUDE

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2. **Linear Algera-2 (Only for DS & AI)**
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   - Projection Matrix
   - Quadratic Form
   - Singular Value Decomposition
   - Partition Matrix
3. **Basic Calculus (Common to ALL Branches)**
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ESE

1. **Linear Algebra-1**
2. **Basic Calculus**
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Note: Common to all branches (Not for CS, DS & AI)

01 CHAPTER

Linear Algebra

Topic 1: Basics of Determinants

1. **[MCQ]**

If the determinant of a 4 × 4 matrix 'A' is 2, then the value of determinant of adj(adj(A)) is

(a) 1024 (b) 256

(c) 32 (d) 512

2. **[MSQ]**

If the adjoint of 3 × 3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$ then the determinant of P is (are)

(a) 2 (b) –2

(c) –1 (d) 1

3. **[MCQ]**

A matrix $A = \begin{bmatrix} p & q & r \\ q & r & p \\ r & p & q \end{bmatrix}$, where p, q and r are real positive numbers. If $pqr = 2$ and $A^TA = I$, then the value of $p^3 + q^3 + r^3$ should be equal to _____.

(a) 3 (b) 4

(c) 7 (d) 10

4. **[MCQ]**

If $ax^4 + bx^3 + cx^2 + dx + e = \begin{vmatrix} 2x & x-1 & x+1 \\ x+1 & x^2-x & x-1 \\ x-1 & x+1 & 3x \end{vmatrix}$

Then the value of e, is:

(a) 2 (b) 1.5

(c) 1 (d) 0

5. **[MCQ]**

The value of the determinant is:

$$\begin{vmatrix} \underset{x\to 0}{Lt}\ \frac{\sin x}{x} & 2 & 4 \\ \underset{x\to 0}{Lt}\ x^2 \cdot \frac{\sin x}{x} & \int_0^{\pi/2} \sin x\, dx & -8 \\ \underset{x\to \infty}{Lt}\ \frac{\sin x}{x} & 0 & \left(\frac{1}{2}\right) \end{vmatrix}$$

(a) $\frac{\sqrt{\pi}}{2}$ (b) $\sqrt{\pi}$

(c) $2\sqrt{\pi}$ (d) π

6. **[NAT]**

Let $A = \begin{bmatrix} 9 & 2 & 7 & 1 \\ 0 & 7 & 2 & 1 \\ 0 & 0 & 11 & 6 \\ 0 & 0 & -5 & 0 \end{bmatrix}$

Then the value of $|\det\ (8I - A)^3|$ is _____. (Enter in integer)

7. **[NAT]**

If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x+1) & x(x+1) \\ 3x(x-1) & x(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix}$

Then f (100) is equal to_____.

(Enter in integer)

8. **[MCQ]**

If the matrix, $M = \left[M_{ij}\right] \forall \begin{matrix} 1 \le i \le 2025, \\ 1 \le j \le 2025 \end{matrix}$

Such that $M_{ij} = \int_i^j \frac{x^2 \ln x}{2x} dx + \lim_{x\to\infty} \frac{\sin(i+j)x}{x}$

Then the determinant of the matrix 'M' is _____.

(a) 1 (b) –1

(c) 2025 (d) 0

9. **[MCQ]**

Let A be a 4 × 4 matrix with real entries. you are given the following information.

- det (A) = 6
- B is a matrix obtained by interchanging two rows of A
- C is obtained by multiplying one row of A by 3
- D is a matrix obtained by adding a multiple of one row of A to another row.

What are determinants of B, C & D?

(a) det $(B) = -6$, det $(C) = 18$, det $(D) = 6$
(b) det $(B) = 6$, det $(C) = 18$, det $(D) = 6$
(c) det $(B) = -6$, det $(C) = 2$, det $(D) = 6$
(d) det $(B) = 6$, det $(C) = 2$, det $(D) = 6$

10. [MCQ]

Let $A \in R^{4\times4}$ be partitioned as: $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

Where, $A_{11} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, $A_{12} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$, $A_{21} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, $A_{22} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

What will be det (A)?

