Part 2

OLYMPIAD WALLAH

JEEnius Edge 11

MATHEMATICS

For JEE, ISI-CMI & Olympiad Maths

Ultimate Resource for JEE, ISI-CMI & Olympiad Maths Preparation

CONTENTS

EXERCISE-1

STRAIGHT OBJECTIVE TYPE

1. Let 'E' be the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ & 'C' be the circle $x^2 + y^2 = 9$. Let P & Q be the points (1, 2) and (2, 1) respectively. Then :

(A) Q lies inside C but outside E
(B) Q lies outside both C & E
(C) P lies inside both C & E
(D) P lies inside C but outside E.

2. The eccentricity of the ellipse $(x - 3)^2 + (y - 4)^2 = \frac{y^2}{9}$ is:

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{3}$
(C) $\frac{1}{3\sqrt{2}}$ (D) $\frac{1}{\sqrt{3}}$

3. The equation, $2x^2 + 3y^2 - 8x - 18y + 35 = K$ represents

(A) no locus if $K > 0$
(B) an ellipse if $K < 0$
(C) a point if $K = 0$
(D) a hyperbola if $K > 0$

4. If the ellipse $\frac{(x-h)^2}{M}+\frac{(y-k)^2}{N}=1$ has major axis on the line $y = 2$, minor axis on the line $x = -1$, major axis has length 10 and minor axis has length 4. The number h, k, M, N (in this order only) are -

(A) –1, 2, 5, 2 (B) –1, 2, 10, 4
(C) 1, –2, 25, 4 (D) –1, 2, 25, 4

5. A circle has the same centre as an ellipse & passes through the foci F_1& F_2 of the ellipse, such that the two curves intersect in 4 points. Let 'P' be any one of their point of intersection. If the major axis of the ellipse is 17 & the area of the triangle PF_1F_2 is 30, then the distance between the foci is:

(A) 11 (B) 12
(C) 13 (D) none

6. The latus rectum of a conic section is the width of the function through the focus. The positive difference between the length of the latus rectum of $3y = x^2 + 4x - 9$ and $x^2 + 4y^2 - 6x + 16y = 24$ is-

(A) $\frac{1}{2}$ (B) 2
(C) $\frac{3}{2}$ (D) $\frac{5}{2}$

7. Imagine that you have two thumbtacks placed at two points, A and B. If the ends of a fixed length of string are fastened to the thumbtacks and the string is drawn taut with a pencil, the path traced by the pencil will be an ellipse. The best way to maximise the area surrounded by the ellipse with a fixed length of string occurs when

I. the two points A and B have the maximum distance between them.
II. two points A and B coincide.
III. A and B are placed vertically.
IV. The area is always same regardless of the location of A and B.

(A) I (B) II
(C) III (D) IV

8. Let S(5,12) and S'(–12,5) are the foci of an ellipse passing through the origin. The eccentricity of ellipse equals -

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{3}}$
(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{2}{3}$

9. The y-axis is the directrix of the ellipse with eccentricity e = 1/2 and the corresponding focus is at(3, 0), equation to its auxiliary circle is

(A) $x^2 + y^2 - 8x + 12 = 0$
(B) $x^2 + y^2 - 8x - 12 = 0$
(C) $x^2 + y^2 - 8x + 9 = 0$
(D) $x^2 + y^2 = 4$

10. Equation of the common tangent to the ellipses, $\frac{x^2}{a^2+b^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}+\frac{y^2}{a^2+b^2}=1$ is-

18. The area of the rectangle formed by the perpendiculars from the centre of the standard ellipse to the tangent and normal at its point whose eccentric angle is $\pi/4$, is :

(A) $\frac{(a^2-b^2)ab}{a^2+b^2}$ (B) $\frac{(a^2-b^2)}{(a^2+b^2)ab}$

(C) $\frac{(a^2-b^2)}{ab(a^2+b^2)}$ (D) $\frac{a^2+b^2}{(a^2-b^2)ab}$

19. If P is any point on ellipse with foci S_1& S_2 and eccentricity is $\frac{1}{2}$ such that $\angle PS_1S_2 = \alpha$, $\angle PS_2S_1 = \beta$, $\angle S_1PS_2 = \gamma$, then $\cot\frac{\alpha}{2}, \cot\frac{\gamma}{2}, \cot\frac{\beta}{2}$ are in

(A) A.P.
(B) G.P.
(C) H.P.
(D) NOT A.P., G.P. & H.P.

EXERCISE-2

1. (a) Find the equation of the ellipse with its centre (1, 2), focus at (6, 2) and passing through the point (4, 6).
(b) An ellipse passes through the points (– 3, 1) & (2, –2) & its principal axis are along the coordinate axes in order. Find its equation.

2. The tangent at any point P of a circle $x^2 + y^2 = a^2$ meets the tangent at a fixed point A (a, 0) in T and T is joined to B, the other end of the diameter through A, prove that the locus of the intersection of AP and BT is an ellipse whose eccentricity is $1/\sqrt{2}$.

3. The tangent at the point α on a standard ellipse meets the auxiliary circle in two points which subtends a right angle at the centre. Show that the eccentricity of the ellipse is $(1 + \sin^2\alpha)^{-1/2}$.

4. If any two chords be drawn through two points on the major axis of an ellipse equidistant from the centre, show that $\tan\frac{\alpha}{2}.\tan\frac{\beta}{2}.\tan\frac{\gamma}{2}.\tan\frac{\delta}{2} = 1$, where α, β, γ, δ are the eccentric angles of the extremities of the chords.

