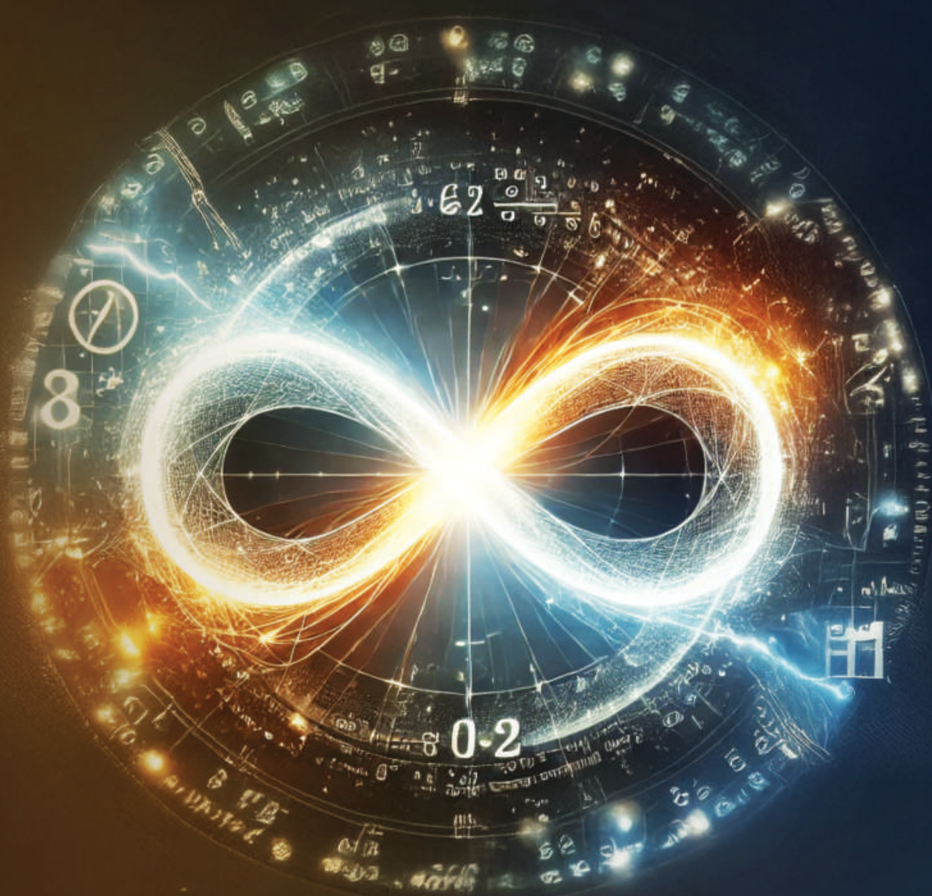
PHYSICS WALLAH

LAKSHYA JEE

CLASS-XII

- Matrices and Determinants
- Relations and Functions
- Inverse Trigonometric Functions



MATHEMATICS

Module 1

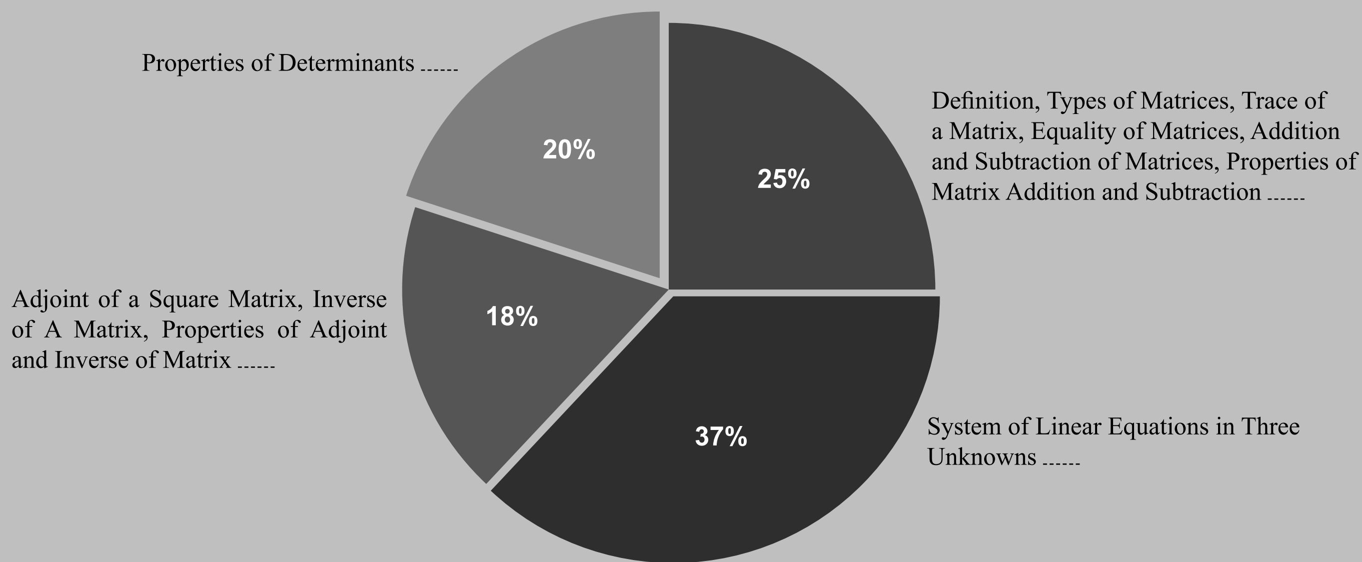
CHAPTER 1

Matrices and Determinants

$M = \begin{bmatrix} m & n \\ o & p \end{bmatrix}$

$|M| = mp - no$

Topicwise Weightage of JEE Main 6 Years Paper (124 Sets)



"How's the Josh?" for these Topics: *Mark your confidence level in the blank space around the topic (Low-L, Medium-M, High-H)*

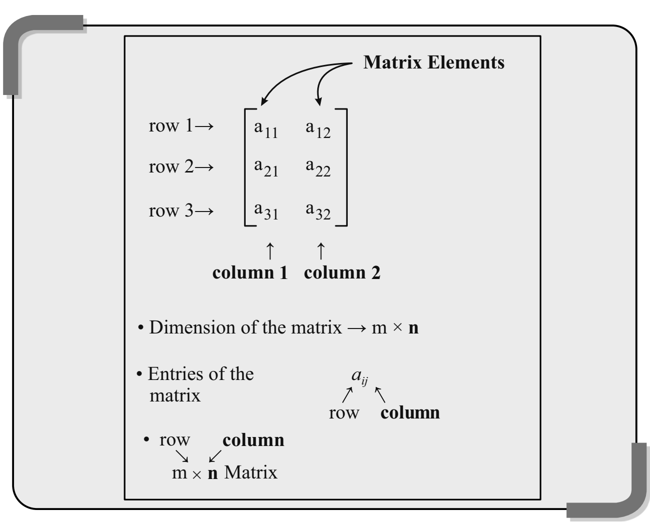
INTRODUCTION

A matrix is a definite collection of quantities like numbers, symbols, or expressions, arranged in a tabular form of rows and columns. Basically it is ordered arrangement of data.

The order of a matrix is written as the number of rows by the number of columns.

For e.g. a 2 × 2 matrix consists of 2 rows and 2 columns. It has a total of 4 elements.

If a matrix has m rows and n columns then the **order** of matrix is written as $m \times n$ and we call it as order m by n



The general $m \times n$ matrix is

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots\ldots & a_{1j} & \ldots\ldots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \ldots\ldots & a_{2j} & \ldots\ldots & a_{2n} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{i1} & a_{i2} & a_{i3} & \ldots\ldots & a_{ij} & \ldots\ldots & a_{in} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{m1} & a_{m2} & a_{m3} & \ldots\ldots & a_{mj} & \ldots\ldots & a_{mn} \end{bmatrix}$$

where a_{ij} denote the element of i^{th} row and j^{th} column. The above matrix is usually denoted as $[a_{ij}]_{m\times n}$.

Now any matrix of order $m \times n$ will have the notation $[a_{ij}]_{m\times n}$.

i.e., $A = [a_{ij}]_{m\times n}$ or $(a_{ij})_{m\times n}$ or $\|a_{ij}\|_{m\times n}$

it is trivial that $1 \le i \le m$ and $1 \le j \le n$

e.g., $A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}, B = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix},$

$$C = \begin{bmatrix} -1 & 3 & c \\ 2 & 9 & 4 \end{bmatrix}, D = [1, 4, 9]$$

PTR (Points to Remember):

1. Generally capital letters of English alphabets are used to denote matrices.
2. The matrix is not a number. It has no numerical value.

Train Your Brain

Example 1: Consider a 3 × 4 matrix $A = [a_{ij}]$, whose elements are given by $a_{ij} = 2i + 3j$, then A =

(a) $\begin{bmatrix} 5 & 8 & 11 & 14 \\ 7 & 10 & 13 & 16 \\ 9 & 12 & 15 & 18 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 8 & 11 & 14 \\ 14 & 10 & 13 & 16 \\ 9 & 12 & 15 & 18 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 8 & 11 & 14 \\ 14 & 10 & 8 & 16 \\ 9 & 12 & 15 & 18 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 8 & 11 & 14 \\ 14 & 10 & 8 & 5 \\ 9 & 12 & 15 & 18 \end{bmatrix}$

Sol. In this problem, i and j are the number of rows and columns respectively. By substituting the respective values of rows and columns in $a_{ij} = 2i + 3j$ we can construct the required matrix.