(a) 12 (b) 13
(c) 8 (d) 6

11. [NAT]

Given the matrix $B = \begin{bmatrix} a & 1 & 2 & 3 \\ 0 & b & 4 & 5 \\ 0 & 0 & c & 6 \\ 0 & 0 & 0 & d \end{bmatrix}$, the determinant is 120 and the trace is 14. The value of $|a - d|$ if $(a < b < c < d)$ are positive integers with , will be ____. (Enter in integer)

12. [NAT]

If matrix 'A' = $[a_{ij}]$ is defined by the element $a_{ij} = \int_i^j \sin^{(j+i)} x\,dx$ where $1 \le i \le 3$; $1 \le j \le 3$.

The determinant of the matrix A is_____. (Round off to one decimal places)

Topic 2: Basics of Matrices

13. [MCQ]

Let $B = \begin{bmatrix} 1+2i & 3 \\ 4 & 2-i \end{bmatrix}$. If the inverse of B exists then which of the following is correct form of B^{-1}?

(a) $\frac{1}{15}\begin{bmatrix} 2-i & -3 \\ -4 & 1+2i \end{bmatrix}$ (b) $\frac{1}{-8+3i}\begin{bmatrix} 2-i & -3 \\ -4 & 1+2i \end{bmatrix}$

(c) $\frac{1}{10}\begin{bmatrix} 2-i & -3 \\ -4 & 1+2i \end{bmatrix}$ (d) $\frac{1}{10}\begin{bmatrix} 2+i & 3 \\ 4 & 1-2i \end{bmatrix}$

14. [MCQ]

If a matrix 'A' shifts the vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ to $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ to $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$, then $A^2 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is:

(a) $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} -2 \\ 0 \end{bmatrix}$
(c) $\begin{bmatrix} 0 \\ -1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 \\ -2 \end{bmatrix}$

15. [NAT]

The minimum number of multiplications required to calculate the product ABC where $A_{3\times4}$, $B_{4\times5}$, $C_{5\times3}$ are the matrices, is _____ (Enter in integer).

16. [NAT]

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 0 & a & 1 \\ b & 3 & -2 \\ -1 & -1 & c \end{bmatrix}$ then $\frac{a+b}{c} =$ _____. (Enter in integer)

17. [MSQ]

Let $[M]$ be a square matrix of order 3 × 3 such that $a_{ij} = \begin{cases} 0, & i \ne j \\ \int_0^{\pi/2} \sin^i x\,dx & i = j \end{cases}$ then which of the following is/are true?

(a) Trace of the matrix M is $\frac{\pi}{4} + \frac{5}{3}$

(b) Trace of the matrix M is $\frac{7\pi}{12} + 1$

(c) Determinant of matrix M is $\frac{\pi^2}{12}$

(d) Determinant of the matrix M is $\frac{\pi}{6}$

18. [MCQ]

If matrix A such that $A^2 = 2A - I$, where I is the identity matrix then for $n \ge 2$, An is equal to

(a) $nA - (n-1)I$ (b) $nA - I$
(c) $2^{n-1}A - (n-1)I$ (d) $2^{n-1}A - I$

19. [MCQ]

For $\alpha, \beta, \gamma \in R$ let $A = \begin{bmatrix} \alpha^2 & 6 & 8 \\ 3 & \beta^2 & 9 \\ 4 & 5 & \gamma^2 \end{bmatrix}$

and $B = \begin{bmatrix} 2\alpha & 3 & 5 \\ 2 & 2\beta & 6 \\ 1 & 4 & 2\gamma - 3 \end{bmatrix}$

If $\text{tr}(A) = \text{tr}(B)$, then the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$ is:

(a) 1 (b) 2
(c) 4 (d) 3

20. [MCQ]

Let A and B be two matrices such that AB exists. Consider the following statements regarding the existence of BA:

1. BA must exist if A and B are both invertible.
2. If AB exists, then BA always exists.
3. If A and B are square matrices of the same order then BA exists.
4. If A has dimensions $m \times n$ and B has deminsion $n \times p$, then BA exists if and only if $p = m$

Which of the above statements is/are correct?