5. If the normal at the point P(θ) to the ellipse $\frac{x^2}{14}+\frac{y^2}{5}=1$, intersects it again at the point Q(2θ), show that $\cos\theta = -(2/3)$.

6. If s, s' are the length of the perpendicular on a tangent from the foci, a, a' are those from the vertices, c is that from the centre and e is the eccentricity of the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then prove that $\frac{ss'-c^2}{aa'-c^2}=e^2$.

7. Find the equations of the lines with equal intercepts on the axes & which touch the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

8. Suppose x and y are real numbers and that $x^2 + 9y^2 - 4x + 6y + 4 = 0$ then find the maximum value of $(4x - 9y)$.

9. A tangent having slope $-\frac{4}{3}$ to the ellipse $\frac{x^2}{18}+\frac{y^2}{32}=1$, intersects the axis of x & y in points A & B respectively. If O is the origin, find the area of triangle OAB.

10. 'O' is the origin & also the centre of two concentric circles having radii of the inner & the outer circle as 'a' & 'b' respectively. A line OPQ is drawn to cut the inner circle in P & the outer circle in Q. PR is drawn parallel to the y-axis & QR is drawn parallel to the x-axis. Prove that the locus of R is an ellipse touching the two circles. If the focii of this ellipse lie on the inner circle, find the ratio of inner: outer radii & find also the eccentricity of the ellipse.

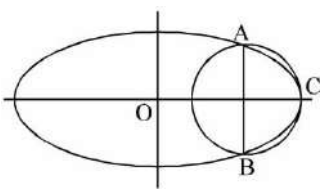
11. Find the equation of the largest circle with centre (1, 0) that can be inscribed in the ellipse $x^2 + 4y^2 = 16$.

12. Let d be the perpendicular distance from the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to the tangent drawn at a point P on the ellipse. If F_1& F_2 are the two foci of the ellipse, then show that $(PF_1 - PF_2)^2 = 4a^2\left[1-\frac{b^2}{d^2}\right]$.

13. Tangents drawn from the point P(2,3) to the circle $x^2 + y^2 - 8x + 6y + 1 = 0$ touch the circle at the points A and B. The circum circle of the ΔPAB cuts the director circle of ellipse $\frac{(x+5)^2}{9}+\frac{(y-3)^2}{b^2}=1$ orthogonally. Find the value of b^2.

14. Find the condition so that the line px + qy = r intersects the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in points whose eccentric angles differ by $\pi/4$.

15. A circle intersects an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ precisely at three points A, B, C as shown in the figure. AB is a diameter of the circle and is perpendicular to the major axis of the ellipse. If the eccentricity of the ellipse is 4/5, find the length of the diameter AB in terms of a.



16. Consider the family of circles, $x^2 + y^2 = r^2$, $2 < r < 5$. If in the first quadrant, the common tangent to a circle of the family and the ellipse $4x^2 + 25y^2 = 100$ meets the co-ordinate axes at A & B, then find the equation of the locus of the mid-point of AB.

17. An ellipse has foci at $F_1(9, 20)$ and $F_2(49, 55)$ in the xy-plane and is tangent to the x-axis. Find the length of its major axis.

18. Point 'O' is the centre of the ellipse with major axis AB & minor axis CD. Point F is one focus of t ellipse. If OF = 6 & the diameter of the inscribed circle of triangle OCF is 2, then find the product (AB) (CD).

19. Find the number of integral values of parameter 'a' for which three chords of the ellipse $\frac{x^2}{2a^2}+\frac{y^2}{a^2}=1$ (other than its diameter) passing through the point $P\left(11a,-\frac{a^2}{4}\right)$ are bisected by the parabola $y^2 = 4ax$.

20. Prove that, in an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, the perpendicular from a focus upon any tangent and the line joining the centre of the ellipse to the point of contact meet on the corresponding directrix.

EXERCISE-3

1. Consider the ellipse $\frac{x^2}{\tan^2\alpha}+\frac{y^2}{\sec^2\alpha}=1$ where $\alpha\in(0,\pi/2)$.
Which of the following quantities would vary as α varies?
(A) degree of flatness
(B) ordinate of the vertex
(C) coordinates of the foci
(D) length of the latus rectum

2. Extremities of the latus rectum of the ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\,(a > b)$ having a given major axis 2a lies on-
(A) $x^2 = a(a - y)$ (B) $x^2 = a(a + y)$
(C) $y^2 = a(a + x)$ (D) $y^2 = a(a - x)$

3. If a number of ellipse (whose axes are x & y axes) be described having the same major axis 2a but a variable minor axis then the tangents at the ends of their latus rectum pass through fixed points which can be -

18. The equation of auxiliary circle of the ellipse is
(A) $x^2 + y^2 - 2x - 4y - 5 = 0$
(B) $x^2 + y^2 - 2x - 4y - 20 = 0$
(C) $x^2 + y^2 + 2x + 4y - 20 = 0$
(D) $x^2 + y^2 + 2x + 4y - 5 = 0$