We have $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$;

$a_{11} = 2 \times 1 + 3 \times 1 = 5$; $a_{12} = 2 \times 1 + 3 \times 2 = 8$

Similarly, $a_{13} = 11$, $a_{14} = 14$, $a_{21} = 7$, $a_{22} = 10$, $a_{23} = 13$, $a_{24} = 16$, $a_{31} = 9$, $a_{32} = 12$, $a_{33} = 15$, $a_{34} = 18$

$$\therefore A = \begin{bmatrix} 5 & 8 & 11 & 14 \\ 7 & 10 & 13 & 16 \\ 9 & 12 & 15 & 18 \end{bmatrix}$$

Example 2: Construct a 3 × 4 matrix, whose elements are given by: $a_{ij} = \frac{1}{2}|-3i + j|$

Sol. $A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

Concept Application

1. The order of $\begin{bmatrix} 3 & -1 & 5 \\ 6 & 2 & -7 \end{bmatrix}$ matrix is ______ .

2. The number of different possible orders of matrices having 18 identical elements are ______ .

TYPES OF MATRIX

Row Matrix

A matrix having only one row is called as row matrix (or row vector). General form of row matrix is

$A = [a_{11}\ a_{12}\ a_{13}\ \ldots\ a_{1n}]$

This is a matrix of order "1 × n" (or a row matrix of order n).

Column Matrix

A matrix having only one column is called *a* column matrix (or column vector).

Column matrix is of the form $A = \begin{bmatrix} a_{11} \\ a_{21} \\ \ldots \\ a_{m1} \end{bmatrix}$.

This is a matrix of order "m × 1" (or a column matrix of order m).

Zero or Null Matrix

$A = [a_{ij}]_{m\times n}$ is called a zero matrix, if $a_{ij} = 0\ \forall\ i$ and j.

e.g., (i) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Singleton Matrix

If in a matrix there is only one element then it is called singleton matrix. Thus, $A = [a_{ij}]_{m\times n}$ is a singleton matrix if $m = n = 1$. E.g. [2], [3], [a], [–3] are singleton matrices.

Horizontal Matrix

A matrix of order $m \times n$ is a horizontal matrix if $n > m$; E.g. $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 1 & 1 \end{bmatrix}$

Vertical Matrix

A matrix of order $m \times n$ is a vertical matrix if $m > n$; E.g. $\begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 3 & 6 \\ 2 & 4 \end{bmatrix}$

Square Matrix

A matrix in which number of rows and columns are equal is called a square matrix. The general form of a square matrix is

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots\ldots & a_{1n} \\ a_{21} & a_{22} & \ldots\ldots & a_{2n} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{n1} & a_{n2} & \ldots\ldots & a_{nn} \end{bmatrix}$$

which we denote as $A = [a_{ij}]_{n\times n}$

This is a matrix of order "n × n" (or a square matrix of order n).

Diagonal Elements: An element of a matrix $A = [a_{ij}]$ is said to be diagonal element if $i = j$. Thus an element whose row suffix equals to the column suffix is a diagonal element e.g., a_{11}, a_{22}, a_{33}, ... are all diagonal element.

Aarambh (Solved Examples)

1. If $A = [a_{ij}]_{3\times3}$, such that $a_{ij} = \begin{cases} 2, \text{when } i = j \\ 0, \text{when } i \neq j \end{cases}$, then

$\log_{1/2}\det(\text{adj (adj }(A)) =$

(*a*) -12 (*b*) -10

(*c*) -13 (*d*) -11

Sol. $A = [a_{ij}]$

$a_{ij} = 2, i = j = 0\ i \neq j$

$|A| = 8$

Now, $\det(\text{adj (adj }(A)) = (8)^4 = 2^{12}$

$\log_{1/2}(2^{12}) = -12$

Therefore, option (*a*) is the correct answer.

2. Find $c^2 + x^2 + y^2$ if the matrix A given by

$$A = \begin{bmatrix} a & 2/3 & 2/3 \\ 2/3 & 1/3 & b \\ c & x & y \end{bmatrix}$$ - is orthogonal.

(*a*) 1 (*b*) 2 (*c*) 0 (*d*) 3

Sol. It is given that the matrix A is orthogonal. Therefore, $AA' = I$

$$\begin{bmatrix} a & 2/3 & 2/3 \\ 2/3 & 1/3 & b \\ c & x & y \end{bmatrix}\begin{bmatrix} a & 2/3 & c \\ 2/3 & 1/3 & x \\ 2/3 & b & y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Comparing the element in the 3rd column of 3rd row, we get $c^2 + x^2 + y^2 = 1$.

Therefore, option (*a*) is the correct answer.

3. Let t be the trace of the matrix

$$A = \begin{bmatrix} \frac{|x+y|}{|x|+|y|} & \alpha_1 & \beta_1 \\ \alpha_2 & \frac{|y+z|}{|y|+|z|} & \beta_2 \\ \alpha_3 & \beta_3 & \frac{|z+x|}{|z|+|x|} \end{bmatrix}, \text{ then}$$

(*a*) $0 \le t \le 3$ (*b*) $1 \le t \le 2$

(*c*) $1 \le t \le 3$ (*d*) $-1 \le t \le 1$

Sol. Now, $|x + y| \le |x| + |y|$

$$\Rightarrow \frac{|x+y|}{|x|+|y|} \le 1$$

Hence, $t \le 3$

Also among the diagonal elements, at least one of the element must be 1.

Therefore, option (*c*) is the correct answer.

4. If $P = \begin{bmatrix} \cos(\pi/6) & \sin(\pi/6) \\ -\sin(\pi/6) & \cos(\pi/6) \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP'$ then $P'Q^{2009}P$ is equal to

(*a*) $\begin{bmatrix} 1 & \sqrt{3}/2 \\ 0 & 2009 \end{bmatrix}$ (*b*) $\begin{bmatrix} 1 & 2009 \\ 0 & 1 \end{bmatrix}$

(*c*) $\begin{bmatrix} \sqrt{3}/2 & 2009 \\ 0 & 1 \end{bmatrix}$ (*d*) $\begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1 & 2009 \end{bmatrix}$

Sol. Now, $P'P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

$$\Rightarrow P'P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \Rightarrow P'P = I \Rightarrow P' = P^{-1}$$

Since $Q = PAP'$

$\therefore\ P'Q^{2009}P$

$= P'[(PAP')(PAP') \ldots 2009 \text{ times}]P$

$$= \underbrace{(P'P)A(P'P)A(P'P)A\ldots(P'P)A(P'P)}_{2009 \text{ times}}$$

$= IA^{2009} = A^{2009}$

$$\therefore\ A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \Rightarrow = A^{2009} = (I + B)^{2009}$$

where $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. As $B^2 = O$, we get $B^r = O$ $\forall\ r \ge 2$.

Thus, by binomial theorem

$$A^{2009} = \begin{bmatrix} 1 & 2009 \\ 0 & 1 \end{bmatrix}$$

Therefore, option (*b*) is the correct answer.

5. If T_p, T_q, T_r are the p^{th}, q^{th} and r^{th} terms of an A.P., then $\begin{vmatrix} T_p & T_q & T_r \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ equals

(*a*) 1 (*b*) -1

(*c*) 0 (*d*) $p + q + r$

Board Level Problems

SINGLE CORRECT TYPE QUESTIONS

1. If a matrix has 36 elements, the number of possible orders it can have, is:

(*a*) 13 (*b*) 3
(*c*) 5 (*d*) 9

2. If $\begin{bmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{bmatrix} = kabc$, then the value of k is:

(*a*) 0 (*b*) 1
(*c*) 2 (*d*) 4

3. If for the matrix A = $\begin{bmatrix} \tan x & 1 \\ -1 & \tan x \end{bmatrix}$, $A + A' = 2\sqrt{3}I$, then the value of $x \in \left[0, \frac{\pi}{2}\right]$ is:

(*a*) 0 (*b*) $\frac{\pi}{4}$
(*c*) $\frac{\pi}{3}$ (*d*) $\frac{\pi}{6}$

4. If A = $[a_{ij}]$ is an identity matrix, then which of the following is true?

(*a*) $a_{ij} = \begin{cases} 0, & \text{if } i = j \\ 1, & \text{if } i \neq j \end{cases}$ (*b*) $a_{ij} = 1, \forall i, j$
(*c*) $a_{ij} = 0, \forall i, j$ (*d*) $a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$

5. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix such that adj $A = A$. Then, $(a + b + c + d)$ is equal to:

(*a*) $2a$ (*b*) $2b$
(*c*) $2c$ (*d*) 0

6. If $\begin{vmatrix} 1 & 3 & 1 \\ k & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \pm 6$, then the value of k is:

(*a*) 2 (*b*) –2
(*c*) ±2 (*d*) ∓2

7. If $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix, then the value of $a + 2b + 3c + 4d$ is:

(*a*) 0 (*b*) 5
(*c*) 10 (*d*) 25

8. The equations $x + 2y + 3z = 1$, $x - y + 4z = 0$, $2x + y + 7z = 1$ have

(*a*) Only one solution (*b*) Two solutions
(*c*) No solution (*d*) Infinitely many solutions

9. If the system of equations $x + ay = 0$, $az + y = 0$ and $ax + z = 0$ has infinite solutions, then the value of a, is

(*a*) –1 (*b*) 1
(*c*) 0 (*d*) No real values

VERY SHORT ANSWER TYPE QUESTIONS

10. If a matrix has 28 elements, what are the possible orders it can have? What if it has 2027 elements?

11. If $A = \begin{bmatrix} 1 & \cot x \\ -\cot x & 1 \end{bmatrix}$, show that

$$A'A^{-1} = \begin{bmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{bmatrix}$$

12. Find values of x, if $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

13. Find |AB|, if $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$.

14. Verify that $A^2 = I$ when $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$.

15. If $A = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$, find a matrix C such that $3A + 5B + 2C$ is a null matrix.