(a) Only 1 and 3 (b) Only 1 and 4
(c) Only 2, 3 and 4 (d) Only 1, 3 and 4

21. [MCQ]

A matrix 'A' is defined by $a_{ij} = i^2 - j^2 \ \forall \ \begin{matrix} 1 \le i \le 2013 \\ 1 \le j \le 2013 \end{matrix}$

The value of sum of all the elements in the matrix is

(a) 1 (b) –1
(c) 0 (d) 5

22. [MCQ]

If a matrix $A = [a_{ij}]$ where $a_{ij} = i^j - j^i$ where $1 \le i \le 3$, $1 \le j \le 3$, then the value of $\sum_{i=1}^{3}\sum_{j=1}^{3} a_{ij}$ is

(a) 207 (b) 0
(c) 217 (d) 343

23. [MCQ]

If $A = A^T$ and $B = -B^T$ then the matrix '$AB + BA$' always is:

(a) Symmetric (b) Skew-Symmetric
(c) Orthogonal (d) Singular

24. [MCQ]

If $A = (a_{ij})_{n\times n}$, where $a_{ij} = i^2 - j^2$ is a square matrix of even order then

(a) A is symmetric and $|A|$ is a perfect square
(b) A is symmetric and $|A| = 0$
(c) A is a skew-symmetric matrix and $|A| = 0$
(d) None of these

Topic 3: Rank of Matrix

25. [MCQ]

Let $v_1 = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. Find the value of the coefficient in the expression $v_1 = v_2 + e$, which minimizes the length of the error.

(a) 13/9 (b) 11/9
(c) 17/9 (d) 2/9

26. [NAT]

Let $v_1 = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$ & $v_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. The error vector $\vec{e} = v_1 - \alpha v_2$ has the modulus $||e||$. Then, at what value of α $||e||$ is minimized?

27. [NAT]

If 'A' $= \begin{bmatrix} 1/9 & 8/9 & -4/9 \\ 4/9 & -4/9 & -7/9 \\ 8/9 & 1/9 & 4/9 \end{bmatrix}$ is an orthogonal matrix and $\vec{x} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ is a vector, then the length of the vector $A\vec{x}$ is_____. (Round off to two decimals)

28. [MCQ]

What will be the value of 'a' such that the rank of 4×4 matrix $A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ a & 2 & 2 & 2 \\ 9 & 9 & a & 3 \end{bmatrix}$ is 3?

(a) $a = \{2, -6\}$ (b) $a = \{1, -6\}$
(c) $a = \{2, 6\}$ (d) $a = \{2, 1\}$

29. [MCQ]

The rank of the matrix $[A]_{3\times 3} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix}$ will be :

(a) 1 (b) 2
(c) 3
(d) The rank cannot be determined

30. [MCQ]

Let A be an $n \times m$ matrix. Consider the following statements:

1. The rank of A is equal to the maximum number of linearly independent rows.
2. The rank of A is equal to the maxinaun number of linearly independent columns.
3. If A is an $n \times m$ matrix with $n > m$ then the rank of A can never exceed m.
4. If A is a Singular Square matrix, then its rank is equal to its size.

Which of the above statement is/are correct?

(a) only 1 and 2 (b) Only 1, 2 and 3
(c) Only 1, 2 and 4 (d) 1, 2, 3 and 4

Topic 4: Non-Homogeneous System

31. [MCQ]

Consider the following system of equations:

$x + y + z = 3$

$2x + 3y + z = 7$

$3x + 4y + 2z = 10$

Which of the following statement is correct?

(a) The system has a unique solution

(b) The system has no solution

(c) The system has infinitely many solutions

(d) The system has exactly two soultion

32. [MCQ]

Which of the following ordered pair (m, n) of the lienar equations:

$x + 2y + 3z = 4$

$3x + 4y + 5z = m$

$4x + 9y + 9z = n$

is consistent.

(a) (4, 1) (b) (5, 2)

(c) (6, 3) (d) For any values of m, n

33. [MCQ]

Consider the following system of linear equations?

$x + 2y + z = 4$

$2x + 4y + 2z = 8$

$3x + 6y + kz = 12$

Which of the following statement is correct?

(a) The system has infinitely many solution for $k = 2$.

(b) The system has no. solution for $k = 3$.

(c) The system has unique solution for $k = 1$

(d) The rank of confined matrix is always 2 regardless of k.