19. The length of semi-minor axis of the ellipse is
(A) 1 (B) $2\sqrt{2}$
(C) $\sqrt{17}$ (D) $\sqrt{19}$

20. The equations of directrices of the ellipse are
(A) $x - y + 2 = 0, x - y - 5 = 0$
(B) $x + y - \frac{21}{2} = 0, x + y + \frac{17}{2} = 0$
(C) $x - y + \frac{3}{2} = 0, x - y - \frac{5}{2} = 0$
(D) $x + y - \frac{31}{2} = 0, x + y + \frac{19}{2} = 0$

MATRIX MATCH TYPE

21. Match the following:

Column-I		Column-II	
(A)	The eccentricity of the ellipse which meets the straight line $2x - 3y = 6$ on the X-axis and the straight line $4x + 5y = 20$ on the Y-axis and whose principal axes lie along the coordinate axes, is	(P)	$\frac{1}{2}$
(B)	A bar of length 20 units moves with its ends on two fixed straight lines at right angles. A point P marked on the bar at a distance of 8 units from one end describes a conic whose eccentricity is	(Q)	$\frac{1}{\sqrt{2}}$
(C)	If one extremity of the minor axis of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the foci form an equilateral triangle, then its eccentricity, is	(R)	$\frac{\sqrt{5}}{3}$
(D)	There are exactly two points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose distance from the centre of the ellipse are greatest and equal to $\sqrt{\frac{a^2 + 2b^2}{2}}$. Eccentricity of this ellipse is equal to	(S)	$\frac{\sqrt{7}}{4}$

JEE MAIN PYQ

1. An ellipse has OB as semi minor axis, F and F ' its focii and the angle FBF' is a right angle. Then the eccentricity of the ellipse is- **[AIEEE-2005]**
(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$
(C) $\frac{1}{4}$ (D) $\frac{1}{\sqrt{3}}$

2. In an ellipse, the distance between its foci is 6 and minor axis is 8. Then its eccentricity is- **[AIEEE-2006]**
(A) $\frac{1}{2}$ (B) $\frac{4}{5}$
(C) $\frac{1}{\sqrt{5}}$ (D) $\frac{3}{5}$

3. A focus of an ellipse is at the origin. The directrix is the line x = 4 and the eccentricity is 1/2. Then the length of the semi-major axis is- **[AIEEE-2008]**
(A) $\frac{8}{3}$ (B) $\frac{2}{3}$
(C) $\frac{4}{3}$ (D) $\frac{5}{3}$

4. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is:- **[AIEEE-2009]**
(A) $4x^2 + 48y^2 = 48$ (B) $4x^2 + 64y^2 = 48$
(C) $x^2 + 16y^2 = 16$ (D) $x^2 + 12y^2 = 16$

5. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point(–3, 1) and has eccentricity $\sqrt{2/5}$ is :- **[AIEEE-2011]**
(A) $3x^2 + 5y^2 - 15 = 0$ (B) $5x^2 + 3y^2 - 32 = 0$
(C) $3x^2 + 5y^2 - 32 = 0$ (D) $5x^2 + 3y^2 - 48 = 0$

6. An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$ as its semi-minor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is 2 at the origin and its axes are the coordinate axes, then the equation of the ellipse is : **[AIEEE-2012]**
(A) $x^2 + 4y^2 = 16$ (B) $4x^2 + y^2 = 4$
(C) $x^2 + 4y^2 = 8$ (D) $4x^2 + y^2 = 8$

16. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ is:

[JEE Main-2018]

(A) $\frac{27}{2}$ (B) 27

(C) $\frac{27}{4}$ (D) 18

17. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is:

[JEE Main-2018]

(A) $\frac{9}{2}$ (B) 6

(C) $\frac{7}{2}$ (D) 4

18. Two sets A and B are as under:

[JEE Main-2018]

$A = \{(a, b) \in R \times R : |a - 5| < 1 \text{ and } |b - 5| < 1\}$;

$B = \{(a, b) \in R \times R : 4(a - 6)^2 + 9(b - 5)^2 \leq 36\}$.

Then:

(A) neither $A \subset B$ nor $B \subset A$

(B) $B \subset A$

(C) $A \subset B$

(D) $A \cap B = \phi$ (an empty set)

19. If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points (1, 2) and (a, b) are perpendicular to each other, then a^2 is equal to: **[JEE Main-2019]**

(A) $\frac{128}{17}$ (B) $\frac{64}{17}$

(C) $\frac{4}{17}$ (D) $\frac{2}{17}$

20. In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $(0,5\sqrt{3})$, then the length of its latus rectum is:

[JEE Main-2019]

(A) 10 (B) 5

(C) 6 (D) 8

21. If the tangent to the parabola $y^2 = x$ at a point (α, β), $(\beta > 0)$ is also a tangent to the ellipse, $x^2 + 2y^2 = 1$, then α is equal to :

[JEE Main-2019]

(A) $2\sqrt{2}-1$ (B) $2\sqrt{2}+1$

(C) $\sqrt{2}+1$ (D) $\sqrt{2}-1$

22. If the line $x - 2y = 12$ is tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $\left(3,\frac{-9}{2}\right)$, then the length of the latus rectum of the ellipse is :

[JEE Main-2019]

(A) $12\sqrt{2}$ (B) 9

(C) $8\sqrt{3}$ (D) 5

23. The tangent and normal to the ellipse $3x^2 + 5y^2 = 32$ at the point P(2, 2) meet the x-axis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is: **[JEE Main-2019]**

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$

(C) $\frac{34}{15}$ (D) $\frac{68}{15}$

24. If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line $2x + y = 4$ and the tangent to the ellipse at P passes through Q(4, 4) then PQ is equal to : **[JEE Main-2019]**