16. Evaluate $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$

LONG ANSWER TYPE QUESTIONS

17. If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations:

$x - 2y = 10$, $2x - y - z = 8$, $-2y + z = 7$

18. Solve the following system of equations, using matrices:

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

where $x, y, z \neq 0$

19. Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.

20. If $A=\begin{bmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}$, find A^{-1} and hence solve the following system of equaitons:

$2x + y - 3z = 13$
$3x + 2y + z = 4$
$x + 2y - z = 8$

21. Consider the matrix $A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and find k such that $A^2 = kA - 7I$.

CASE STUDY BASED QUESTIONS

CASE STUDY-I

If A and B are square matrix of order 3 given by

$$A=\begin{bmatrix} 1 & 2 & 4 \\ 4 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix},\ B=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

On the basis of above information, answer the following questions:

22. $|\text{adj}\ (\text{adj}\ A)|$ is equal to

(*a*) 10^2 (*b*) 100^3
(*c*) 10^4 (*d*) None of these

23. $|\text{adj}\ (AB)|$ is equal to

(*a*) 100
(*b*) 1000
(*c*) 10^4
(*d*) None of these

CASE STUDY-II

Let A and B are two matrices of same order i.e. 3×3 where

$$A=\begin{bmatrix} 1 & -3 & 2 \\ 2 & k & 5 \\ 4 & 2 & 1 \end{bmatrix}\ B=\begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 4 \\ 3 & 3 & 5 \end{bmatrix}$$

On the basis of above information, answer the following questions:

24. If $k = 2$ then $\text{tr}(AB) + \text{tr}(BA)$ is equal to

(*a*) 66 (*b*) 42
(*c*) 84 (*d*) 63

25. If $C = A + 3B$ and $\text{tr}(C) = 0$, then k is equal to

(*a*) –10 (*b*) –20
(*c*) –29 (*d*) –39

Prarambh (Topicwise)

TYPES OF MATRICES, ADDITION, SUBTRACTION AND EQUALITY OF MATRICES

1. The number of different possible orders of matrices having 12 identical elements is

(*a*) 3 (*b*) 1 (*c*) 6 (*d*) 2

2. In an upper triangular matrix $A = [a_{ij}]_{n \times n}$ the elements $a_{ij} = 0$ for

(*a*) $i < j$ (*b*) $i = j$ (*c*) $i > j$ (*d*) $i \leq j$

3. $\begin{bmatrix} x^2+x & x \\ 3 & 2 \end{bmatrix}+\begin{bmatrix} 0 & -1 \\ -x+1 & x \end{bmatrix}=\begin{bmatrix} 0 & -2 \\ 5 & 1 \end{bmatrix}$

then x is equal to

(*a*) – 1 (*b*) 2
(*c*) 1 (*d*) No value of x

4. Which of the following is a diagonal matrix

(*a*) $\begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ (*b*) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(*c*) $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (*d*) None of these

TRACE OF MATRIX

5. If trace of matrix A=$\begin{bmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ -5 & 1 & x \end{bmatrix}$ is 5 then x is

(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4

MULTIPLICATION OF MATRICES AND IDENTITY MATRIX

6. If $\begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix} X = \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$, then $X =$

(*a*) $\begin{bmatrix} -3 & 4 \\ 14 & -13 \end{bmatrix}$ (*b*) $\begin{bmatrix} 3 & -4 \\ -14 & 13 \end{bmatrix}$

(*c*) $\begin{bmatrix} 3 & 4 \\ 14 & 13 \end{bmatrix}$ (*d*) $\begin{bmatrix} -3 & 4 \\ -14 & 13 \end{bmatrix}$

7. If $M = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $M^2 - \lambda M - I_2 = O$, then $\lambda =$

(*a*) –2 (*b*) 2

(*c*) –4 (*d*) 4

8. If $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ and A^2 is the identity matrix, then $x =$

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 0

9. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ and $B = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$, then the correct relation is

(*a*) $A^2 = B^2$

(*b*) $A + B = B - A$

(*c*) $AB = BA$

(*d*) $AB = 0$

10. If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ and I is the identity matrix of order 2, then $(A - 2I)(A - 3I) =$

(*a*) I (*b*) O

(*c*) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ (*d*) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

11. If A and B are two matrices such that AB = B and BA = A, then $A^2 + B^2 =$

(*a*) 2AB (*b*) 2BA

(*c*) A + B (*d*) AB

12. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$, then

(*a*) $A^3 + 3A^2 + A - 9I_3 = 0$

(*b*) $A^3 - 3A^2 + A + 9I_3 = 0$

(*c*) $A^3 + 3A^2 - A + 9I_3 = 0$

(*d*) $A^3 - 3A^2 - A + 9I_3 = 0$

TRANSPOSE OF MATRICES AND OTHER PROPERTIES

13. If $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 5 \\ 2 & -5 & 0 \end{bmatrix}$, then

(*a*) $A' = A$ (*b*) $A' = -A$

(*c*) $A' = 2A$ (*d*) $A' = -2A$

14. For the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & \lambda & 5 \end{bmatrix}$ to be non-singular, λ should not be equal to

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

15. If $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$, then $(AB)^T =$

(*a*) $\begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$ (*b*) $\begin{bmatrix} -3 & 10 \\ -2 & 7 \end{bmatrix}$

(*c*) $\begin{bmatrix} -3 & 10 \\ 7 & -2 \end{bmatrix}$ (*d*) $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

16. Which of the following is incorrect?

(*a*) $A^2 - B^2 = (A + B)(A - B)$

(*b*) $(A^T)^T = A$

(*c*) $(AB)^n = A^n B^n$, where A, B commute

(*d*) $(A - I)(I + A) = O \Leftrightarrow A^2 = I$

SYMMETRIC & SKEW-SYMMETRIC MATRIX

17. If A is a skew- symmetric matrix of odd order, then trace of A is

(*a*) 1 (*b*) –1 (*c*) $|A|$ (*d*) 2

18. For what value of x, is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetric matrix?

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

19. Out of the following a skew-symmetric matrix is

(*a*) $\begin{bmatrix} 0 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & 0 \end{bmatrix}$ (*b*) $\begin{bmatrix} 1 & 4 & 5 \\ -4 & 1 & -6 \\ -5 & 6 & 1 \end{bmatrix}$

(*c*) $\begin{bmatrix} 1 & 4 & 5 \\ -4 & 2 & -6 \\ -5 & 6 & 3 \end{bmatrix}$ (*d*) $\begin{bmatrix} i+1 & 4 & 5 \\ -4 & i & -6 \\ -5 & 6 & i \end{bmatrix}$

62. If $\Delta(x)=\begin{vmatrix}\sin^2 x & \log\cos x & \log\tan x\\ n^2 & 2n-1 & 2n+1\\ 1 & -2\log 2 & 0\end{vmatrix}$ then evaluate

$\int_0^{\pi/2}\Delta(x)dx$

(*a*) –1 (*b*) 0 (*c*) 1 (*d*) 2

63. Let $\lim_{x\to\infty}\frac{f(x)}{x^2}$ then find

$f(x)=\begin{vmatrix}x(x+1) & x^2-1 & x^2+5x+6\\ 1 & 2 & 3\\ 1 & -1 & 1\end{vmatrix}$

(*a*) –1 (*b*) 0 (*c*) 1 (*d*) 4

64. The determinant $\begin{vmatrix}\cos(\theta+\phi) & -\sin(\theta+\phi) & \cos 2\phi\\ \sin\theta & \cos\theta & \sin\phi\\ -\cos\theta & \sin\theta & \cos\phi\end{vmatrix}$ is

(*a*) 0

(*b*) independent of θ

(*c*) independent of ϕ

(*d*) independent of θ and ϕ both

65. If $l_i^2+m_i^2+n_i^2=1$ and $l_il_j+m_im_j+n_in_j=0\ \forall\ i,j\in\{1,2,3\}$, $i\neq j$ and $\Delta=\begin{vmatrix}l_1 & m_1 & n_1\\ l_2 & m_2 & n_2\\ l_3 & m_3 & n_3\end{vmatrix}$ then

(*a*) $|\Delta|=3$ (*b*) $|\Delta|=2$

(*c*) $|\Delta|=1$ (*d*) $\Delta=0$

66. If $f_r(x)$, $g_r(x)$, $h_r(x)$, $r=1,2,3$ are polynomials in x such that $f_r(a)=g_r(a)=h_r(a)$, $r=1,2,3$ and $F(x)=\begin{vmatrix}f_1(x) & f_2(x) & f_3(x)\\ g_1(x) & g_2(x) & g_3(x)\\ h_1(x) & h_2(x) & h_3(x)\end{vmatrix}$ then value of $F'(x)$ at $x=a$ is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 0