34. [MCQ]

A system of linear equation given below.

$x + 2y + \mu z = \lambda$

$x + y + z = 6$

$x + 2y + 3z = 10$

The system has no solution if

(a) $\lambda = 10, \mu = 3$

(b) $\mu \neq 3, \lambda = 10$

(c) $\mu = 3, \lambda \neq 10$

(d) $\mu \neq 3$

35. [NAT]

A d.c circuit involves 3 closed loops. Applying Kirchoff's laws to the closed loops give the following equations for current flow in milliamperes.

$2I_1 + 3I_2 - 4I_3 = 26$

$I_1 - 5I_2 - 3I_3 = -87$

$-7I_1 + 2I_2 + 6I_3 = 12$

The value of 'I_3' in milli amperes is _____. (Enter an integer)

36. [MCQ]

Consider the system of equations:

$4x - 2y + 6z = 5$

$2x + 2y + 4z = 3$

$10x - 2y + \lambda z = b$

If the system has infinitely many solutions, then the values of λ & b will be:

(a) $\lambda = 14, b = 13$ (b) $\lambda = 13, b = 12$

(c) $\lambda = 14, b = 10$ (d) $\lambda = 16, b = 13$

37. [MCQ]

Consider the following linear system of equations:

$2x + 4y - 6z = p$

$4x + 6y + 6z = q$

$10x + 18y - 12z = r$

Which of the following option is correct?

(a) The system is consistent for all values of p, q, r

(b) The system is consistent if $p + q + r = 0$

(c) The system is consistent if $p - q + 2r = 0$

(d) The system is consistent if $3p + q - r = 0$

38. [MCQ]

A company needs to allocate n resources to m tasks. The allocation is modeled as a system of linear equations with m equations and n variables. Consider the following scenarios.

1. If the number of task m is less than the number of resources n, then there may exist infinitely many ways to allocate recourses.
2. If $m > n$, the allocation becomes impossible for any configuration.
3. If $m = n$, the allocation can be determined uniquely if the allocation matrix is invertible.

Which of the following is correct?

(a) Only 1 is correct

(b) Only 3 is correct

(c) Both 1 and 3 are correct

(d) None of these are correct

39. [MCQ]

Consider the system of equations:

$x + y - z = 4$

$x + 3y + z = 10$

$x - y + 2z = 3$

Which of the following statement is correct?

(a) The system has no solutions
(b) The system has infinitely many solutions
(c) The system has a unique solutions
(d) The system has inconsistent equations

40. [MCQ]

For what value of λ, the given system have a solution:

$2x + y + z = 1$

$2x + 2y + 4z = \lambda$

$6x + 6y + 12z = \lambda^2$

(a) $\frac{7 \pm \sqrt{17}}{2}$ (b) $\frac{5 \pm \sqrt{17}}{2}$

(c) $\frac{1 \pm \sqrt{17}}{2}$ (d) 0, 3

41. [MCQ]

Consider the following equations:

$x + 2y - z = 4$

$2x + 5y + kz = 10$

$3x + 7y + 3z = 14$

For what value of 'k' the above system of equation has infinitely many solutions?

(a) $k \neq 4$ (b) $k = 4$
(c) $k = 2$ (d) $k \neq 2$

42. [MCQ]

Given the matrix A as follows:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{bmatrix}$$

What can be concluded about the system $Ax = b$ for any vector b?

(a) The system has a unique solution for any b.
(b) The system has no solution for any b.
(c) The system has infinitely many solutions for any b.
(d) The system may have no solutions or infinitely many solutions depending on b.

43. [MCQ]

Consider the system of equations:

$x + 3z = 5$

$-2x + 5y - z = 0$

$-x + 4y + z = 4$

Which of the following statements is correct about the consistency of the system?

(a) The system is consistent with a unique solution.
(b) The system is inconsistent and has no solution.
(c) The system is consistent and has infinitely many solution.
(d) The system has a solution only if $x = 0$.

44. [MSQ]

Consider the following system of equation:

$x + y + z = 1$

$x + 2y + 4z = n$

$x + 4y + 10z = n^2$

For what values of 'n' does the system have infinite solutions and what is the corresponding general solution for x, y and z?