(A) $\frac{\sqrt{221}}{2}$ (B) $\frac{5\sqrt{5}}{2}$

(C) $\frac{\sqrt{157}}{2}$ (D) $\frac{\sqrt{61}}{2}$

25. An ellipse, with foci at (0, 2) and (0, – 2) and minor axis of length 4, passes through which of the following points? **[JEE Main-2019]**

(A) $\left(2,\sqrt{2}\right)$ (B) $\left(1,2\sqrt{2}\right)$

(C) $\left(\sqrt{2},2\right)$ (D) $\left(2,2\sqrt{2}\right)$

53. If the length of the latus rectum of the ellipse $x^2 + 4y^2 + 2x + 8y - \lambda = 0$ is 4 , and l is the length of its major axis, then $\lambda + 1$ is equal to **[JEE Main-2022]**

54. Let the tangents at the points P and Q on the ellipse $\frac{x^2}{2} + \frac{y^2}{4} = 1$ meet at the point $R(\sqrt{2}, 2\sqrt{2} - 2)$. If S is the focus of the ellipse on its negative major axis, then $SP^2 + SQ^2$ is equal to **[JEE Main-2022]**

55. If the tangent at a point P on the parabola $y^2 = 3x$ is parallel to the line $x + 2y = 1$ and the tangents at the points Q and R on the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ are perpendicular to the line $x - y = 2$, then the area of the triangle PQR is : **[JEE Main-2023]**

(A) $\frac{3}{2}\sqrt{5}$ (B) $5\sqrt{3}$

(C) $\frac{9}{\sqrt{5}}$ (D) $3\sqrt{5}$

56. If the maximum distance of normal to the ellipse $\frac{x^2}{4} + \frac{y^2}{b^2} = 1, b < 2$, from the origin is 1, then the eccentricity of the ellipse is : **[JEE Main-2023]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$

(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{\sqrt{3}}{4}$

57. Let C be the largest circle centred at $(2,0)$ and inscribed in the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$.

If $(1,\alpha)$ lies on C , then $10\alpha^2$ is equal to _____ **[JEE Main-2023]**

58. Let a tangent to the curve $9x^2 + 16y^2 = 144$ intersect the coordinate axes at the points A and B . Then. the minimum length of the line segment AB is **[JEE Main-2023]**

59. The line $x = 8$ is the directrix of the ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with the corresponding focus $(2,0)$. If the tangent to E at the point P in the first quadrant passes through the point $(0, 4\sqrt{3})$ and intersects the x -axis at Q , then $(3PQ)^2$ is equal to **[JEE Main-2023]**

60. Let P be a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Let the line passing through P and parallel to y-axis meet the circle $x^2 + y^2 = 9$ at point Q such that P and Q are on the same side of the x-axis. Then, the eccentricity of the locus of the point R on PQ such that PR : RQ = 4 : 3 as P moves on the ellipse, is : **[JEE Main-2024]**

(A) $\frac{\sqrt{13}}{7}$ (B) $\frac{11}{19}$

(C) $\frac{13}{21}$ (D) $\frac{\sqrt{139}}{23}$

61. The length of the chord of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, whose mid point is $\left(1, \frac{2}{5}\right)$, is equal to: **[JEE Main-2024]**

(A) $\frac{\sqrt{1741}}{5}$ (B) $\frac{\sqrt{1541}}{5}$

(C) $\frac{\sqrt{2009}}{5}$ (D) $\frac{\sqrt{1691}}{5}$

62. If the length of the minor axis of an ellipse is equal to half of the distance between the foci, then the eccentricity of the ellipse is: **[JEE Main-2024]**

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\frac{2}{\sqrt{5}}$ (D) $\frac{\sqrt{5}}{3}$

63. Let A(α, 0) and B(0, β) be the points on the line $5x + 7y = 50$. Let the point P divide the line segment AB internally in the ratio 7:3. Let $3x-25 = 0$ be a directrix of the ellipse $E:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the corresponding focus be S. If from S, the perpendicular on the x-axis passes through P, then the length of the latus rectum of E is equal to, **[JEE Main-2024]**

(A) $\frac{32}{5}$ (B) $\frac{25}{9}$

(C) $\frac{32}{9}$ (D) $\frac{25}{3}$

64. Let the ellipse $E_1:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$ and $E_2:\frac{x^2}{A^2}+\frac{y^2}{B^2}=1,\ A<B$ have same eccentricity $\frac{1}{\sqrt{3}}$. Let the product of their lengths of latus rectums be $\frac{32}{\sqrt{3}}$, and the distance between the foci of E_1 be 4. If E_1 and E_2 meet at A, B, C and D, then the area of the quadrilateral ABCD equals: **[JEE Main-2025]**

(A) $\frac{12\sqrt{6}}{5}$ (B) $6\sqrt{6}$

(C) $\frac{18\sqrt{6}}{5}$ (D) $\frac{24\sqrt{6}}{5}$

65. If $\alpha x+\beta y=109$ is the equation of the chord of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$, whose mid point is $\left(\frac{5}{2},\frac{1}{2}\right)$, then $\alpha+\beta$ is equal to :

[JEE Main-2025]