CRAMER'S RULE: SYSTEM OF LINEAR EQUATIONS

67. $x+ky-z=0$, $3x-ky-z=0$ and $x-3y+z=0$ has non-zero solution for $k=$

(*a*) –1 (*b*) 0 (*c*) 1 (*d*) 2

68. The number of solutions of equations $x+4y-z=0$, $3x-4y-z-0$, $x-3y+z=0$ is

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) Infinite

69. If the system of equations, $x+2y-3z=1$, $(k+3)z=3$, $(2k+1)x+z=0$ is inconsistent, then the value of k is

(*a*) –3 (*b*) 1/2

(*c*) 0 (*d*) 2

70. If the system of equation $3x-2y+z=0$, $\lambda x-14y+15z=0$, $x+2y+3z=0$ have a non-trivial solution, then $\lambda=$

(*a*) 5 (*b*) –5

(*c*) –29 (*d*) 29

71. The existence of the unique solution of the system $x+y+z=\lambda$, $5x-y+\mu z=10$, $2x+3y-z=6$ depends on

(*a*) μ only (*b*) λ only

(*c*) λ and μ both (*d*) Neither λ nor μ

Prabal (JEE Main Level)

1. If α and β are the roots of the equation

$$[1\ \ 5]\begin{bmatrix}1 & 3\\ -4 & 7\end{bmatrix}^2\begin{bmatrix}\frac{7}{19} & -\frac{3}{19}\\ \frac{4}{19} & \frac{1}{19}\end{bmatrix}^4\begin{bmatrix}1 & 3\\ -4 & 7\end{bmatrix}^2\begin{bmatrix}x^2-5x+5\\ -3\end{bmatrix}=[-4]$$

then the value of (2 – α) (2 – β) is

(*a*) 51 (*b*) –12

(*c*) 12 (*d*) –7

2. Let $S=\left\{\begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix}: a_{ij}\in\{-1,0,1\}\right\}$

then the number of symmetric matrices with trace equals zero, is

(*a*) 729 (*b*) 189

(*c*) 162 (*d*) 27

3. If $A_1, A_3, \ldots\ldots\ldots\ A_{2n-1}$ are n skew-symmetric matrices of same order, then $B=\sum_{r=1}^{n}(2r-1)(A_{2r-1})^{2r-1}$ will be

(*a*) Symmetric

(*b*) Skew-symmetric

(*c*) Neither sysmmetric nor skew-symmetric

(*d*) Data is insufficient

4. A skew-symmetric matrix A satisfies the relation $A^2+I=O$, where I is a unit matrix. Then A is

(*a*) Idempotent matrix (*b*) Orthogonal matrix

(*c*) Nilpotent matrix (*d*) Non-periodic matrix

5. A, B, C are three matrices of the same order such that any two are symmetric and the 3rd one is skew-symmetric. If X = ABC + CBA and Y = ABC – CBA, then $(XY)^T$ is

(*a*) Symmetric (*b*) Skew-symmetric

(*c*) I – XY (*d*) – YX

6. If A and B are square matrices of order 3 such that $|A| = 3$ and $|B| = 2$, then value of $|A^{-1}\text{adj } B^{-1}\text{adj}(3A^{-1})|$ is equal to

(*a*) 27 (*b*) $\frac{27}{4}$

(*c*) $\frac{1}{108}$ (*d*) $\frac{1}{4}$

7. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{bmatrix}, B=\begin{bmatrix}1 & 0\\ -1 & 1\end{bmatrix}$, C = ABA^T, then A^TC^nA equals to ($n \in I^+$)

(*a*) $\begin{bmatrix}-n & 1\\ 1 & 0\end{bmatrix}$ (*b*) $\begin{bmatrix}1 & -n\\ 0 & 1\end{bmatrix}$

(*c*) $\begin{bmatrix}0 & 1\\ 1 & -n\end{bmatrix}$ (*d*) $\begin{bmatrix}1 & 0\\ -n & 1\end{bmatrix}$

8. Which of the following is/are correct?

(*a*) If A and B are two square matrices of order 3 and A is a non-singular matrix such that AB = O, then B must not be a null matrix.

(*b*) If A, B, C are three square matrices of order 2 and det.(A) = 2, det.(B) = 3, det.(C) = 4, then the value of det. (3ABC) is 216.

(*c*) If A is a square matrix of order 3 and det. (A) = $\frac{1}{2}$, then det. (adj. A^{-1}) is 8.

(*d*) Every skew-symmetric matrix is singular.

9. If A and B are two square matrices such that AB = A and BA = B, then A and B are

(*a*) Idempotent matrices (*b*) Involutory matrices

(*c*) Orthogonal matrices (*d*) Nilpotent matrices

10. Let three matrices

$A=\begin{bmatrix}2 & 1\\ 4 & 1\end{bmatrix}; B=\begin{bmatrix}3 & 4\\ 2 & 3\end{bmatrix}$ and $C=\begin{bmatrix}3 & -4\\ -2 & 3\end{bmatrix}$, then

$$tr(A)+tr\left(\frac{ABC}{2}\right)+tr\left(\frac{A(BC)^2}{4}\right)+tr\left(\frac{A(BC)^3}{8}\right)+\ldots\infty=$$

(*a*) 6 (*b*) 9

(*c*) 12 (*d*) None of these

11. Let $P=\begin{bmatrix}1 & 0 & 0\\ a & 1 & 0\\ b & c & 1\end{bmatrix}$ and I be the identity matrix of order 3.

If $Q = [q_{ij}]$ is a matrix such that $P^{15} + Q = I$, then

(*a*) $q_{23} = q_{11}$ (*b*) $q_{32} = 15c$

(*c*) $q_{21} = 15a$ (*d*) $q_{22} \neq 0$

12. For a given matrix A = $\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$, which of the following statement holds good?

(*a*) $A = A^{-1}\ \forall\theta \in R$

(*b*) A is symmetric, for $\theta = (2n + 1)\frac{\pi}{2}, n \in I$

(*c*) A is an orthogonal matrix for $\theta \in R$

(*d*) A is a skew symmetric, for $\theta = n\pi;\ n \in I$

13. If matrix $A = [a_{ij}]_{3\times3}$, matrix $B = [b_{ij}]_{3\times3}$, where $a_{ij} + a_{ji} = 0$ and $b_{ij} - b_{ji} = 0\ \forall\ i, j$, then $A^{2022} B^{2021}$ is

(*a*) Singular (*b*) Zero matrix

(*c*) Skew-Symmetric matrix (*d*) None of these

14. If $A=\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & a & 1\end{bmatrix}, B=\text{Adj }A=\begin{bmatrix}-1 & 1 & -1\\ 8 & -6 & c\\ -5 & 3 & -1\end{bmatrix}$, then

(*a*) $a = 1, c = 2$ (*b*) $a = 2, c = -\frac{1}{2}$

(*c*) $a = -1, c = 2$ (*d*) $a = \frac{1}{2}, c = \frac{1}{2}$

15. If AB = A and BA = B, then incorrect statement is

(*a*) $A^2B = A^2$ (*b*) $B^2A = B^2$

(*c*) ABA = A (*d*) BAB = B + I

16. Let A = $[a_{ij}]_{3\times3}$ be such that $a_{ij}=\begin{cases}3; \text{ when } i=j\\ 0; i\neq j\end{cases}$, then $\left\{\frac{\det(\text{adj}(\text{adj}A))}{2025}\right\}$ equals (where {.} denotes fractional part function)

(*a*) $\frac{3}{25}$ (*b*) $\frac{11}{25}$

(*c*) $\frac{11}{10}$ (*d*) $\frac{14}{10}$

17. If P is an orthogonal matrix and $Q = PAP^T$ and $X = P^T Q^{1000} P$, then X^{-1} is, where A is involutary matrix:

(*a*) A (*b*) I

(*c*) A^{999} (*d*) None of these

18. If n_1, n_2 are integers then $\begin{vmatrix}n_1 & n_2\\ 1 & 1\end{vmatrix}\begin{vmatrix}n_1 & n_2+1\\ -1 & 1\end{vmatrix}$ is always multiple of

(*a*) 2 (*b*) 4

(*c*) 3 (*d*) 5

45. If P is idempotent and Q is involuntary matrices of order 3 and $(I-P)^3-(I-Q)^3=n_1I+n_2P+n_3Q$ then find $n_1+n_2+n_3$

46. Elements of a matrix A of order 10×10 are defined as $a_{ij}=\omega^{i+j}$ (where ω is due cube root of unity), then $|P|$ is (where P = trace (A) of the matrix)

47. The number of all 4×4 matrices A, with enteries from the set $\{0, 1, 2, 3\}$ such that the sum of the diagonal elements of AA^T is 4, is

48. If A is a matrix such that $A\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}-1\\2\end{bmatrix}$ and $A^2\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$, then sum of square of elements in A is

49. If $x, y, z\in R$ & $\Delta=\begin{vmatrix}x & x+y & x+y+z\\2x & 5x+2y & 7x+5y+2z\\3x & 7x+3y & 9x+7y+3z\end{vmatrix}=-16$ then value of x is

50. If $0\le\theta\le\pi/2$, find the number of roots of

$$\Delta(\theta)=\begin{vmatrix}\cos^2 2\theta & \cos^2 4\theta & \cos^2 6\theta\\ \sin 2\theta & \sin 4\theta & \sin 6\theta\\ 1 & 1 & 1\end{vmatrix}$$

51. If x, y, z are in A.P, lying between 1 and 9, and $x51$, $y41$ and $z31$ are three digit numbers, then value of $\begin{vmatrix}5 & 4 & 3\\ x51 & y41 & z31\\ x & y & z\end{vmatrix}$ is

52. If $x>m, y>n, z>r$ $(x, y, z>0)$ such that $\begin{vmatrix}x & n & r\\ m & y & r\\ m & n & z\end{vmatrix}=0$ then find the greatest value of $\dfrac{27xyz}{(x-m)(y-n)(z-r)}$.