(a) $n = 1$, with the solution $x = 1 + 2z$, $y = -3z$, $z = z$
(b) $n = 1$, with the solution $x = 2z$, $y = 1 - 3z$, $z = z$
(c) $n = 2$, with the solution $x = 1 + 2z$, $y = -3z$, $z = z$
(d) $n = 2$, with the solution $x = 2z$, $y = 1 - 3z$, $z = z$

45. [MCQ]

If A is 4×5 matrix and the system of equations $AX = B$ is inconsistent then the highest possible rank of A will be

(a) $\rho(A) \leq 2$ (b) $\rho(A) \leq 3$
(c) $\rho(A) \leq 4$ (d) $\rho(A) \leq 5$

Topic 5: Homogeneous System

46. [MCQ]

Consider a 3×5 matrix A defined as follows

$$A = \begin{bmatrix} 1 & 4 & 5 & a & 18 \\ 0 & 1 & 7 & 19 & b \\ 0 & 0 & 1 & 11 & 15 \end{bmatrix}$$

Where a and b are real numbers. Choose the correct statement regarding the matrix A based on values of a and b.

(a) There are specific values of a and b for which the columns of A become linearly independent.
(b) There are values of a and b for which the equation $Ax = 0$ has only the trivial solution $x = 0$.
(c) For any values of a and b, the rows of A span a 3-d subspace in R^5.
(d) There exist values of a and b such that the rank A is equal to 2.

47. [MCQ]

Consider the following system of linear equations:

$2x + 2py + pz = 0$

$2x + 2qy + qz = 0$

$2x + 2ry + rz = 0$

where $p, q, r \in R$ are non-zero and distinct: has a non-zero solution, then choose correct option.

(a) $p + q + r = 0$
(b) $p + q + r = 1$
(c) p, q, r can be any combination except 0
(d) $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ are in $A.P.$

ANSWER KEY

1.	(d)	**2.**	(a, b)	**3.**	(c)
4.	(d)	**5.**	(b)	**6.**	(216)
7.	**(0)**	**8.**	(d)	**9.**	(a)
10.	(b)	**11.**	(3)	**12.**	(–0.001 to 0.001)
13.	(b)	**14.**	(a)	**15.**	(96)
16.	(–0.001 to 0.001)	**17.**	(a, d)	**18.**	(a)
19.	(d)	**20.**	(d)	**21.**	(c)
22.	(b)	**23.**	(b)	**24.**	(d)
25.	(a)	**26.**	**(2)**	**27.**	(3.7 to 3.78)
28.	(a)	**29.**	(b)	**30.**	(b)
31.	(c)	**32.**	(d)	**33.**	(a)
34.	(c)	**35.**	(8 to 10)	**36.**	(d)
37.	(d)	**38.**	(c)	**39.**	(c)
40.	(d)	**41.**	(b)	**42.**	(d)
43.	(b)	**44.**	(a, d)	**45.**	(b)
46.	(c)	**47.**	(c)	**48.**	(a, d)
49.	(a)	**50.**	(c)	**51.**	(1)
52.	(a)	**53.**	(d)	**54.**	(0)
55.	(a, b)	**56.**	(c)	**57.**	(12)
58.	(b)	**59.**	(a)	**60.**	(a)
61.	(d)	**62.**	(a)	**63.**	(d)
64.	(a)	**65.**	(b)	**66.**	(a)
67.	(a)	**68.**	(c)	**69.**	(a)
70.	(a, c)	**71.**	(c)	**72.**	(d)
73.	(–0.3 to –0.4)	**74.**	(b)	**75.**	(c)
76.	(56)	**77.**	(b)	**78.**	(a)
79.	(d)	**80.**	(a)	**81.**	(c)
82.	(16)	**83.**	(a)	**84.**	(b)
85.	(c)	**86.**	(a)	**87.**	(c)
88.	(a)	**89.**	(d)	**90.**	(a)
91.	(b)	**92.**	(a)	**93.**	(a, b, c)
94.	(a, b, c, d)	**95.**	(c)	**96.**	(a)
97.	(d)	**98.**	(c)	**99.**	(c)
100.	(d)	**101.**	(37)	**102.**	(a, b, c)
103.	(b)	**104.**	(c)	**105.**	(d)
106.	(a)	**107.**	(b)	**108.**	(a)
109.	(c, d)	**110.**	(c)	**111.**	(b)
112.	(a)	**113.**	(d)	**114.**	(b)
115.	(d)	**116.**	(4)	**117.**	(b, d)
118.	(b)	**119.**	(b)	**120.**	(a)
121.	(a)	**122.**	(b)	**123.**	(b)
124.	(a, b, d)	**125.**	(4)	**126.**	(d)
127.	(c)	**128.**	(b)	**129.**	(c)