(A) 58 (B) 46

(C) 37 (D) 72

66. Let $E_1:\frac{x^2}{9}+\frac{y^2}{4}=1$ be an ellipse. Ellipses E_1 's are constructed such that their centres and eccentricities are same as that of E_1, and the length of minor axis of E_i is the length of major axis of $E_{i+1}(i\ge 1)$. If A_i is the area of the ellipse E_i, then $\frac{5}{\pi}\left(\sum_{i=1}^{\infty}A_i\right)$, is equal to

[JEE Main-2025]

67. If the midpoint of a chord of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ is $\left(\sqrt{2},4/3\right)$, and the length of the chord is $\frac{2\sqrt{\alpha}}{3}$, then α is : **[JEE Main-2025]**

(A) 20 (B) 22

(C) 18 (D) 26

68. Let the product of the focal distances of the point $\left(\sqrt{3},\frac{1}{2}\right)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,(a>b)$, be $\frac{7}{4}$. Then the absolute difference of the eccentricities of two such ellipses is

[JEE Main-2025]

(A) $\frac{1-\sqrt{3}}{\sqrt{2}}$ (B) $\frac{3-2\sqrt{2}}{2\sqrt{3}}$

(C) $\frac{3-2\sqrt{2}}{3\sqrt{2}}$ (D) $\frac{1-2\sqrt{2}}{\sqrt{3}}$

69. The equation of the chord, of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, whose mid-point is (3, 1) is:

[JEE Main-2025]

(A) $48x + 25y = 169$

(B) $5x + 16y = 31$

(C) $25x + 101y = 176$

(D) $4x + 122y = 134$

70. The length of the chord of the ellipse $\frac{x^2}{4}+\frac{y^2}{2}=1$, whose mid-point is $\left(1,\frac{1}{2}\right)$, is:

[JEE Main-2025]

(A) $\frac{5}{3}\sqrt{15}$ (B) $\frac{1}{3}\sqrt{15}$

(C) $\frac{2}{3}\sqrt{15}$ (D) $\sqrt{15}$

ISI PYQ

1. Let AB be a fixed line segment. Let P be a moving point such that $\angle APB$ is equal to a constant acute angle. Then, which of the following curves does the point P move along? **[ISI – 2007]**

(A) a circle
(B) an ellipse
(C) the boundary of the common region of 2 identical intersecting circles with centres outside the common region
(D) the boundary of the union of 2 identical intersecting circles with centres outside the common region

2. Let P be a point on the ellipse $x^2 + 4y^2 = 4$ which does not lie on the axes. If the normal at the point P intersects the major and minor axes at C and D respectively, then the ratio PC : PD equals **[ISI – 2007]**

(A) 2 (B) 1/2
(C) 4 (D) 1/4

3. The minimum area of the triangle formed by any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the coordinate axes is **[ISI – 2015]**

(A) ab (B) $\frac{a^2+b^2}{2}$
(C) $\frac{(a+b)^2}{4}$ (D) $\frac{a^2+ab+b^2}{3}$

4. Let $a,b,c,d>0$, be any real numbers. Then the maximum possible value of $cx+dy$, over all points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, must be **[ISI – 2021]**

(A) $\sqrt{a^2c^2+b^2d^2}$ (B) $\sqrt{a^2b^2+c^2d^2}$
(C) $\sqrt{\frac{a^2c^2+b^2d^2}{a^2+b^2}}$ (D) $\sqrt{\frac{a^2b^2+c^2d^2}{c^2+d^2}}$

JEE ADVANCED PYQ

1. (a) If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in G, P with the same common ratio, then the points (x_1, y_1), (x_2, y_2) & (x_3, y_3);
(A) lie on a straight line
(B) lie on an ellipse
(C) lie on a circle
(D) are vertices of a triangle.

(b) On the ellipse, $4x^2+9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$ are:

(A) $\left(\frac{2}{5},\frac{1}{5}\right)$ (B) $\left(-\frac{2}{5},\frac{1}{5}\right)$
(C) $\left(-\frac{2}{5},-\frac{1}{5}\right)$ (D) $\left(\frac{2}{5},-\frac{1}{5}\right)$

(c) Consider the family of circles, $x^2+y^2=r^2$, $2 < r < 5$. If in the first quadrant, the common tangent to a circle of the family and the ellipse $4x^2+25y^2=100$ meets the co-ordinate axes at A & B, then find the equation of the locus of the mid-point of AB. **[JEE '99, 2+3+10 (out of 200)]**

2. Find the equation of the largest circle with center (1, 0) that can be inscribed in the ellipse $x^2+4y^2=16$. **[REE '99,6]**

3. Let A B C be an equilateral triangle inscribed in the circle $x^2+y^2=a^2$, Suppose perpendiculars from, A, B, C to the major axis of the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $(a > b)$ meet the ellipse respectively at P, Q, R so that P, Q, R lie on the same side of the major axis as A, B, C respectively. Prove that the normals to the ellipse drawn at the points P, Q and R are concurrent. **[JEE '2000,7]**

4. Let C_1 and C_2 be two circles with C_2 lying inside C_1, A circle C lying inside C_1 touches C_1 internally and C_2 externally. Identify the locus of the center of C. **[JEE '2001, 5]**

14. In a triangle ABC with fixed base BC, the vertex A moves such that $\cos B + \cos C = 4\sin^2\frac{A}{2}$. If a, b and c denote the lengths of the sides of the triangle opposite to the angles A, B and C respectively, then **[JEE 2009, 4]**

(A) $b + c = 4a$

(B) $b + c = 2a$

(C) locus of point A is an ellipse.