53. If the value of the determinant

$$\begin{vmatrix}\sqrt{(13)}+\sqrt{3} & 2\sqrt{5} & \sqrt{5}\\ \sqrt{(15)}+\sqrt{(26)} & 5 & \sqrt{(10)}\\ 3+\sqrt{(65)} & \sqrt{(15)} & 5\end{vmatrix}$$

$=a(\sqrt{b})(c-\sqrt{6})(a,b,c\in I)$ then $a+b+c$ is

54. If $\Delta=\begin{vmatrix}f(x) & f\left(\frac{1}{x}\right)+f(x)\\ 1 & f\left(\frac{1}{x}\right)\end{vmatrix}=0$

where it is given $f(2)=17$, then $f(5)$ is equal to:

55. Let $a=\lim\limits_{x\to 1}\dfrac{x}{\ln x}-\dfrac{1}{x\ln x}$;

$b=\lim\limits_{x\to 0}\dfrac{x^3-16x}{4x+x^2}$;

$c=\lim\limits_{x\to 0}\dfrac{\ln(1+\sin x)}{x}$ and

$d=\lim\limits_{x\to -1}\dfrac{(x+1)^3}{3[\sin(x+1)-(x+1)]}$, then value of det($A$) where $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$

56. If $a_1, a_2, a_3, 5, 4, a_6, a_7, a_8, a_9$ are in H.P. and $\Delta=\begin{vmatrix}a_1 & a_2 & a_3\\ 5 & 4 & a_6\\ a_7 & a_8 & a_9\end{vmatrix}$ then find the value of $[\Delta]$ is, where [.] denotes G.I.F.

57. If $\begin{vmatrix}(\beta+\gamma-\alpha-\delta)^4 & (\beta+\gamma-\alpha-\delta)^2 & 1\\ (\gamma+\alpha-\beta-\delta)^4 & (\gamma+\alpha-\beta-\delta)^2 & 1\\ (\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta-\gamma-\delta)^2 & 1\end{vmatrix}$

$=-k(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\beta-\gamma)(\beta-\delta)(\gamma-\delta)$. Then the value of $(k)^{1/2}$ is

Parikshit (JEE Advanced Level)

SINGLE CORRECT TYPE QUESTIONS

1. Let $M=\begin{bmatrix}a & b & c\\ d & e & f\\ 1 & 1 & 1\end{bmatrix}$ and $N=\dfrac{M^2}{2}$. If $(a-b)^2+(d-e)^2=36$, $(b-c)^2+(e-f)^2=64$, $(a-c)^2+(d-f)^2=100$, then value of $|N|$ is equal to

(*a*) 1152 (*b*) 48

(*c*) 144 (*d*) 288

2. If $\left\{\begin{bmatrix}5 & 1 & 4\\ 7 & 6 & 2\\ 1 & 3 & 5\end{bmatrix}\begin{bmatrix}1 & 6 & -7\\ 6 & 2 & 4\\ -7 & 4 & 3\end{bmatrix}\begin{bmatrix}5 & 7 & 1\\ 1 & 6 & 3\\ 4 & 2 & 5\end{bmatrix}\right\}^{2020}$

$=\begin{bmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{bmatrix}$ then the value of $2|a_2-b_1|+3|a_3-c_1|$ $+4|b_3-c_2|$ is equal to

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

3. If $P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$,

$P_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$,

$P_5 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $P_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$tr[(P_1^{-1})^T (P_2^{-2})^T + (P_3^{-1})^T (P_4) + P_6^T P_5]$

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 5

4. Identify the correct statement:
   (*a*) D is a 3 × 3 diagonal matrix. AD = DA for every matrix A of order 3 × 3
   (*b*) If system of *n* simultaneous linear equations has a unique solution, then coefficient matrix is non singular
   (*c*) If A^{-1} exists, $(\text{adj}A)^{-1}$ may or may not exist
   (*d*) $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 0 \end{bmatrix}$, then $F(x) \cdot F(y) = F(x - y)$

5. Characteristic equation of matrix A of order 3, is (where S_1 = Sum of the main diagonal elements, $S_3 = |A|$, S_2 = Sum of the minors of the main diagonal elements)
   (*a*) $\lambda^3 - S_1\lambda^2 - S_2\lambda + S_3 = 0$
   (*b*) $\lambda^3 - S_1\lambda^2 + S_2\lambda + S_3 = 0$
   (*c*) $\lambda^3 - S_1\lambda^2 - S_2\lambda - S_3 = 0$
   (*d*) $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

6. Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$. Find sum of elements of P such that $BPA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$
   (*a*) 1 (*b*) –1
   (*c*) 0 (*d*) 2

7. If A is matrix such that $A^2 + A + 2I = O$, then which of the following is INCORRECT?
   (*a*) A is non-singular (*b*) $A \neq O$
   (*c*) A is symmetric (*d*) $A^{-1} = -\frac{1}{2}(A + I)$
   (Where I is unit matrix of order 2 and O is null matrix of order 2)

8. Let A, B, C be three square matrices such that $ABC + AB + BC + AC + A + B + C = O$. If A and BC commute, then
   (*a*) $A(B + C)$ is singular
   (*b*) $A + C$ and $B + C$ commute
   (*c*) $AB + BC + CA$ is null matrix
   (*d*) A and $B + C$ commute

9. Let I_n is a $(n \times n)$ identity matrix and O_n is a $(n \times n)$ null matrix consider, $A = \begin{bmatrix} B & O_2 & O_2 \\ O_2 & B & O_2 \\ O_2 & O_2 & B \end{bmatrix}$, $B = \begin{bmatrix} I_1 & O_1 \\ O_1 & I_1 \end{bmatrix}$ then which is correct
   (*a*) det $(A) = B$
   (*b*) det (det $(A)) = M$ then, $M \cdot M^T \neq M^T \cdot M$
   (*c*) det (B) = det (I_1)
   (*d*) None of these

10. If $A = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}$, then $|A^2 + 3A + 2I|$ + tr$(B + C)$ where B and C are matrices of order 2 with integer elements and $A = B^3 + C^3$, is
    (*a*) –3 (*b*) 3
    (*c*) 0 (*d*) 2

11. If a square matrix has entries either 1 or –1, it will be called special matrix if product of elements of any row or column is –1 so number of such special matrices will be
    (*a*) 2^{n-1} (*b*) $2^{(n-1)^2}$
    (*c*) 2^{n-2} (*d*) $2^{(n-2)^2}$

12. Let A, B, C, D be (not necessarily square) real matrices such that $A^T = BCD$; $B^T = CDA$; $C^T = DAB$ and $D^T = ABC$ for the matrix $S = ABCD$, then which of the following is/are true
    (*a*) $S^3 = S$
    (*b*) $S^2 = S^4$
    (*c*) $S = S^2$
    (*d*) None of these

13. If $\begin{vmatrix} 1 + a_1 + b_1 & a_1 + b_2 & a_1 + b_3 \\ a_2 + b_1 & 1 + a_2 + b_2 & a_2 + b_3 \\ a_3 + b_1 & a_3 + b_2 & 1 + a_3 + b_3 \end{vmatrix}$
    $= k + \sum_{i=1}^{3}(a_i + b_i) + \sum_{1 \le i < j \le 3}\sum (a_i - a_j)(b_j - b_i)$; then find the value of k
    (*a*) 0 (*b*) 1
    (*c*) 2 (*d*) 8

14. If $n \in N$ and $\Delta_n = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$ then
    $\lim_{n \to \infty} \frac{(3n^3 - 5)\Delta_n}{\Delta_{n+1}}$ equals
    (*a*) $\frac{3}{2}$ (*b*) $\frac{5}{2}$
    (*c*) $-\frac{5}{2}$ (*d*) 3

PYQ's (Past Year Questions)

DEFINITION, TYPES OF MATRICES, TRACE OF A MATRIX, ALGEBRA OF MATRIX

1. Consider the matrix $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Given below are two statements:

Statement-I: $f(-x)$ is the inverse of the matrix $f(x)$.

Statement-II: $f(x)f(y) = f(x + y)$.