130. (a) **131.** (b) **132.** (d)
133. (a) **134.** (d) **135.** (a, b)
136. (b) **137.** (a) **138.** (a)
139. (a, d) **140.** (a) **141.** (a)
142. (a, d) **143.** (12) **144.** (c)
145. (20) **146.** (a) **147.** (–6)
148. (b) **149.** (d) **150.** (d)
151. (a) **152.** (b, d) **153.** (a, c)

SOLUTIONS

1. (d)

Given, $|A| = 2$ and $n = 4$

For a matrix $A_{n\times n}$

$|\text{adj A}| = |A|^{n-1}$ and $|\text{adj (adj } A)| = |A|^{(n-1)^2}$

$\Rightarrow \quad |\text{adj (adj } A)| = 2^{(4-1)^2} = 2^9 = 512$

$\therefore \quad |\text{adj (adj } A)| = 512$

2. (a, b)

Given:

$$\text{Adj } P = \begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$$

$$|\text{Adj } P| = \begin{vmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{vmatrix} \qquad R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - R_1$$

$$|\text{Adj } P| = \begin{vmatrix} 1 & 4 & 4 \\ 0 & -7 & -1 \\ 0 & -3 & -1 \end{vmatrix} = 1(7-3) = 4$$

$\because \quad |\text{Adj } P|_{3\times 3} = |P|^{3-1}$ (Property of Adjoint)

On comparing it, $|P|^2 = 4 \Rightarrow |P| = \pm 2$

3. (c)

Given, $A = \begin{bmatrix} p & q & r \\ q & r & p \\ r & p & q \end{bmatrix} \to$ Symmetric matrix $(A = A^T)$

$p, q, r \in \text{R}^+$; $p\,q\,r = 2$

$A^T A = I \Rightarrow A^2 = I$

$\Rightarrow |A|^2 = |I| = 1$

$$\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix}^2 = 1$$

$$\left(p\left(qr - p^2\right) - q\left(q^2 - rp\right) + r\left(pq - r^2\right)\right)^2 = 1$$

$$\left(3pqr - \left(p^3 + q^3 + r^3\right)\right)^2 = 1$$

$6 - (p^3 + q^3 + r^3) = \pm 1$

$p^3 + q^3 + r^3 = 5$ (or) 7

4. (d)

Given:

$$ax^4 + bx^3 + cx^2 + dx + e = \begin{vmatrix} 2x & x-1 & x+1 \\ x+1 & x^2 - x & x-1 \\ x-1 & x+1 & 3x \end{vmatrix}$$

Put $x = 0$

$$e = \begin{vmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{vmatrix}$$

Using $C_2 \to C_2 + C_3$

$$e = \begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & -1 \\ -1 & 1 & 0 \end{vmatrix}$$

$\Rightarrow \quad e = 1 - 1 = 0$

5. (b)

$$\begin{vmatrix} \underset{x\to 0}{Lt}\ \dfrac{\sin x}{x} & 2 & 4 \\ \underset{x\to 0}{Lt}\ x^2 \dfrac{\sin x}{x} & \displaystyle\int_0^{\pi/2} \sin x\,dx & -8 \\ \underset{x\to \infty}{Lt}\ \dfrac{\sin x}{x} & 0 & \sqrt{\dfrac{1}{2}} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 & 4 \\ 0 & 1 & -8 \\ 0 & 0 & \sqrt{\pi} \end{vmatrix} = 1(\sqrt{\pi} - 0) = \sqrt{\pi}$$

6. (216)