(D) locus of point A is a pair of straight lines.

Comprehension: 15 to 17

Tangents are drawn from the point P(3,4) to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ touching the ellipse at points A and B. **[JEE 2010, 3+3+3]**

15. The coordinates of A and B are

(A) (3,0) and (0,2)

(B) $\left(-\frac{8}{5},\frac{2\sqrt{261}}{15}\right)$ and $\left(-\frac{9}{5},\frac{8}{5}\right)$

(C) $\left(-\frac{8}{5},\frac{2\sqrt{161}}{15}\right)$ and (0, 2)

(D) (3,0) and $\left(-\frac{9}{5},\frac{8}{5}\right)$

16. The orthocenter of the triangle PAB is

(A) $\left(5,\frac{8}{7}\right)$ (B) $\left(\frac{7}{5},\frac{25}{8}\right)$

(C) $\left(\frac{11}{5},\frac{8}{5}\right)$ (D) $\left(\frac{8}{25},\frac{7}{5}\right)$

17. The equation of the locus of the point whose distances from the point P and the line AB are equal, is –

(A) $9x^2 + y^2 – 6xy – 54x – 62y + 241 = 0$

(B) $x^2 + 9y^2 + 6xy – 54x + 62y – 241 = 0$

(C) $9x^2 + 9y^2 – 6xy – 54x – 62y – 241 = 0$

(D) $x^2 + y^2 – 2xy + 27x + 31y – 120 = 0$

18. The ellipse E_1: $\frac{x^2}{9}+\frac{y^2}{4}=1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0,4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is - **[JEE 2012, 3M, 1M]**

(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$ (D) $\frac{3}{4}$

19. A vertical line passing through the point (h,0) intersects the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ at the points P and Q.Let the tangents to the ellipse at P and Q meet at the point R. If $\Delta(h)$ = area of the triangle PQR, $\Delta_1 = \max\limits_{1/2\le h\le 1}\Delta(h)$ and $\Delta_2 = \min\limits_{1/2\le h\le 1}\Delta(h)$, then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 =$ **[JEE-Advanced 2013, 4, (–1)]**

20. Match the following:

	List-I	List-II	
P.	Let $y(x) = \cos(3\cos^{-1}x)$, $x \in [-1, 1]$, $x \neq \pm\frac{\sqrt{3}}{2}$. Then $\frac{1}{y(x)}\left\{(x^2-1)\frac{d^2y(x)}{dx^2}+x\frac{dy(x)}{dx}\right\}$ equals	**1.**	1
Q.	Let $A_1, A_2, \ldots\ldots, A_n$ ($n > 2$) be the vertices of a regular polygon of n sides with its centre at the origin. $K \times \overrightarrow{a_k}$ be the position vector of the point A_k, $k = 1, 2, \ldots\ldots n$. If $\left\lvert\sum_{k=1}^{n-1}\left(\overrightarrow{a_k}\times\overrightarrow{a_{k+1}}\right)\right\rvert = \left\lvert\sum_{k=1}^{n-1}\left(\overrightarrow{a_k}.\overrightarrow{a_{k+1}}\right)\right\rvert$, then the minimum value of n is	**2.**	2
R.	If the normal from the point P(h, 1) on the ellipse $\frac{x^2}{6}+\frac{y^2}{3}=1$ is perpendicular to the line $x + y = 8$, then the value of h is	**3.**	8
S.	Number of positive solutions satisfying the equation $\tan^{-1}\left(\frac{1}{2x+1}\right)+\tan^{-1}\left(\frac{1}{4x+1}\right)=\tan^{-1}\left(\frac{2}{x^2}\right)$ is	**4.**	9

Codes:

	P	Q	R	S
(A)	4	3	2	1
(B)	2	4	3	1
(C)	4	3	1	2
(D)	2	4	1	3

[JEE(Advanced)-2014, 3(–1)]

R_n : rectangle of largest area, with sides parallel to the axes, inscribed in E_n , $n > 1$

Then which of the following options is/are correct ? **[JEE(Advanced)-2019]**

(A) $\sum_{n=1}^{N}$(area of R_n) < 24, for each positive integer N

(B) The eccentricities of E_{18} and E_{19} are NOT equal

(C) The distance of a focus from the centre in E_9 is $\frac{\sqrt{5}}{32}$

(D) The length of latus rectum of E_9 is $\frac{1}{6}$

27. Let a, b and λ be positive real numbers. Suppose P is an end point of the latus rectum of the parabola $y^2 = 4\lambda x$, and suppose the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through the point P. If the tangents to the parabola and the ellipse at the point P are perpendicular to each other, then the eccentricity of the ellipse is:

[JEE Advance 2020]

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$

(C) $\frac{1}{3}$ (D) $\frac{2}{5}$

28. Let E be the ellipse $\frac{X^2}{16}+\frac{Y^2}{9}=1$. For any three distinct points P, Q and Q' on E, let M(P, Q) be the midpoint of the line segment joining P and Q, and M(P, Q') be the mid-point of the line segment joining P and Q'. Then the maximum possible value of the distance between M(P, Q) and M(P, Q'), as P, Q and Q' vary on E, is _____.