In the light of the above statements, choose the correct answer from the options given below: **[27 Jan, 2024 (Shift-I)]**

(*a*) Statement-I is false but Statement-II is true

(*b*) Both Statement-I and Statement-II are false

(*c*) Statement-I is true but Statement-II is false

(*d*) Both Statement-I and Statement-II are true

2. Let A be a 2 × 2 real matrix and I be the identity matrix of order 2. If the roots of the equation $|A - xI| = 0$ be –1 and 3, then the sum of the diagonal elements of the matrix A^2 is ... **[27 Jan, 2024 (Shift-II)]**

3. Let $R = \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$ be a non-zero 3 × 3 matrix, where $x\sin\theta = y\sin\left(\theta + \frac{2\pi}{3}\right) = z\sin\left(\theta + \frac{4\pi}{3}\right) \neq 0$, $\theta \in (0, 2\pi)$. For a square matrix M, let trace (M) denote the sum of all the diagonal entries of M. Then, among the statements:

I. Trace (R) = 0

II. If trace (adj(adj(R))) = 0, then R has exactly one non-zero entry. **[30 Jan, 2024 (Shift-II)]**

(*a*) Both (I) and (II) are true (*b*) Neither (I) nor (II) is true

(*c*) Only (II) is true (*d*) Only (I) is true

4. Let $A = \begin{bmatrix} 2 & a & 0 \\ 1 & 3 & 1 \\ 0 & 5 & b \end{bmatrix}$. If $A^3 = 4A^2 - A - 21I$, where I is the identity matrix of order 3 × 3, then $2a + 3b$ is equal to: **[08 April, 2024 (Shift-I)]**

(*a*) –10 (*b*) –13 (*c*) –9 (*d*) –12

5. Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$. If the sum of the diagonal elements of A^{13} is 3^n, then n is equal to _____ . **[08 April, 2024 (Shift-I)]**

6. Consider the matrices: $A = \begin{bmatrix} 2 & -5 \\ 3 & m \end{bmatrix}$, $B = \begin{bmatrix} 20 \\ m \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \end{bmatrix}$. Let the set of all m, for which the system of equations $AX = B$ has a negative solution (i.e., $x < 0$ and $y < 0$), be the interval (a, b).

Then $8\int_a^b |A|\,dm$ is equal to _____. **[09 April, 2024 (Shift-II)]**

7. If $A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$, then: **[1 Feb, 2023 (Shift-II)]**

(*a*) $A^{30} - A^{25} = 2I$

(*b*) $A^{30} + A^{25} + A = I$

(*c*) $A^{30} + A^{25} - A = I$

(*d*) $A^{30} = A^{25}$

8. The set of all values of $t \in$ R, for which the matrix

$$\begin{bmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^t & e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{bmatrix}$$

Invertible, is **[29 Jan, 2023 (Shift-II)]**

(*a*) $\left\{(2k+1)\frac{\pi}{2}, k \in Z\right\}$ (*b*) $\left\{k\pi + \frac{\pi}{4}, k \in Z\right\}$

(*c*) $\{k\pi, k \in Z\}$ (*d*) R

9. Let $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and Q = PAP^T. If $P^T Q^{2007} P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $2a + b - 3c - 4d$ equal to **[8 April, 2023 (Shift-I)]**

(*a*) 2004 (*b*) 2005

(*c*) 2007 (*d*) 2006

10. Let A be a matrix of order 2 × 2, whose entries are from the set {0, 1, 2, 3, 4, 5}. If the sum of all the entries of A is a prime number p, $2 < p < 8$, then the number of such matrices A is **[27 June, 2022 (Shift-II)]**

11. Let $A = \begin{bmatrix} 1 & a & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}, a, b \in \mathbb{R}$. If for some $n \in$ N,

$A^n = \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 96 \\ 0 & 0 & 1 \end{bmatrix}$ then $n + a + b$ is equal to _____ **[25 July, 2022 (Shift-II)]**

12. Let $A = \begin{pmatrix} 1+i & 1 \\ -i & 0 \end{pmatrix}$ where $i = \sqrt{-1}$. Then, the number of elements in the set $\{n \in \{1, 2, ..., 100\} : A^n = A\}$ is ______ **[28 June, 2022 (Shift-II)]**

84. Let S denote the set of all real values of λ such that the system of equations **[1 Feb, 2023 (Shift-I)]**

$\lambda x + y + z = 1$

$x + \lambda y + z = 1$

$x + y + \lambda z = 1$

is inconsistent, then $\sum_{\lambda \in S}\left(|\lambda|^2 + |\lambda|\right)$ is equal to

(*a*) 2 (*b*) 12 (*c*) 4 (*d*) 6

85. For the system of linear equations **[10 April, 2023 (Shift-I)]**

$2x - y + 3z = 5$

$3x + 2y - z = 7$

$4x + 5y + \alpha z = \beta$

Which of the following is NOT correct ?

(*a*) The system has infinitely many solutions for $\alpha = -5$ and $\beta = 9$

(*b*) The system has a unique solution for $\alpha \neq -5$ and $\beta = 8$

(*c*) The system has infinitely many solutions for $\alpha = -6$ and $\beta = 9$

(*d*) The system is inconsistent for $\alpha = -5$ and $\beta = 8$

86. The number of real values λ, such that the system of linear equations

$2x - 3y + 5z = 9$

$x + 3y - z = -18$

$3x - y + (\lambda^2 - |\lambda|)z = 16$ has no solution, is

[25 July, 2022 (Shift-II)]

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 4

87. Let the system of linear equations $x + 2y + z = 2$, $\alpha x + 3y - z = \alpha$, $-\alpha x + y + 2z = -\alpha$ be inconsistent. Then α is equal to: **[27 June, 2022 (Shift-I)]**

(*a*) $\frac{5}{2}$ (*b*) $-\frac{5}{2}$ (*c*) $\frac{7}{2}$ (*d*) $-\frac{7}{2}$

88. If the system of linear equations

$2x + 2ay + az = 0$

$2x + 3by + bz = 0$

$2x + 4cy + cz = 0$

where $a, b, c \in R$ are non-zero and distinct; has a non-zero solution, then **[7 Jan, 2020 (Shift-I)]**

(*a*) a, b, c are in A.P.

(*b*) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

(*c*) $a + b + c = 0$

(*d*) a, b, c are in G.P.

PW Challengers

SINGLE CORRECT TYPE QUESTIONS

1. A matrix 'M' describes the reflection of a point (x, y) about the line $y = (2-\sqrt{3})x$ The matrix 'M' also satisfies the relation AM – MA = I, where A is a non-singular matrix. If A^{-1} is a symmetrix matrix with entries x and y then

(*a*) $x = 2, y = 0$ (*b*) $x = 2, y = 2$

(*c*) $x = 1, y = 1$ (*d*) $x = 2, y = 2$

2. An $n \times n$ matrix is formed using only 0, 1 and –1 as its elements. Number of such matrices which are skew symmetric is

(*a*) $\frac{n(n-1)}{2}$ (*b*) $(n-1)^2$

(*c*) $2^{\frac{n(n-1)}{2}}$ (*d*) $3^{\frac{n(n-1)}{2}}$

3. Number of skew-symmetric matrices of order 3 whose elements are 0, 0, 0, 1, –1, 2, –2, 3, –3 is

(*a*) 8 (*b*) 12 (*c*) 12 (*d*) 48

4. If a square matrix has entries either 1 or –1, it will be called special matrix if product of elements of any row or column is –1 so number of such special matrices will be

(*a*) 2^{n-1} (*b*) $2^{(n-1)^2}$

(*c*) 2^{n-2} (*d*) $2^{(n-2)^2}$

5. The number of 4th order matrices with elements {–3, –2, –1, 0, 1, 2, 3} such that matrix is either symmetric or skew symmetric

(*a*) 7^{16} (*b*) $7^{10} + 7^6 + 1$

(*c*) $7^{10} + 7^6 - 1$ (*d*) None of these

6. If $A = \lim_{n \to \infty} \begin{bmatrix} 1 & \frac{x}{n} \\ -\frac{x}{n} & 1 \end{bmatrix}^n$ then $|A|$ is

(*a*) 0 (*b*) 1 (*c*) 6 (*d*) 2

7. If $\begin{vmatrix} (x^2+1)^2 & (xy+1)^2 & (xz+1)^2 \\ (xy+1)^2 & (y^2+1)^2 & (yz+1)^2 \\ (xz+1)^2 & (yz+1)^2 & (z^2+1) \end{vmatrix} = k(x-y)^2(x-z)^2(y-z)^2$.

Find the value of k.

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 5

8. Find sum of all numbers in the interval [–2025, 2025] that can be equal to the determinant of an 11×11 matrix with entries equal to 1 or –1.

(*a*) 2048 (*b*) 1024

(*c*) 4096 (*d*) 0

9. For any integers $x_1, x_2, \ldots, x_n$ and positive integers $k_1, k_2, \ldots, k_n$, the determinant $\begin{vmatrix} x_1^{k_1} & x_2^{k_1} & \ldots & x_n^{k_1} \\ x_1^{k_2} & x_2^{k_2} & \ldots & x_n^{k_2} \\ \vdots & \vdots & \vdots & \vdots \\ x_1^{k_n} & x_2^{k_n} & \ldots & x_n^{k_n} \end{vmatrix}$ is

(*a*) divisible by $n!$
(*b*) divisible by $(n + 1)!$
(*c*) 0
(*d*) divisible by $(n + 2)!$

10. Let A be the $n \times n$ matrix whose entry in the i^{th} row and the j^{th} column is $\dfrac{1}{\min(i, j)}$ for $1 \le i, j \le n$. Compute det(A).

(*a*) $\dfrac{(-1)^{n-1}}{(n-1)!n!}$ (*b*) $\dfrac{(-1)^{n-1}}{(n+1)!n!}$
(*c*) 0 (*d*) None of these

MULTIPLE CORRECT TYPE QUESTIONS

11. Let M be a 3 × 3 non-singular matrix with det. (M) = 4. If M^{-1} *adj*(*adj* M) = k^2I, then the value of 'k' may be:

(*a*) +2 (*b*) 4
(*c*) –2 (*d*) –4

12. Let $A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$ If trace (A) = 9 and a, b, c are positive integers such that $ab + bc + ca = 26$.