Given:

$$A=\begin{bmatrix}9&2&7&1\\0&7&2&1\\0&0&11&6\\0&0&-5&0\end{bmatrix}$$

Now,

$$8I-A=\begin{bmatrix}8&0&0&0\\0&8&0&0\\0&0&8&0\\0&0&0&8\end{bmatrix}\begin{bmatrix}9&2&7&1\\0&7&2&1\\0&0&11&6\\0&0&-5&0\end{bmatrix}$$

$$=\begin{bmatrix}-1&-2&-7&-1\\0&1&-2&-1\\0&0&-3&-6\\0&0&5&8\end{bmatrix}$$

$$\Rightarrow \quad |8I-A|=\begin{vmatrix}-1&-2&-7&-1\\0&1&-2&-1\\0&0&-3&-6\\0&0&5&8\end{vmatrix}$$

$$\Rightarrow \quad \left|(8I-A)^T\right|=\begin{vmatrix}-1&0&0&0\\-2&1&0&0\\-7&2&-3&5\\-1&1&-6&8\end{vmatrix}$$

$\because \quad |8I-A|=\left|(8I-A)^T\right|$

$$\Rightarrow \quad |8I-A|=-1\begin{vmatrix}1&0&0\\2&-3&5\\1&-6&8\end{vmatrix}$$

$= -1 \times 1 \times (6) = -6$

$\therefore \quad \det(A^n) = \det(a)^n$

$$\Rightarrow \quad \left|(8I-A)^3\right|=|8I-A|^3=(-6)^3=-216$$

$$\therefore \quad \left|\det(8I-A)^3\right|=|-216|=216$$

7. (0)

Given:

$$f(x)=\begin{bmatrix}1&x&x+1\\2x&x(x+1)&x(x+1)\\3x(x-1)&x(x-1)(x-2)&x(x+1)(x-1)\end{bmatrix}$$

$$=x(x+1)(x-1)\begin{vmatrix}1&1&1\\2x&(x-1)&x\\3x&(x-2)&x\end{vmatrix}$$

$C_1 \to C_1 - C_3$ & $C_2 \to C_2 - C_3$

$$=x(x+1)(x-1)\begin{vmatrix}0&0&1\\x&-1&x\\2x&-2&x\end{vmatrix}$$

$= x(x+1)(x-1)\,0$

$\therefore f(x) = 0$ for all values of x

$\therefore f(100) = 0$

8. (d)

Given:

$$M=\left[M_{ij}\right]\ \forall\ \begin{matrix}1\le i\le 2025,\\1\le j\le 2025\end{matrix}$$

$$\lim_{x\to\infty}\frac{\sin(i+j)x}{x}=0\ \forall\ i,j\in N.$$

$$\therefore \quad M_{ij}=\frac{1}{2}\int_i^j x\ln x\,dx$$

$$\Rightarrow \quad M_{ji}=-M_{ij} \qquad \left(\because \int_a^b f(x)dx=-\int_b^a f(x)dx\right)$$

Given matrix is a skew-symmetric matrix of odd order.

So, determinant is zero.

9. (a)

$\det(B) = -\det(A) = -6$

$\det(C) = 3 \times \det(A) = 3 \times 6 = 18$

$\det(D) = \det(A) = 6$

Hence, option 'a' is correct

10. (b)

$$A=\begin{bmatrix}2&1&0&1\\1&3&2&0\\1&0&1&1\\0&2&0&1\end{bmatrix}$$

$$\det(A)=2\times\begin{vmatrix}3&2&0\\0&1&1\\2&0&1\end{vmatrix}-1\begin{vmatrix}1&2&0\\1&1&1\\0&0&1\end{vmatrix}+0-1\begin{vmatrix}1&3&2\\1&0&1\\0&2&0\end{vmatrix}$$

$= 2(3(1-0) - 2(-2) + 0) - (1(1) - 2(1))$
$\quad - 1(1(-2) - 3(0) + 2(2-0))$

$= 2(3+4) + 1 - (2)$

$= 13$

11. (3)

Given:

$$B=\begin{bmatrix}a&1&2&3\\0&b&4&5\\0&0&c&6\\0&0&0&d\end{bmatrix}$$

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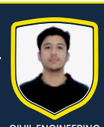

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