[JEE Advance 2021]

29. Consider the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$. Let $H(\alpha,0), 0<\alpha<2$, be a point. A straight line drawn through H parallel to y-axis crosses the ellipse and its auxiliary circle at points E and F respectively, in the first quadrant. The tangents to the ellipse at the point E intersects the positive x-axis at a point G. Suppose the straight line joining F and the origin makes an angle π with the positive x-axis. **[JEE Advance 2022]**

List-I		List-II	
(I)	If $\phi=\frac{\pi}{4}$, then the area of the triangle FGH is	(P)	$\frac{(\sqrt{3}-1)^4}{8}$
(II)	If $\phi=\frac{\pi}{3}$, then the area of the triangle FGH is	(Q)	1
(III)	If $\phi=\frac{\pi}{6}$, then the area of the triangle FGH is	(R)	$\frac{3}{4}$
(IV)	If $\phi=\frac{\pi}{12}$, then the area of the triangle FGH is	(S)	$\frac{1}{2\sqrt{3}}$
		(T)	$\frac{3\sqrt{3}}{2}$

The correct option is:

(A) (I) → (R); (II) → (S); (III) → (Q); (IV) → (P)

(B) (I) → (R); (II) → (T); (III) → (S); (IV) → (P)

(C) (I) → (Q); (II) → (T); (III) → (S); (IV) → (P)

(D) (I) → (Q); (II) → (S); (III) → (Q); (IV) → (P)

30. Consider the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$. Let $S(p,q)$ be a point in the first quadrant such that $\frac{p^2}{9}+\frac{q^2}{4}>1$. Two tangents are drawn from S to the ellipse, of which one meets the ellipse at one end point of the minor axis and the other meets the ellipse at a point T in the fourth quadrant. Let R be the vertex of the ellipse with positive x-coordinate and O be the center of the ellipse. If the area of the triangle ΔORT is $\frac{3}{2}$, then which of the following options is correct?

[JEE Advance 2024]

(A) $q=2, p=3\sqrt{3}$ (B) $q=2, p=4\sqrt{3}$

(C) $q=1, p=5\sqrt{3}$ (D) $q=1, p=6\sqrt{3}$

ADVANCED & OLYMPIAD CHALLENGER QUESTIONS

1. PG is the normal to a standard ellipse at P, G being on the major axis. GP is produced outwards to Q so that PQ = GP. Show that the locus of Q is an ellipse whose eccentricity is $\frac{a^2-b^2}{a^2+b^2}$.

2. A normal inclined at 45° to the axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is drawn. It meets the x-axis & the y-axis in P & Q respectively. If C is the centre of the ellipse, show that the area of triangle CPQ is $\frac{\left(a^2-b^2\right)^2}{2\left(a^2+b^2\right)}$ sq. units.

3. A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at P & Q. Prove that the tangents at P & Q of the ellipse $x^2 + 2y^2 = 6$ are at right angles.

4. A tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ touches at the point P on it in the first quadrant & meets the coordinate axes in A & B respectively. If P divides AB in the ratio 3 : 1 reckoning from the x-axis find the equation of the tangent.

5. Let P_i and P_i' be the feet of the perpendiculars drawn from foci S, S' on a tangent T_i to an ellipse whose length of semi-major axis is 20. If $\sum_{i=1}^{10}(SP_i)(S'P_i')=2560$, then find the value of 100e.

6. Consider an ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ with centre C and a point P on it with eccentric angle $\frac{\pi}{4}$. Normal drawn at P intersects the major and minor axes in A and B respectively. N_1 and N_2 are the feet of the perpendiculars from the foci S_1 and S_2 respectively on the tangent at P and N is the foot of the perpendicular from the centre of the ellipse on the normal at P. Tangent at P intersects the axis of x at T.

Match the entries of Column-I with the entries of Column-II.

Column-I		Column-II	
(A)	(CA)(CT) is equal to	(P)	9
(B)	(PN)(PB) is equal to	(Q)	16
(C)	$(S_1N_1)(S_2N_2)$ is equal to	(R)	17
(D)	$(S_1P)(S_2P)$ is equal to	(S)	25

7. Rectangle ABCD has area 200. An ellipse with area 200π passes through A and C and has foci at B and D. Find the perimeter of the rectangle.

8. Consider the parabola $y^2 = 4x$ and the ellipse $2x^2 + y^2 = 6$, intersecting at P and Q.

(a) Prove that the two curves are orthogonal.

(b) Find the area enclosed by the parabola and the common chord of the ellipse and parabola.

(c) If tangent and normal at the point P on the ellipse intersect the x-axis at T and G respectively then find the area of the triangle PTG.

9. Consider the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with centre 'O' where $a > b > 0$. Tangent at any point P on the ellipse meets the coordinate axes at X and Y and N is the foot of the perpendicular from the origin on the tangent at P. Minimum length of XY is 36 and maximum length of PN is 4.

(a) Find the eccentricity of the ellipse.

(b) Find the maximum area of an isosceles triangle inscribed in the ellipse if one of its vertex coincides with one end of the major axis of the ellipse.

(c) Find the maximum area of the triangle OPN.

10. A straight line AB touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ & the circle $x^2 + y^2 = r^2$; where $a > r > b$. A focal chord of the ellipse, parallel to AB intersects the circle in P &Q, find the length of the perpendicular drawn from the centre of the ellipse to PQ. Hence show that PQ = 2b.

11. A ray emanating from the point (– 4, 0) is incident on the ellipse $9x^2 + 25y^2 = 225$ at the point P with abscissa 3. Find the equation of the reflected ray after first reflection.