Let A_1 denotes the adjoint of A; A_2 denote adjoint of A_1 ...... and so on.

If value of *det* (A_4) is M then

(*a*) $M = 3^{48}$
(*b*) $M = 3^{24}$
(*c*) Last two digits of M is 61
(*d*) *Det* (A) = –27

13. If p, q, r, s are in A.P. and $f(x)$

$$= \begin{vmatrix} p+\sin x & q+\sin x & p-r+\sin x \\ q+\sin x & r+\sin x & -1+\sin x \\ r+\sin x & s+\sin x & s-q+\sin x \end{vmatrix}$$

such that $\int_0^2 f(x)dx = -4$ then the common difference of the A.P. can be:

(*a*) –1 (*b*) 1/2 (*c*) –1/2 (*d*) 1

14. If $\begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2 \\ xz-y^2 & xy-z^2 & yz-x^2 \\ xy-z^2 & yz-x^2 & zx-y^2 \end{vmatrix} = \begin{vmatrix} r^2 & u^2 & u^2 \\ u^2 & r^2 & u^2 \\ u^2 & u^2 & r^2 \end{vmatrix}$, then

(*a*) $r^2 = x + y + z$ (*b*) $r^2 = x^2 + y^2 + z^2$
(*c*) $u^2 = yz + zx + xy$ (*d*) $u^2 = xyz$

15. Let $A = \begin{bmatrix} a & b & c \\ p & q & r \\ 1 & 1 & 1 \end{bmatrix}$ and $B = A^2$.

If $(a-b)^2 + (p-q)^2 = 25$, $(b-c)^2 + (q-r)^2 = 36$ and $(c-a)^2 + (r-p)^2 = 49$, then det $\left(\dfrac{B}{2}\right)$ is divisible by

(*a*) 12 (*b*) 11
(*c*) 15 (*d*) 6

16. If $T_n = \begin{vmatrix} n-1 & n-2 & n-6 \\ 2n-4 & 2n-6 & 2n-11 \\ 3n-9 & 3n-12 & 3n-18 \end{vmatrix}$ where $n \in$ N, then which of the following is/are true?

(*a*) $\prod_{n=1}^{m} T_n = 6^m$ (*b*) $\sum_{n=1}^{10} T_n = 60$

(*c*) $\dfrac{T_{n+1}}{T_n} = 2$ (*d*) $T_{100} \cdot T_{101} = 48$

17. If $(x_1 - x_2)^2 + (y_1 - y_2)^2 = a^2$;
$(x_2 - x_3)^2 + (y_2 - y_3)^2 = b^2$;
$(x_3 - x_1)^2 + (y_3 - y_1)^2 = c^2$; and

$$4\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = \lambda(\lambda^3 - (\lambda_1 + \lambda_2 + \lambda_3)\lambda^2 + \lambda(\lambda_1\lambda_2 + \lambda_2\lambda_3$$

$+ .. - \lambda_1\lambda_2\lambda_3))$, then

(*a*) $\lambda > \dfrac{3}{2}(\lambda_1\lambda_2\lambda_3)^{\frac{1}{3}}$
(*b*) $\lambda_1\lambda_2\lambda_3 = 8abc$
(*c*) $\sum \lambda_1\lambda_2 = 4\sum ab$
(*d*) $2\lambda = \lambda_1 + \lambda_2 + \lambda_3$

INTEGER TYPE QUESTIONS

18. Let A_n and B_n be square matrices of order 3, which are defined as

$A_n = [a_{ij}]$ and $B_n = [b_{ij}]$ where $a_{ij} = \dfrac{2i+j}{3^{2n}}$ and $b_{ij} = \dfrac{3i-j}{2^{2n}}$ for all i and j, $1 \le i, j \le 3$.

if $l = \lim\limits_{n\to\infty} \text{Tr}\cdot(3A_1 + 3^2 A_2 + 3^3 A_3 + \ldots\ldots\ldots\ldots + 3^n A_n)$ and $m = \lim\limits_{n\to\infty} \text{Tr}.(2B_1 + 2^2 B_2 + 2^3 B_3 + \ldots.. + 2^n B_n)$ then the value of $(l + m)$ is equal to.

[Note: Tr. (P) denotes the trace of matrix P.]

19. Let A be a non singular square matrix of order 2, such that |A + |A| Adj (A)| = 0, then the value of |A – |A| Adj (A)| is equal to

(where adj (A) is the adjoint of matrix A)

38. The minimum value of a 3 × 3 minus special determinant is

(*a*) –6 (*b*) –4

(*c*) –2 (*d*) 0

PASSAGE-III

Paragraph (39 to 40):

If $g(x) = (c_1 - x)(c_2 - x)(c_3 - x)$

$$f(x) = \begin{vmatrix} x+c_1 & x+a & x+a \\ x+b & x+c_2 & x+a \\ x+b & x+b & x+c_3 \end{vmatrix}$$

39. Coefficient of x in $f(x)$ is

(*a*) $\dfrac{g(a)-f(b)}{b-a}$ (*b*) $\dfrac{g(-a)-g(-b)}{b-a}$

(*c*) $\dfrac{g(a)-g(b)}{b-a}$ (*d*) None of these

40. Which of the following is not a constant term in $f(x)$

(*a*) $\dfrac{bg(a)-ag(b)}{b-a}$ (*b*) $\dfrac{bg(a)-af(-b)}{b-a}$

(*c*) $\dfrac{bf(-a)-ag(b)}{b-a}$ (*d*) None of these

ANSWER KEY

CONCEPT APPLICATION

1. [2 × 3] **2.** [6] **3.** (*c*) **4.** (*a*) **5.** (*d*) **6.** (*a*) **7.** (*c*) **8.** [14] **9.** (*b*) **10.** (*b*)

11. (*c*) **12.** (*c*) **13.** (*b*) **14.** (*d*) **15.** (*d*) **16.** (*b*) **17.** (*b*) **18.** (*c*) **19.** (*d*) **20.** (*b*)

21. (*d*) **22.** (*c*) **23.** (*b*) **24.** (*d*) **25.** (*b*) **26.** (*b*) **27.** (*d*) **28.** (*a*) $m = 1$; (*b*) Column Matrix; (*c*) $a_{ij} = 0\ \forall\ i, j$; (*d*) $m = n = 1$; (*e*) $n > m$; (*f*) Vertical Matrix; (*g*) $m = n$; (*h*) $a_{ij} = 0$, when $i \neq j$; (*i*) Scalar Matrix;

(*j*) $a_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$; (k) Upper Triangular Matrices; (*l*) $a_{ij} = 0$ when $i < j$; (*m*) $|A| = 0$; (*n*) $|A| \neq 0$; (*o*) $a_{ij} = a_{ji}$;

(*p*) Skew-Symmetric Matrices; (q) $A = A^q$; (*r*) Skew-Hermitian Matrix; (*s*) $AA^T = I_n = A^TA$; (*t*) $A^2 = A$; (*u*) $A^2 = I$, $A^{-1} = A$;

(*v*) there exists $p \in N$ such that $A^p = O$, $A^{p-1} \neq 0$; (*w*) $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{r \times s}$ where, $a_{ij} = b_{ij}$, $m = r$, and $n = s$

29. $A^{-1} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & -\frac{1}{4} & 0 \\ -\frac{1}{6} & 0 & -\frac{1}{6} \end{bmatrix}$, $x = 1, y = 2, z = 2.$ **30.** $A^{-1} = \frac{1}{17}\begin{bmatrix} -2 & \frac{1}{2} & \frac{5}{2} \\ 7 & -6 & 4 \\ -2 & 9 & -6 \end{bmatrix}$ **31.** (*c*) **32.** (*d*) **33.** [–10]

34. (*a*) **35.** (*c*) **36.** (*b*) **37.** (*c*) **38.** (*b*) **39.** (*a*) **40.** (*b*) **41.** (*c*) **42.** (*a*) **43.** (*d*)

44. (*b*) **45.** (*b*) **46.** (*a*) **47.** (*b*) **50.** (*a*) **51.** (*b*) **52.** (*c*) **53.** (*c*) **54.** (*b*) **55.** (*d*)

56. (*a,b,c*) **57.** (*a*) **58.** (*a*) **59.** (*d*) **60.** (*c*) **61.** (*a, b*) **62.** (*c*) **63.** (*c*) **64.** [0] **65.** [1]

68. (*c*) **69.** (*b*) **70.** (*d*) **72.** (*a*) **73.** (*b*) **74.** (*c*) **75.** (*b*) **76.** (*b*) **77.** (*b*) **78.** {0,8}

79. (*c*)

BOARD LEVEL PROBLEMS

1. (*d*) **2.** (*d*) **3.** (*c*) **4.** (*d*) **5.** (*d*) **6.** (*d*) **7.** (*d*) **8.** (*d*) **9.** (*a*) **22.** (*c*)

23. (*d*) **24.** (*c*) **25.** (*c*)

PRARAMBH (TOPICWISE)