12. Prove that the equation to the circle, having double contact with the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (with eccentricity e) at the ends of a latus rectum, is $x^2 + y^2 – 2ae^3x = a^2 (1 – e^2 – e^4)$.

13. The tangents from (x_1, y_1) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ intersect at right angles. Show that the normals at the points of contact meet on the line $\frac{y}{y_1}=\frac{x}{x_1}$.

14. If the tangent at any point of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ makes an angle α with the major axis and an angle β with the focal radius of the point of contact then show that the eccentricity 'e' of the ellipse is given by the absolute value of $\frac{\cos\beta}{\cos\alpha}$.

ADDITIONAL EXERCISE

SUBJECTIVE TYPE

1. An ellipse $x^2 + 4y^2 = 4$ is rotated anticlockwise through a right angle in its own plane about its centre. If the locus of the point of intersection of a tangent to ellipse in its origin position with the tangent at the same point of the ellipse in its new position is given by the curve $(x^2 + y^2)^2 = \lambda(x^2 + y^2) + \mu xy$ where λ and μ are positive integers. Find the value of $(\lambda + \mu)$.

2. Line L_1 is parallel to the line L_2. Slope of L_1 is 9. Also L_3 is parallel to L_4. Slope of L_4 is $\frac{-1}{25}$. All these lines touch the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$. Find the area of the parallelogram formed by these lines.

3. Given the equation of the ellipse $\frac{(x-3)^2}{16}+\frac{(y+4)^2}{49}=1$, a parabola is such that its vertex is the lowest point of the ellipse and it passes through the ends of the minor axis of the ellipse. The equation of the parabola is in the form $16y = a(x – h)^2 – k$. Determine the value of $(a + h + k)$.

4. Variable pairs of chords at right angles are drawn through any point P (with eccentric angle $\pi/4$) on the ellipse $\frac{x^2}{4} + y^2 = 1$, to meet the ellipse at two points say A and B. If the line joining A and B passes through a fixed point Q(a, b) such that $a^2 + b^2$ has the value equal to $\frac{m}{n}$, where m, n are relatively prime positive integers, find $(m + n)$.

Answer Key

EXERCISE -1

1.	D	**2.**	B	**3.**	C	**4.**	D	**5.**	C	**6.**	A	**7.**	B
8.	C	**9.**	A	**10.**	B	**11.**	A	**12.**	D	**13.**	A	**14.**	A
15.	C	**16.**	(a) D; (b) A,B,C,D	**17.**	C	**18.**	A	**19.**	A				

EXERCISE -2

1. (a) $20x^2 + 45y^2 - 40x - 180y - 700 = 0$; (b) $3x^2 + 5y^2 = 32$ 7. $x + y - 5 = 0$, $x + y + 5 = 0$

8. 16 **9.** 24 sq. units **10.** $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ **11.** $(x-1)^2 + y^2 = \frac{11}{3}$

13. 54 **14.** $a^2p^2 + b^2q^2 = r^2\sec^2\frac{\pi}{8} = \left(4 - 2\sqrt{2}\right)r^2$ **15.** $\frac{18a}{17}$

16. $25y^2 + 4x^2 = 4x^2y^2$ **17.** 85 **18.** 65 **19.** 2

EXERCISE -3

1.	A,B,D	**2.**	A,B	**3.**	A,C	**4.**	A,B	**5.**	A,B,C	**6.**	A,C		
7.	A,B,C	**8.**	A,D	**9.**	A,B	**10.**	B,C,D	**11.**	B,C	**12.**	A	**13.**	B
14.	C	**15.**	B	**16.**	D	**17.**	A	**18.**	B	**19.**	C	**20.**	D

21. (A) S, (B) R, (C) P, (D) Q

JEE MAIN PYQ

1.	A	**2.**	D	**3.**	A	**4.**	D	**5.**	C	**6.**	A	**7.**	C
8.	A	**9.**	D	**10.**	C	**11.**	D	**12.**	A	**13.**	D	**14.**	C
15.	C	**16.**	B	**17.**	A	**18.**	C	**19.**	D	**20.**	B	**21.**	C
22.	B	**23.**	D	**24.**	B	**25.**	C	**26.**	D	**27.**	C	**28.**	B
29.	A	**30.**	B	**31.**	C	**32.**	C	**33.**	A	**34.**	A	**35.**	C
36.	A	**37.**	C	**38.**	C	**39.**	D	**40.**	3	**41.**	3	**42.**	A
43.	A	**44.**	B	**45.**	C	**46.**	B	**47.**	B	**48.**	A	**49.**	B
50.	B	**51.**	4	**52.**	2929	**53.**	75	**54.**	13	**55.**	D	**56.**	C
57.	118	**58.**	7	**59.**	39	**60.**	A	**61.**	D	**62.**	C	**63.**	A
64.	D	**65.**	A	**66.**	54	**67.**	B	**68.**	B	**69.**	A	**70.**	C

OLYMPIAD WALLAH

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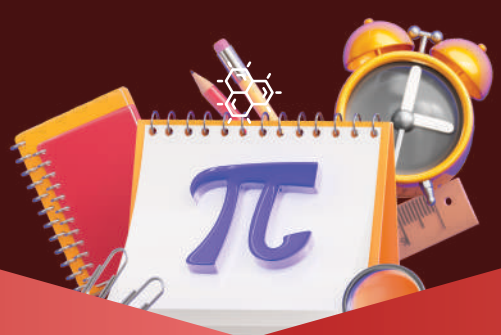

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