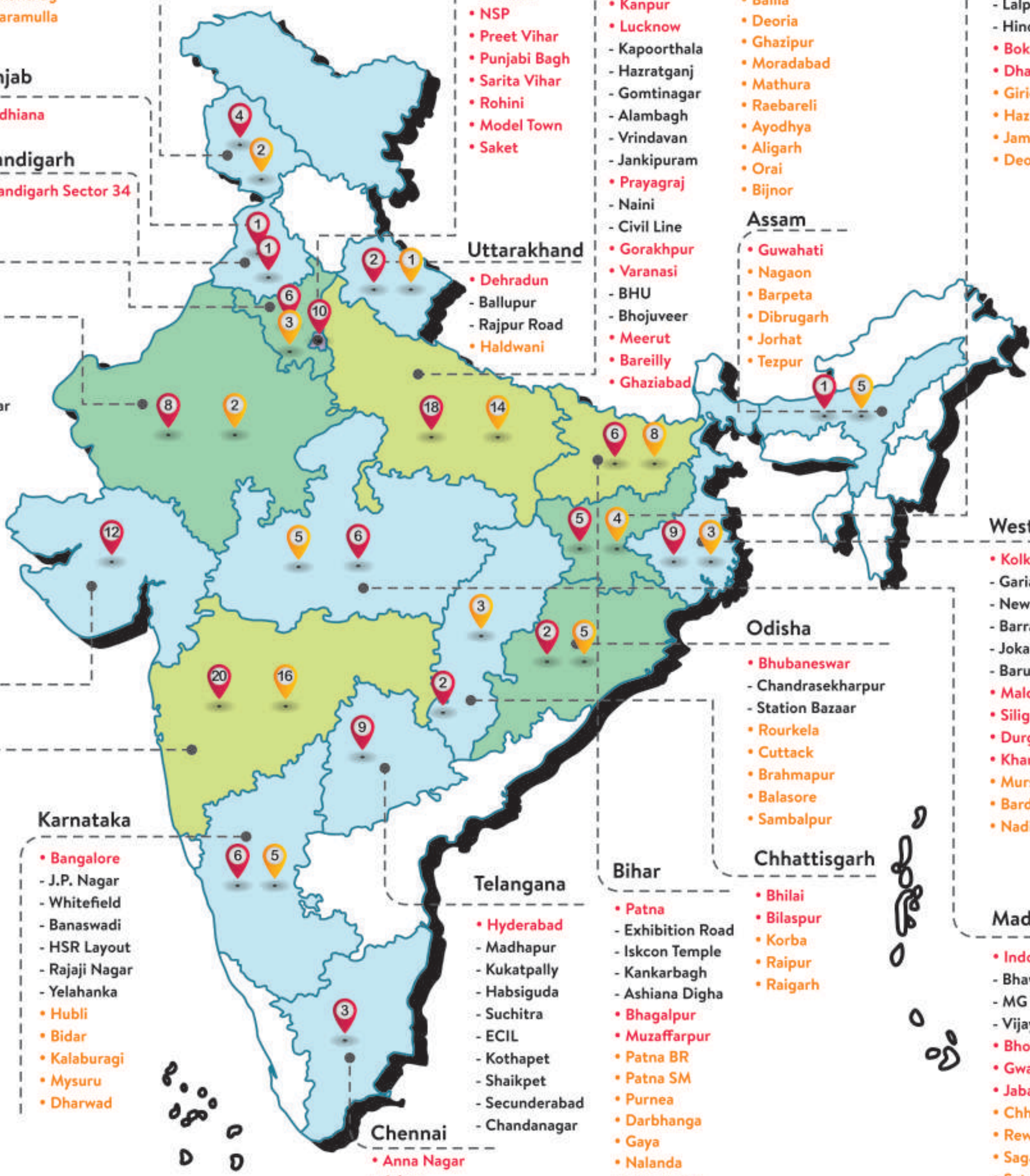
1. (*c*) **2.** (*c*) **3.** (*a*) **4.** (*b*) **5.** (*b*) **6.** (*a*) **7.** (*d*) **8.** (*d*) **9.** (*c*) **10.** (*b*)

11. (*c*) **12.** (*d*) **13.** (*b*) **14.** (*d*) **15.** (*b*) **16.** (*a*) **17.** (*c*) **18.** (*b*) **19.** (*a*) **20.** (*a*)

21. (*c*) **22.** (*a*) **23.** (*a*) **24.** (*a*) **25.** (*c*) **26.** (*d*) **27.** (*c*) **28.** (*a*) **29.** (*d*) **30.** (*d*)

VIDYAPEETH Is Now Present In 140+ Cities Across India

IIT-JEE | NEET | FOUNDATION



Jammu & Kashmir
- Srinagar
  - Rajbagh
  - Barzulla
- Jammu
  - Paloura
  - Channi Rama
- Anantnag
- Baramulla

Haryana
- Faridabad
- Panchkula
- Hisar
- Rohtak
- Gurugram
  - South City
  - Sector 16
- Narnaul
- Sonipat
- Sirsa

Punjab
- Ludhiana

Chandigarh
- Chandigarh Sector 34

Delhi
- Janakpuri
- Kalusarai
- Dwarka
- NSP
- Preet Vihar
- Punjabi Bagh
- Sarita Vihar
- Rohini
- Model Town
- Saket

Uttar Pradesh
- Noida
  - Block C
  - Gaur City
- Greater Noida
- Kanpur
- Lucknow
  - Kapoorthala
  - Hazratganj
  - Gomtinagar
  - Alambagh
  - Vrindavan
  - Jankipuram
- Prayagraj
  - Naini
  - Civil Line
- Gorakhpur
- Varanasi
  - BHU
  - Bhojuveer
- Meerut
- Bareilly
- Ghaziabad
- Agra
- Indrapuram
- Jhansi
- Jaunpur
- Ballia
- Deoria
- Ghazipur
- Moradabad
- Mathura
- Raebareli
- Ayodhya
- Aligarh
- Orai
- Bijnor

Jharkhand
- Ranchi
  - Harmu
  - Lalpur
  - Hinoo
- Bokaro
- Dhanbad
- Giridih
- Hazaribagh
- Jamshedpur
- Deoghar

Uttarakhand
- Dehradun
  - Ballupur
  - Rajpur Road
- Haldwani

Assam
- Guwahati
- Nagaon
- Barpeta
- Dibrugarh
- Jorhat
- Tezpur

Gujarat
- Surat
  - Adajan
  - Vesu
- Bothra
  - Dahod
  - Satellite
- Bharuch
- Gandhinagar
- Ahmedabad
  - Satellite
  - Maninagar
  - Nikol
  - Gota
- Vadodara
  - Manjalpur
  - Vadiwadi

Rajasthan
- Jaipur
  - Tonk Road
  - RS Circle
  - Pratap Nagar
- Jodhpur
  - Akhaliya
  - Sardarpura
- Kota
- Udaipur
- Sikar
- Alwar
- Nagaur

West Bengal
- Kolkata
  - Gariahat
  - New Town
  - Barrackpore
  - Joka
  - Baruipur
- Malda
- Siliguri
- Durgapur
- Kharagpur
- Murshidabad
- Bardhaman
- Nadia

Odisha
- Bhubaneswar
  - Chandrasekharpur
  - Station Bazaar
- Rourkela
- Cuttack
- Brahmapur
- Balasore
- Sambalpur

Maharashtra
- Pune
  - FC Road
  - Hadapsar
  - Pimpri
  - Kothrud
  - Baner
  - Viman Nagar
- Chhatrapati Sambhajinagar
- Nanded
- Nagpur
- Nashik
- Mumbai
  - Thane
  - Kalyan
  - Andheri
  - Kalyan Palava City
  - Borivali
  - Panvel
  - Juinagar
  - Virar
- Latur
- Akola
- Ahmednagar
- Buldhana
- Amravati
- Baramati
- Beed
- Chandrapur
- Dhule
- Gondia
- Hingoli
- Jalgaon
- Jalna
- Kolhapur
- Satara
- Sangli
- Solapur
- Yavatmal

Karnataka
- Bangalore
  - J.P. Nagar
  - Whitefield
  - Banaswadi
  - HSR Layout
  - Rajaji Nagar
  - Yelahanka
- Hubli
- Bidar
- Kalaburagi
- Mysuru
- Dharwad

Telangana
- Hyderabad
  - Madhapur
  - Kukatpally
  - Habsiguda
  - Suchitra
  - ECIL
  - Kothapet
  - Shaikpet
  - Secunderabad
  - Chandanagar

Chennai
- Anna Nagar
- Adyar
- Medvakkam

Bihar
- Patna
  - Exhibition Road
  - Iskcon Temple
  - Kankarbagh
  - Ashiana Digha
- Bhagalpur
- Muzaffarpur
- Patna BR
- Patna SM
- Purnea
- Darbhanga
- Gaya
- Nalanda
- Samastipur
- Motihari

Chhattisgarh
- Bhilai
- Bilaspur
- Korba
- Raipur
- Raigarh

Madhya Pradesh
- Indore
  - Bhawarkuan
  - MG Road
  - Vijay Nagar
- Bhopal
- Gwalior
- Jabalpur
- Chhatarpur
- Rewa
- Sagar
- Satna
- Chhindwara

VIDYAPEETH | VP PATHSHALA

PHYSICS WALLAH PUBLICATION

Visit Your Vidyapeeth — SCAN ME!

To share Feedback — SCAN ME!

ISBN 978-93-48141-99-6

SKU Code - 51b3e58c-15f1-4f75-8327-683f281f9015